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Robust stability of uncertain impulsive dynamical systems ☆

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Abstract

This paper studies robust stability of uncertain impulsive dynamical systems. By introducing the concepts of uniformly positive definite matrix functions and Hamilton–Jacobi/Riccati inequalities, several criteria on robust stability, robust asymptotic stability and robust exponential stability are established. An example is also worked through to illustrate our results.

Keywords: Uncertain impulsive systems; Uncertainty; Robust stability; Hamilton–Jacobi/Riccati inequality; Interval matrix

1. Introduction

Impulsive dynamical systems have been widely studied in recent years; see [1–5] and references cited therein. Such systems arise in many applied fields such as control technology, communication networks, and biological population management. Since impulsive dynamical systems provide a natural framework for mathematical modelling of many physical phenomena, their study is assuming a greater importance. For the basic concept and theorems of impulsive dynamical systems, we refer the reader to [1,2]. On the other hand, uncertainties happen frequently in various engineering, biological, and economical sys-

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tems due to modelling errors, measurement inaccuracy, linear approximation, and so on. It is well known that uncertainties often result in instability. Therefore, robustness analysis of uncertain systems is very important. Several interesting results have been established in [6–8] for continuous dynamical systems. But so far very few robust stability results for uncertain impulsive dynamical systems have been reported.

In this paper, we shall investigate the robust stability properties of uncertain impulsive dynamical systems. By utilizing the ideas developed in [4], we shall establish several criteria on robust stability, robust asymptotic stability and robust exponential stability. The organization of this paper is as follows. In Section 2, we introduce the concept of uniformly positive definite matrix function and some other notations. We state and prove our main results in Section 3, where both linear and nonlinear uncertain impulsive dynamical systems are considered. By using Riccati and Hamilton–Jacobi inequalities, we establish several robust stability criteria. Finally, we work through an example to illustrate the applicability of our results.

2. Preliminaries

Let $\mathbb{R}^n$ denote the n-dimensional Euclidean space. Let $\mathbb{R}_+ = [0, +\infty)$, $\mathbb{N} = \{1, 2, \ldots\}$. Denote by $\varkappa$ the class of functions $\phi : \mathbb{R}_+ \to \mathbb{R}_+$, which are continuous, strictly increasing and $\phi(0) = 0$, $\varkappa_0$ the class of continuous functions $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\psi(s) = 0$ if and only if $s = 0$, and PC the class of functions $\lambda : \mathbb{R}_+ \to \mathbb{R}_+$, where λ is continuous everywhere except t_k $(k \in \mathbb{N})$ at which λ is left continuous and the right limit $\lambda(t_k^+)$ exists. In this paper, we let $S_\rho = \{x \in \mathbb{R}^n \colon \|x\| \leqslant \rho\}$.

Consider the uncertain impulsive dynamical systems of the form

$$\begin{aligned} \dot{x} &= f(t,x) + g(t,x), \quad t \neq t_k, \\ \Delta x &= I_k(x) + J_k(x), \quad t = t_k,\ k \in \mathbb{N}, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $f, g : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$, $I_k, J_k : \mathbb{R}^n \to \mathbb{R}^n$, and $\Delta x = x(t_k^+) - x(t_k^-)$, where $x(t_k^+)$ is the right limit of $x(t)$ at $t = t_k$, and $x(t_k^-)$ is the left limit. The functions $g(t,x)$, $J_k(x)$ represent structural uncertainty or uncertain perturbation characterized by

$$g \in \Omega_g = \bigl\{g \colon\ g(t,x) = e_g(t,x) \cdot \delta_g(t,x),\ \bigl\|\delta_g(t,x)\bigr\| \leqslant \bigl\|m_g(t,x)\bigr\|\bigr\}$$

and

$$J_k \in \Omega_J = \bigl\{J_k \colon\ J_k(x) = e_k(x) \cdot \delta_k(x),\ \bigl\|\delta_k(x)\bigr\| \leqslant \bigl\|m_k(x)\bigr\|\bigr\}, \quad k \in \mathbb{N},$$

where $e_g : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}^{n\times m}$ and $e_k : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are known matrix functions whose entries are smooth functions of the state, and δ_g, δ_k are unknown vector-valued functions whose norm are bounded, respectively, by the norm of the vector-valued functions $m_g(t,x), m_k(x)$, respectively. Here, $m_g : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^m$, $m_k : \mathbb{R}^n \to \mathbb{R}^m$ $(k \in \mathbb{N})$ are given smooth functions, and $\|\cdot\|$ stands for the Euclidean norm in $\mathbb{R}^n$.

Let $t_0 \in \mathbb{R}_+$ and $x_0 \in \mathbb{R}^n$. Denote by $x(t, t_0, x_0)$ the solution of (1) satisfying the initial condition $x(t_0^+) = x_0$. We assume, for simplicity, that the functions $f(t,x)$, $g(t,x)$, $I_k(x)$ and $J_k(x)$, $k \in \mathbb{N}$ satisfy all the required conditions [1] so that all solutions $x(t) = x(t, t_0, x_0)$ of (1) exist for all $t \geqslant t_0$.

We also assume $f(t,0)=0$, $\delta_g(t,0)=0$, $I_k(0)=0$, $\delta_k(0)=0$ for all $t\in\mathbb{R}_+$, $k\in\mathbb{N}$. Hence, $x=0$ is a solution of system (1).

Definition 2.1. Let $X:\mathbb{R}_+\to\mathbb{R}^{n\times n}$ be an $n\times n$ matrix function. Then $X(t)$ is said to be

(i) a positive definite matrix function if for any $t\in\mathbb{R}_+$, $X(t)$ is a positive definite matrix;
(ii) a positive definite matrix function bounded above if it is a positive definite matrix function and there exists a positive real number $M>0$ such that

$$\lambda_{\max}\big(X(t)\big)\leqslant M,\quad t\in\mathbb{R}_+,\tag{2}$$

where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue;
(iii) a uniformly positive definite matrix function if it is a positive definite matrix function and there exists a positive real number $m>0$ such that

$$\lambda_{\min}\big(X(t)\big)\geqslant m,\quad t\in\mathbb{R}_+,\tag{3}$$

where $\lambda_{\min}(\cdot)$ is the minimum eigenvalue of matrix $(\cdot)$.

Definition 2.2. Let $V:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}_+$; then V is said to belong to class ν_0 if

(i) V is continuous in $(t_{k-1},t_k]\times\mathbb{R}^n$ and for each $x\in\mathbb{R}^n$, $t\in(t_{k-1},t_k]$, $k\in\mathbb{N}$,

$$\lim_{\substack{(t,y)\to(t_{k-1}^+,x)\\ t>t_{k-1}}} V(t,y)=V\big(t_{k-1}^+,x\big)\tag{4}$$

exists;
(ii) V is locally Lipschitzian in x.

Definition 2.3. For $(t,x)\in(t_{k-1},t_k]\times\mathbb{R}^n$, we define

$$D^+V(t,x)=\lim_{h\to0^+}\sup\frac{1}{h}\Big[V\big(t+h,x+h\big(f(t,x)+g(t,x)\big)\big)-V(t,x)\Big].\tag{5}$$

Definition 2.4. The uncertain impulsive dynamical system (1) is called robustly stable, robustly asymptotically stable, and robustly exponentially stable, respectively, if for any $g\in\Omega_g$, $J_k\in\Omega_J$ $(k\in\mathbb{N})$, the trivial solution $x=0$ of system (1) is stable, asymptotically stable, and exponentially stable, respectively.

3. Robust stability criteria

The present section consists of three parts. In Part A, we summarize the existing stability results given in [4] for the nominal system of system (1). In Part B, we establish some robust stability criteria for linear uncertain impulsive dynamical systems. The corresponding results for nonlinear uncertain impulsive dynamical systems are given in Part C.

Part A. Stability results of nominal impulsive systems

The nominal impulsive system of system (1) is given by

$$\begin{aligned} &\dot{x} = f(t,x), \quad t \neq t_k, \\ &\Delta x = I_k(x), \quad t = t_k,\ k \in \mathbb{N}, \\ &x(t_0^+) = x_0. \end{aligned} \tag{6}$$

For system (6), we summarize the following general results.

Proposition 3.1 [4]. *Assume that*

(i) *there exists* ρ_0 *with* $0 < \rho_0 \leqslant \rho$ *such that* $x \in S_{\rho_0}$ *implies that* $x + I_k(x) \in S_\rho$ *for all* $k \in \mathbb{N}$;

(ii) $V \in \nu_0$, *and there exist* $a, b \in \varkappa$, *such that*

$$b(\|x\|) \leqslant V(t,x) \leqslant a(\|x\|), \tag{7}$$

where $(t,x) \in \mathbb{R}_+ \times S_\rho$;

(iii)
$$V(t_k^+, x_k + I_k(x_k)) \leqslant \psi_k(V(t_k, x_k)), \tag{8}$$

where $\psi_k \in \varkappa_0$, $k \in \mathbb{N}$;

(iv) *there exist* $c \in \varkappa$, $p \in PC$ *such that*

$$D^+V(t,x) \leqslant p(t) \cdot c(V(t,x)), \tag{9}$$

where $(t,x) \in (t_k, t_{k+1}] \times S_\rho$, $k \in \mathbb{N}$;

(v) *there exists a constant* $\sigma > 0$ *such that for all* $z \in (0,\sigma)$,

$$\int_{t_k}^{t_{k+1}} p(s)\,ds + \int_z^{\psi_k(z)} \frac{ds}{c(s)} \leqslant -r_k \tag{10}$$

for some constants $r_k \in \mathbb{R}$ *and* $k \in \mathbb{N}$.

Then the system (6) *is stable if* $r_k \geqslant 0$ *for all* $k \in \mathbb{N}$, *and asymptotically stable if in addition* $\sum_{k=1}^{\infty} r_k = +\infty$.

Proposition 3.2 [4]. *Assume that conditions* (i)–(iii) *of Proposition* 3.1 *hold. Suppose further that*

(iv*) *there exist* $c \in \varkappa$, $\lambda \in PC$ *such that*

$$D^+V(t,x) \leqslant -\lambda(t) \cdot c(V(t,x)), \tag{11}$$

where $(t,x) \in (t_k, t_{k+1}] \times S_\rho$, $k \in \mathbb{N}$;

(v*) *there exists a constant* $\sigma > 0$ *such that for all* $z \in (0, \sigma)$,

$$-\int_{t_k}^{t_{k+1}} \lambda(s)\,ds + \int_{z}^{\psi_k(z)} \frac{ds}{c(s)} \leqslant -r_k \tag{12}$$

for some constants $r_k \in \mathbb{R}$ *and* $k \in \mathbb{N}$.

Then system (6) *is stable if* $r_k \geqslant 0$ *for all* $k \in \mathbb{N}$, *and asymptotically stable if in addition* $\sum_{k=1}^{\infty} r_k = +\infty$.

Part B. Robust stability results for linear uncertain impulsive dynamical systems

The time-varying linear uncertain impulsive dynamical system is of the form

$$\begin{aligned}
&\dot{x} = A(t)x + B(t)x, \quad t \neq t_k,\\
&\Delta x = C(t)_k x + D(t)_k x, \quad t = t_k,\\
&x\left(t_0^+\right) = x_0, \quad k \in \mathbb{N},
\end{aligned} \tag{13}$$

where $A(t), C(t)_k \in \mathbb{R}^{n\times n}$ are known matrices, and $B(t), D(t)_k \in \mathbb{R}^{n\times n}$ $(k \in \mathbb{N})$ are interval matrices, i.e., $B(t) \in N[P(t), Q(t)] = \{B(t) \in \mathbb{R}^{n\times n}\colon\ B(t) = (b(t)_{ij})_{n\times n},\ p(t)_{ij} \leqslant b(t)_{ij} \leqslant q(t)_{ij},\ i, j = 1, 2, \ldots, n\}$, $D(t)_k \in N[P(t)_k, Q(t)_k]$, where $P(t) = (p(t)_{ij})_{n\times n}$, $P(t)_k = (p(t)_{k_{ij}})_{n\times n}$, $Q(t) = (q(t)_{ij})_{n\times n}$, $Q(t)_k = (q(t)_{k_{ij}})_{n\times n}$, $k \in \mathbb{N}$, are known matrices.

In order to obtain robust stability results for system (13), we shall first establish some lemmas.

Lemma 3.1. *Let* $X(t) \in \mathbb{R}^{n\times n}$ *be a positive definite matrix function and* $Y(t) \in \mathbb{R}^{n\times n}$ *a symmetric matrix. Then for any* $x \in \mathbb{R}^n$, $t \in \mathbb{R}_+$ *the following inequality holds*:

$$x^T Y(t) x \leqslant \lambda_{\max}\left(X(t)^{-1} Y(t)\right) \cdot x^T X(t) x. \tag{14}$$

Proof. It follows from the properties of positive definite matrices. □

Lemma 3.2. *Let* $B(t) \in N[P(t), Q(t)]$, *where* $P = (p(t)_{ij})_{n\times n}$ *and* $Q = (q(t)_{ij})_{n\times n}$ *are known matrices. Then* $B(t)$ *can be written as*

$$B(t) = B(t)_0 + E(t)\Sigma(t)F(t), \tag{15}$$

where

$$\begin{aligned}
&B(t)_0 = \frac{1}{2}\left(P(t) + Q(t)\right), \qquad H(t) = \left(h(t)_{ij}\right)_{n\times n} = \frac{1}{2}\left(Q(t) - P(t)\right),\\
&\Sigma(t) \in \Sigma^* = \Big\{\Sigma(t) \in \mathbb{R}^{n^2\times n^2}\colon\ \Sigma(t) = \operatorname{diag}\big(\varepsilon(t)_{11}, \ldots, \varepsilon(t)_{1n}, \ldots, \varepsilon(t)_{n1}, \ldots,\\
&\qquad \varepsilon(t)_{nn}\big),\ \left|\varepsilon(t)_{ij}\right| \leqslant 1,\ i, j = 1, 2, \ldots, n\Big\},\\
&E(t) = \left(\sqrt{h(t)_{11}}\,e_1, \ldots, \sqrt{h(t)_{1n}}\,e_1, \ldots, \sqrt{h(t)_{n1}}\,e_n, \ldots, \sqrt{h(t)_{nn}}\,e_n\right) \in \mathbb{R}^{n\times n^2},
\end{aligned}$$

$$F(t) = \left(\sqrt{h(t)_{11}}\, e_1, \ldots, \sqrt{h(t)_{1n}}\, e_n, \ldots, \sqrt{h(t)_{n1}}\, e_1, \ldots, \sqrt{h(t)_{nn}}\, e_n\right)^T \in \mathbb{R}^{n^2 \times n},$$
$$e_i = (0, \ldots, 0, 1, 0, \ldots, 0)^T \in \mathbb{R}^n, \quad i = 1, 2, \ldots, n.$$

Proof. For any $B(t) \in N[P(t), Q(t)]$, we have

$$\begin{aligned} b(t)_{ij} &= \frac{1}{2}\left(p(t)_{ij} + q(t)_{ij}\right) + \varepsilon(t)_{ij} \cdot \frac{1}{2}(q_{ij} - p_{ij}) \\ &= \frac{1}{2}\left(p(t)_{ij} + q(t)_{ij} + \varepsilon(t)_{ij} h(t)_{ij}\right) \end{aligned}$$

for some $\varepsilon(t)_{ij} \in \mathbb{R}$ satisfying $|\varepsilon(t)_{ij}| \leqslant 1$, $i, j = 1, 2, \ldots, n$, $t \in \mathbb{R}_+$. Thus we can express $B(t)$ by

$$B(t) = B(t)_0 + \sum_{i,j=1}^{n} \varepsilon(t)_{ij} B(t)_{ij}, \tag{16}$$

where $B(t)_{ij} \in \mathbb{R}^{n \times n}$ whose entry in position (i, j) is $h(t)_{ij}$ and all other entries are zero, $i, j = 1, 2, \ldots, n$. Since $\sum_{i,j=1}^{n} \varepsilon(t)_{ij} B(t)_{ij} = E(t)\Sigma(t)F(t)$, we get Eq. (15) for $B(t)$. □

Remarks. (1) Clearly, for any $\Sigma(t) \in \Sigma^*$, we get

$$\begin{aligned} &\Sigma(t)\Sigma(t)^T = \Sigma(t)^T \Sigma(t) \leqslant I, \\ &E(t)E(t)^T = \operatorname{diag}\left\{\sum_{j=1}^{n} h(t)_{1j} \ldots \sum_{j=1}^{n} h(t)_{nj}\right\} \in \mathbb{R}^{n \times n}, \\ &F(t)^T F(t) = \operatorname{diag}\left\{\sum_{j=1}^{n} h(t)_{j1} \ldots \sum_{j=1}^{n} h(t)_{jn}\right\} \in \mathbb{R}^{n \times n}, \end{aligned}$$

where I is the $n \times n$ identity matrix.

(2) By Lemma 3.2, system (13) can be rewritten as

$$\begin{aligned} &\dot{x} = A(t)_0 x + E(t)\Sigma(t)F(t)x, \quad t \neq t_k, \\ &\Delta x = \tilde{C}(t)_k x + \tilde{E}(t)_k \tilde{\Sigma}(t)_k \tilde{F}(t)_k x, \quad t = t_k, \\ &x\left(t_0^+\right) = x_0, \quad k \in \mathbb{N}, \end{aligned} \tag{17}$$

where $A(t)_0 \stackrel{\Delta}{=} A(t) + B(t)_0$, $\tilde{C}(t)_k \stackrel{\Delta}{=} C(t)_k + D(t)_{k_0}$, $D(t)_k = D(t)_{k_0} + \tilde{E}(t)_k \tilde{\Sigma}(t)_k \tilde{F}(t)_k$. Here, $B(t)_0$, $E(t)$, $\Sigma(t)$, $F(t)$, $D(t)_{k_0}$, $\tilde{E}(t)_k$, $\tilde{\Sigma}(t)_k$, and $\tilde{F}(t)_k$ ($k \in \mathbb{N}$) are defined as in Lemma 3.2.

Lemma 3.3. *If $\Sigma(t) \in \Sigma^*$, then for any positive scalar function $\lambda(t) > 0$ and for any $\xi \in \mathbb{R}^{n^2}$, $\eta \in \mathbb{R}^{n^2}$ the following inequality holds*:

$$2\xi^T \Sigma(t)\eta \leqslant \lambda(t)^{-1} \xi^T \xi + \lambda(t)\eta^T \eta. \tag{18}$$

Proof. It follows from the Schwarz inequality and $\Sigma(t)\Sigma(t)^T = \Sigma(t)^T\Sigma(t) \leqslant I$. □

Theorem 3.1. *Assume that there exist scalar functions $\lambda(t) > 0$, $\alpha(t) \geqslant 0$ and a uniformly positive definite matrix function $X(t)$ bounded above such that*

(i) *$X(t)$ is differentiable at $t \neq t_k$ and the Riccati inequality holds*

$$\dot{X} + XA_0 + A_0^T X + \lambda^{-1} XEE^T X + \lambda F^T F \leqslant \alpha X \quad \text{for all } t \neq t_k,\ k \in \mathbb{N}; \tag{19}$$

(ii) *there exist some $r_k \in \mathbb{R}$, $\xi_k > 0$ $(k \in \mathbb{N})$ such that*

$$\int_{t_k}^{t_{k+1}} \alpha(s)\,ds + \ln\beta_k \leqslant -r_k \quad \text{for all } k \in \mathbb{N}, \tag{20}$$

where

$$\beta_k = \lambda_{\max}\big\{X(t_k)^{-1}\big[\big(I + C(t_k)_k^T\big)\big(X(t_k) + \xi_k^{-1} X(t_k)\tilde{E}(t_k)_k \tilde{E}(t_k)_k^T X(t_k)\big) \\ \times \big(I + C(t_k)_k\big) + \big(\xi_k + \lambda_{\max}\big(\tilde{E}(t_k)_k^T X(t_k)\tilde{E}(t_k)_k\big)\big)\tilde{F}(t_k)_k^T \tilde{F}(t_k)_k\big]\big\}.$$

Then the system (17) *is robustly stable if $r_k \geqslant 0$ for all $k \in \mathbb{N}$, and it is robustly asymptotically stable if in addition $\sum_{k=1}^{\infty} r_k = +\infty$.*

Proof. To prove this theorem, we only need to check all the conditions of Proposition 3.1. Let $V(t,x) = x^T X(t)x$. Clearly, V belongs to ν_0 and

$$\lambda_{\min}\big(X(t)\big)\cdot \big\|x(t)\big\|^2 \leqslant V \leqslant \lambda_{\max}\big(X(t)\big)\cdot\big\|x(t)\big\|^2, \quad (t,x) \in \mathbb{R}_+ \times \mathbb{R}^n. \tag{21}$$

Since $X(t)$ is a uniformly positive definite matrix function and bounded above, there exist positive real numbers $M \geqslant m > 0$ such that

$$m \leqslant \lambda_{\min}\big(X(t)\big) \leqslant \lambda_{\max}\big(X(t)\big) \leqslant M. \tag{22}$$

Define $a(s) = M\cdot s^2$ and $b(s) = m\cdot s^2$, $s \in \mathbb{R}_+$; then $a, b \in \varkappa$ and from (21) and (22), we have

$$b\big(\|x\|\big) \leqslant V(t,x) \leqslant a\big(\|x\|\big). \tag{23}$$

Hence, condition (ii) of Proposition 3.1 holds.

Denote $x_k = x(t_k)$, $X_k = X(t_k)$, $\tilde{C}_k = \tilde{C}(t_k)_k$, $\tilde{E}_k = \tilde{E}(t_k)_k$, $\tilde{\Sigma}_k = \tilde{\Sigma}(t_k)_k$, $\tilde{F}_k = \tilde{F}(t_k)_k$. When $t = t_k$, by Lemmas 3.1 and 3.3, we get

$$\begin{aligned} V\big(t_k^+, x_k + I_k(x_k)\big) &= x_k^T\big[(I+\tilde{C}_k) + \tilde{E}_k\tilde{\Sigma}_k\tilde{F}_k\big]^T X_k\big[(I+\tilde{C}_k) + \tilde{E}_k\tilde{\Sigma}_k\tilde{F}_k\big]x_k \\ &= x_k^T\big\{\big(I+\tilde{C}_k^T\big)X_k(I+\tilde{C}_k) + (I+\tilde{C}_k)^T X_k\tilde{E}_k\tilde{\Sigma}_k\tilde{F}_k \\ &\quad + (\tilde{E}_k\tilde{\Sigma}_k\tilde{F}_k)^T X_k(I+\tilde{C}_k) + (\tilde{E}_k\tilde{\Sigma}_k\tilde{F}_k)^T X_k(\tilde{E}_k\tilde{\Sigma}_k\tilde{F}_k)\big\}x_k \\ &\leqslant x_k^T\big\{\big(I+\tilde{C}_k^T\big)X_k(I+\tilde{C}_k) + \xi_k^{-1}(I+\tilde{C}_k)^T X_k\tilde{E}_k\tilde{E}_k^T X_k(I+\tilde{C}_k) + \xi_k\tilde{F}_k^T\tilde{F}_k\big\}x_k \\ &\quad + \lambda_{\max}\big(\tilde{E}_k^T X_k\tilde{E}_k\big)\cdot x_k^T\tilde{F}_k^T\tilde{F}_k x_k \end{aligned}$$

$$
\begin{aligned}
&= x_k^T\big\{(I+\tilde C_k)^T\big[X_k+\xi_k^{-1}X_k\tilde E_k\tilde E_k^T X_k\big](I+\tilde C_k)\\
&\quad+\big(\xi_k+\lambda_{\max}\big(\tilde E_k^T X_k\tilde E_k\big)\big)\cdot\tilde F_k^T\tilde F_k\big\}x_k\\
&\leqslant \beta_k V(t_k,x_k). \qquad (24)
\end{aligned}
$$

Let $\psi_k(s)=\beta_k\cdot s$, $s\in\mathbb{R}_+$. Then, it is easy to see $\psi_k\in\varkappa_0$. From (24), condition (iii) of Proposition 3.1 is satisfied.

Denote $x=x(t)$, $X=X(t)$, $E=E(t)$, $\Sigma=\Sigma(t)$, $F=F(t)$. Using Lemma 3.3 and condition (i), for $t\neq t_k$, we get

$$
\begin{aligned}
D^+V(t,x) &= \dot x^T Xx + x^T\dot Xx + x^T X\dot x\\
&= x^T\big(\dot X+A_0^T X+XA_0\big)x+2x^T XE\Sigma Fx\\
&\leqslant x^T\big(\dot X+A_0^T X+XA_0+\lambda^{-1}XEE^T X+\lambda F^T F\big)x\\
&\leqslant \alpha(t)\cdot x^T Xx=\alpha(t)\cdot V(t,x). \qquad (25)
\end{aligned}
$$

Thus, letting $c(s)=s$, $p(t)=\alpha(t)$, $s\in\mathbb{R}_+$, we get

$$D^+V(t,x)\leqslant p(t)\cdot c\big(V(t,x)\big). \qquad (26)$$

Hence, by (26), condition (iv) of Proposition 3.1 is also satisfied. By $r_k\geqslant 0$ and (20), we have $\beta_k\leqslant 1$ for all $k\in\mathbb{N}$. In view of (24), condition (i) of Proposition 3.1 is satisfied. The condition (v) of Proposition 3.1 is satisfied as well by using (20) and $\psi_k(s)=\beta_k\cdot s$ and $c(s)=s$. Therefore, all conditions of Proposition 3.1 are satisfied. Hence the theorem is true and the proof is complete. □

Theorem 3.2. *Assume that there exist scalar functions $\lambda(t)>0$, $\mu(t)\geqslant 0$ and a uniformly positive definite matrix function $X(t)$ bounded above such that*

(i) *$X(t)$ is differentiable at $t\neq t_k$ and the following Riccati inequality holds*:

$$\dot X+XA_0+A_0^T X+\lambda^{-1}XEE^T X+\lambda F^T F\leqslant -\mu X,\quad t\neq t_k,\ k\in\mathbb{N}; \qquad (27)$$

(ii) *there exist some $r_k\in\mathbb{R}$, $\xi_k>0$ ($k\in\mathbb{N}$) such that*

$$-\int_{t_k}^{t_{k+1}}\mu(s)\,ds+\ln\beta_k\leqslant -r_k,\quad k\in\mathbb{N}, \qquad (28)$$

where

$$
\begin{aligned}
\beta_k=\lambda_{\max}\big\{&X(t_k)^{-1}\big[\big(I+C(t_k)_k^T\big)\big(X(t_k)+\xi_k^{-1}X(t_k)\tilde E(t_k)_k\tilde E(t_k)_k^T X(t_k)\big)\\
&\times\big(I+C(t_k)_k\big)+\big(\xi_k+\lambda_{\max}\big(\tilde E(t_k)_k^T X(t_k)\tilde E(t_k)_k\big)\big)\tilde F(t_k)_k^T\tilde F(t_k)_k\big]\big\}.
\end{aligned}
$$

Then the system (17) *is robustly stable if $r_k\geqslant 0$ for all $k\in\mathbb{N}$, and it is robustly asymptotically stable if in addition $\sum_{k=1}^{\infty}r_k=+\infty$.*

Proof. It follows from Proposition 3.2 and similar arguments to those used in the proof of Theorem 3.1. The details are omitted. □

Theorem 3.3. *Assume that*

(i) *there exist a scalar function* $\lambda(t) > 0$ *and an uniformly positive definite matrix function* $X(t)$ *bounded above and* $X(t)$ *is differentiable at* $t \neq t_k$ *such that the following Riccati inequality holds*:

$$\dot{X} + XA_0 + A_0^T X + \lambda^{-1} XEE^T X + \lambda F^T F < 0, \tag{29}$$

and that $-(\dot{X} + XA_0 + A_0^T X + \lambda^{-1} XEE^T X + \lambda F^T F)$ *is a uniformly positive definite matrix function*;

(ii) $\prod_{k=1}^{\infty} \beta_k$ *converges, where*

$$\begin{aligned}\beta_k = \lambda_{\max}\big\{ X(t_k)^{-1}\big[\big(I + C(t_k)_k^T\big)\big(X(t_k) + \xi_k^{-1} X(t_k)\tilde{E}(t_k)_k \tilde{E}(t_k)_k^T X(t_k)\big) \\ \times \big(I + C(t_k)_k\big) + \big(\xi_k + \lambda_{\max}\big(\tilde{E}(t_k)_k^T X(t_k)\tilde{E}(t_k)_k\big)\big)\tilde{F}(t_k)_k^T \tilde{F}(t_k)_k\big]\big\}.\end{aligned}$$

Then the system (17) *is robustly exponentially stable.*

Proof. Using Theorem 3.1, we see that system (17) is robustly asymptotically stable. Let $Y(t) = -(\dot{X} + XA_0 + A_0^T X + \lambda^{-1} XEE^T X + \lambda F^T F)$. Then by condition (i), $Y(t)$ is a uniformly positive definite matrix function. Moreover, since $X(t)$ is a uniformly positive definite matrix function bounded above, there exist positive real numbers $\sigma_1, \sigma_2, \sigma_3$ satisfying

$$\sigma_1 \leqslant \lambda_{\min}\big(Y(t)\big), \quad \sigma_3 \leqslant \lambda_{\min}\big(X(t)\big) \leqslant \lambda_{\max}\big(X(t)\big) \leqslant \sigma_2, \quad t \in \mathbb{R}_+.$$

Let $V(t,x) = x^T X(t) x$. Then, for any $t \neq t_k$, we have

$$\begin{aligned} D^+ V(t,x) &= x^T\big(\dot{X} + A_0^T X + XA_0\big)x + 2x^T XE\Sigma Fx \\ &\leqslant x^T\big(\dot{X} + A_0^T X + XA_0 + \lambda^{-1} XEE^T X + \lambda F^T F\big)x \\ &= -x^T Y(t)x \leqslant -\sigma \cdot x^T X(t)x = -\sigma \cdot V(t,x), \end{aligned} \tag{30}$$

where $\sigma \stackrel{\Delta}{=} \sigma_1/\sigma_2 > 0$. When $t = t_k$, by the similar proof of (24) of Theorem 3.1, we get

$$V\big(t_k^+, x_k^+\big) \leqslant \beta_k \cdot V(t_k, x_k). \tag{31}$$

From (30) and (31), for $t_k < t \leqslant t_{k+1}$, we get

$$\begin{aligned} V\big(t, x(t)\big) &\leqslant V\big(t_k^+, x_k^+\big) \cdot e^{-\sigma(t-t_k)} \leqslant \beta_k \cdot V(t_k, x_k) \cdot e^{-\sigma(t-t_k)} \\ &\leqslant V(t_0, x_0) \cdot \prod_{i=1}^{k} \beta_i \cdot e^{-\sigma(t-t_0)}. \end{aligned} \tag{32}$$

Hence, for all $t \in (t_k, t_{k+1}]$, $k \in \mathbb{N}$, we get

$$\begin{aligned} \|x(t)\| &\leqslant \left[\frac{V(t_0, x_0)}{\lambda_{\min}(X(t))} \cdot \prod_{i=1}^{k} \beta_i\right]^{1/2} \cdot e^{-(\sigma/2)(t-t_0)} \\ &\leqslant \|x_0\| \left[\frac{\sigma_2}{\sigma_3} \cdot \prod_{i=1}^{k} \beta_i\right]^{1/2} \cdot e^{-(\sigma/2)(t-t_0)} \end{aligned} \tag{33}$$

and

$$\left\|x\left(t_k^+\right)\right\| \leqslant \left(\frac{\beta_k}{\sigma_3}\right)^{1/2} \cdot \left\|x(t_k)\right\|. \tag{34}$$

Hence, from (33) and (34), system (17) is robustly exponentially stable and the proof is complete. □

Remark. As a special case, we can get some corresponding results for the time-invariant linear uncertain impulsive dynamical systems, i.e., all matrices $A_0, E, F, \Sigma, \tilde{C}_k, \tilde{\Sigma}_k, \tilde{F}_k$ in (17) are constant matrices. To save space, we just give one result here and other corresponding results are omitted.

Corollary 3.1. *Assume that system* (17) *is time-invariant. Suppose further that*

(i) *there exist real numbers* $\lambda, \varepsilon > 0$ *and a positive definite matrix* X *such that the following algebraic Riccati equation holds*:

$$XA_0 + A_0^T X + \lambda^{-1} XEE^T X + \lambda F^T F + \varepsilon I = 0, \tag{35}$$

or

(i′) A_0 *is asymptotically stable matrix and*

$$\left\|F(sI - A_0)^{-1}E\right\|_\infty < 1; \tag{36}$$

(ii) *condition* (ii) *of Theorem* 3.3 *holds.*

Then system (17) *is robustly exponentially stable.*

Proof. From [9], it is easy to see that condition (i) or (i′) is equivalent to condition (i) of Theorem 3.3. Thus the corollary is true. The proof is complete. □

Part C. Robust stability results for nonlinear uncertain impulsive dynamical systems

For uncertain impulsive system (1), we establish some general results as follows.

Theorem 3.4. *Assume that there is* $V \in \nu_0$ *such that* $V(t,x)$ *is differentiable on* $(t_{k-1}, t_k) \times \mathbb{R}^n$ *for any* $k \in \mathbb{N}$, *and conditions* (i), (ii) *and* (v) *of Proposition* 3.1 *hold. Suppose further that*

(i) *there exist functions* $P_{1_k} : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^{1\times m}$, $P_{2_k} : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^{m\times m}$ *with* $P_{2_k}(t,x) \geqslant 0$, *and for* $t \in \mathbb{R}_+$, $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $k \in \mathbb{N}$,

$$\begin{aligned} &V\left(t, x + I_k(x) + e_k(x)y\right) \\ &\quad \leqslant V\left(t, x + I_k(x)\right) + P_{1_k}(t,x)y + y^T P_{2_k}(t,x)y; \end{aligned} \tag{37}$$

(ii) *there are positive constants* ε_k $(k \in \mathbb{N})$ *such that*

$$V\big(t_k^+, x_k + I_k(x_k)\big) + \varepsilon_k^{-1} P_{1_k} P_{1_k}^T + \big(\varepsilon_k + \lambda_{\max}(P_{2_k})\big) m_k^T m_k \leqslant \psi\big(V(t_x, x_k)\big), \tag{38}$$

where $\psi_k \in \varkappa_0$, $P_{1_k} = P_{1_k}(t_k, x_k)$, $P_{2_k} = P_{2_k}(t_k, x_k)$, $m_k = m_k(x_k)$, $k \in \mathbb{N}$;

(iii) *there exist* $c \in \varkappa$, $p \in PC$ *and scalar function* $\lambda_k \in C[\mathbb{R}^n, \mathbb{R}_+]$ *such that*

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{\lambda_k^2}{2} \frac{\partial V}{\partial x} e_g e_g^T \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda_k^2} m_g^T m_g \leqslant p(t) \cdot c\big(V(t,x)\big), \tag{39}$$

where $(t, x) \in (t_k, t_{k+1}] \times S_\rho$, $k \in \mathbb{N}$.

Then system (1) *is robustly stable if* $r_k \geqslant 0$ *for all* $k \in \mathbb{N}$, *and it is robustly asymptotically stable if in addition* $\sum_{k=1}^{\infty} r_k = +\infty$.

Proof. From Proposition 3.1, we only need to verify conditions (iii) and (iv) of Proposition 3.1. When $t = t_k$, $k \in \mathbb{N}$, by conditions (i), (ii) and Lemma 3.3, we get

$$\begin{aligned}
&V\big(t_k^+, x_k + I_k(x_k) + J_k(x_k)\big) \\
&\quad\leqslant V\big(t_k^+, x_k + I_k(x_k)\big) + P_{1_k}\delta_k(x_k) + \delta_k(x_k)^T P_{2_k}\delta_k(x_k) \\
&\quad\leqslant V\big(t_k^+, x_k + I_k(x_k)\big) + \varepsilon_k^{-1} P_{1_k} P_{1_k}^T + \varepsilon_k m_k^T m_k + \lambda_{\max}(P_{2_k}) m_k^T m_k \\
&\quad= V\big(t_k^+, x_k + I_k(x_k)\big) + \varepsilon_k^{-1} P_{1_k} P_{1_k}^T + \big(\varepsilon_k + \lambda_{\max}(P_{2_k})\big) m_k^T m_k \\
&\quad\leqslant \psi_k\big(V(t_k, x_k)\big).
\end{aligned} \tag{40}$$

Let $V = V(t,x)$, $f = f(t,x)$, $g = g(t,x)$, $m_g = m_g(t,x)$, $\delta_g = \delta_g(t,x)$, and $\lambda_k = \lambda(t)$. Then for $t \neq t_k$, $k \in \mathbb{N}$, in view of inequality (39), we have

$$\begin{aligned}
D^+ V\big(t, x(t)\big) &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}(f + g) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{\partial V}{\partial x} e_g \delta_g \\
&= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{\lambda_k^2}{2} \frac{\partial V}{\partial x} e_g e_g^T \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda_k^2} m_g^T m_g \\
&\quad - \frac{1}{2}\left\{\lambda_k \frac{\partial V}{\partial x} e_g - \frac{1}{\lambda_k}\delta_g^T\right\} \cdot \left\{\lambda_k e_g^T \frac{\partial V^T}{\partial x} - \frac{1}{\lambda_k}\delta_g\right\} \\
&\quad - \frac{1}{2\lambda_k^2}\big\{m_g^T m_g - \delta_g^T \delta_g\big\} \\
&\leqslant \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{\lambda_k^2}{2} \frac{\partial V}{\partial x} e_g e_g^T \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda_k^2} m_g^T m_g \\
&\leqslant p(t) \cdot c\big(V\big(t, x(t)\big)\big).
\end{aligned} \tag{41}$$

Thus, by (40) and (41), conditions (iii) and (iv) of Proposition 3.1 are satisfied. Therefore, by Proposition 3.1, system (1) is robustly asymptotically stable and the proof is complete. □

Theorem 3.5. *Assume that there is $V \in \nu_0$ such that $V(t,x)$ is differentiable on $(t_{k-1}, t_k) \times \mathbb{R}^n$ for any $k \in \mathbb{N}$, and conditions* (i) *and* (ii) *of Proposition* 3.1 *and conditions* (i) *and* (ii) *of Theorem* 3.4 *hold. Suppose further that*

(i′) *there exist $c \in \varkappa$, $\lambda \in PC$ and scalar functions $\lambda_k \in C[\mathbb{R}^n, \mathbb{R}_+]$ such that*

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{\lambda_k^2}{2} \frac{\partial V}{\partial x} e_g e_g^T \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda_k^2} m_g^T m_g \leqslant -\lambda(t) \cdot c\big(V(t,x)\big); \tag{42}$$

(ii′) *there exists a constant $\sigma > 0$ such that for all $z \in (0, \sigma)$,*

$$-\int_{t_{k-1}}^{t_k} \lambda(s)\, ds + \int_{z}^{\psi_k(z)} \frac{ds}{c(s)} \leqslant -r_k \tag{43}$$

for some constant r_k and $k \in \mathbb{N}$.

Then the system (1) *is robustly stable if $r_k \geqslant 0$ for all $k = 1, 2, \ldots,$ and it is robustly asymptotically stable if in addition $\sum_{k=1}^{\infty} r_k = +\infty$.*

Proof. It follows from Proposition 3.2 and similar arguments to those used in the proof of Theorem 3.4. The details are omitted. □

Corollary 3.2. *Assume that all conditions of Theorem* 3.5 *hold. Moreover, if $\lambda(t) \equiv 0$, $t \in \mathbb{R}_+$, and $\psi_k(s) = \mu_k \cdot s$, $s \in \mathbb{R}_+$, for some positive constants μ_k $(k \in \mathbb{N})$, then system* (1) *is robustly stable if $\mu_k \leqslant 1$ for all $k \in \mathbb{N}$, and robustly asymptotically stable if in addition $\sum_{k=1}^{\infty} \ln \mu_k = -\infty$.*

Proof. The result is a direct consequence of Theorem 3.5. □

Theorem 3.6. *Assume that there is $V \in \nu_0$ such that $V(t,x)$ is differentiable on $(t_{k-1}, t_k) \times \mathbb{R}^n$ for any $k \in \mathbb{N}$, and condition* (i) *of Theorem* 3.4 *holds. Suppose further that*

(i) *there are positive real numbers μ_1, μ_2 such that*

$$\mu_1 \|x(t)\|^2 \leqslant V(t,x) \leqslant \mu_2 \|x(t)\|^2, \quad (t,x) \in \mathbb{R}_+ \times S_\rho; \tag{44}$$

(ii) *there are positive constants ε_k $(k \in \mathbb{N})$ such that*

$$V\big(t_k^+, x_k + I_k(x_k)\big) + \varepsilon_k^{-1} P_{1_k} P_{1_k}^T + \big(\varepsilon_k + \lambda_{\max}(P_{2_k})\big) m_k^T m_k \leqslant V(t_x, x_k), \tag{45}$$

where $P_{1_k} = P_{1_k}(t_k, x_k)$, $P_{2_k} = P_{2_k}(t_k, x_k)$, $m_k = m_k(x_k)$, $k \in \mathbb{N}$;

(iii) *there are* $\varepsilon > 0$ *and scalar functions* $\lambda_k \in C[\mathbb{R}^n, \mathbb{R}_+]$ *such that*

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{\lambda_k^2}{2} \frac{\partial V}{\partial x} e_g e_g^T \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda_k^2} m_g^T m_g + \frac{\varepsilon^2}{2} x^T x \leqslant 0, \tag{46}$$

where $(t, x) \in (t_k, t_{k+1}] \times S_\rho$, $k \in \mathbb{N}$.

Then system (1) *is robustly exponentially stable.*

Proof. From the assumptions, it is easy to get the following inequalities:

$$\|x(t)\| \leqslant \|x_0\| \sqrt{\frac{\mu_2}{\mu_1}} \exp\left\{-\frac{1}{2}\left(\frac{\varepsilon}{\mu_2}\right)^2 (t - t_0)\right\}, \quad t \in (t_k, t_{k+1}], \tag{47}$$

and

$$\|x(t_k^+)\| \leqslant \sqrt{\frac{\mu_2}{\mu_1}} \|x(t_k)\|, \quad k \in \mathbb{N}. \tag{48}$$

From (47) and (48), system (1) is robustly exponentially stable and the proof is complete. □

4. Example

Finally, we shall discuss an example to illustrate our results.

Example 4.1. Consider the following uncertain impulsive dynamical system:

$$\begin{aligned} &\dot{x} = f(t, x) + g(t, x), \quad t \in (k, k+1], \\ &\Delta x = \begin{pmatrix} -1 + \frac{1}{k+2} & 0 \\ 0 & -1 + \frac{1}{k+2} \end{pmatrix} x_k, \quad t = k, \ k \in \mathbb{N}, \end{aligned} \tag{49}$$

where

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \qquad f(t, x) = \begin{pmatrix} -x_1 + x_2(x_1^2 + x_2^2) \\ -x_2 + x_1(x_1^2 + x_2^2) \end{pmatrix},$$

$$g(t, x) \in \Omega_g = \left\{ g\colon\ g = e_g \cdot \delta_g,\ e_g = \begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix},\ \|\delta_g\| \leqslant \|m_g\|, \right.$$
$$\left. m_g = \begin{pmatrix} x_1 + x_2 \\ \sqrt{2} x_1 x_2 \end{pmatrix} \right\}.$$

Let $V(t, x) = (1/2)(x_1^2 + x_2^2)$. Then, obviously, $V \in \nu_0$ and V is differentiable at any $t \in \mathbb{R}^+$. For any $t \in (k, k+1]$ and $\lambda_k = 1$, we get

$$\begin{aligned} &\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{\lambda_k^2}{2} \frac{\partial V}{\partial x} e_g e_g^T \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda_k^2} m_g^T m_g \\ &\quad = -x_1^2 - x_2^2 + 2x_1 x_2(x_1^2 + x_2^2) + \frac{1}{2}(x_1^4 + x_2^4) + \frac{1}{2}\left\{(x_1 + x_2)^2 + 2x_1^2 x_2^2\right\} \end{aligned}$$

$$= x_1x_2 - \frac{1}{2}\big(x_1^2 + x_2^2\big) + 2x_1x_2\big(x_1^2 + x_2^2\big) + \frac{1}{2}\big(x_1^2 + x_2^2\big) \leqslant 4V^2. \tag{50}$$

Hence, if let $p(t) = 1$, $c(s) = 4s^2$, then the Hamilton–Jacobi inequality

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{\lambda_k^2}{2}\frac{\partial V}{\partial x} e_g e_g^T \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda_k^2} m_g^T m_g \leqslant p(t) \cdot c\big(V(t,x)\big)$$

is satisfied. Furthermore, when $t = k$, $k \in \mathbb{N}$, we get

$$\begin{aligned} V\big(t_k^+, x_k^+\big) &= \frac{1}{2}\left(\frac{1}{k+2}\right)^2 \cdot \big(x_1^2(t_k) + x_2^2(t_k)\big) \\ &= \left[\frac{1}{k+2}\right]^2 \cdot V(t_k, x_k) \leqslant \psi_k\big(V(t_k, x_k)\big), \end{aligned}$$

where $\psi_k(s) = (1/(k+2))^2 \cdot s$, $s \in \mathbb{R}_+$. Set $\sigma = 1$; then, for any $z \in (0, \sigma)$,

$$\begin{aligned} \int_k^{k+1} p(s)\,ds + \int_z^{\psi_k(z)} \frac{1}{c(s)}\,ds &= 1 + \int_z^{\psi_k(z)} \frac{1}{4s^2}\,ds = 1 + \frac{1}{4z}\big[1 - (k+2)^2\big] \\ &\leqslant 1 + \frac{1}{4}\big[1 - (k+2)^2\big] = -\frac{1}{4}\big[(k+2)^2 - 5\big]. \end{aligned} \tag{51}$$

Hence, setting $r_k = (1/4)[(k+2)^2 - 5]$, we get $\int_k^{k+1} p(s)\,ds + \int_z^{\psi_k(z)} (1/c(s))\,ds \leqslant -r_k$, and $r_k \geqslant 0$ for all $k \in \mathbb{N}$. Clearly, $\sum_{k=1}^{\infty} r_k = (1/4)\sum_{k=1}^{\infty}[(k+2)^2 - 5] = +\infty$. Therefore, by Theorem 3.4, system (49) is robustly asymptotically stable.

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