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"Induced" $\mathcal{N}=4$ conformal supergravity

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ABSTRACT

We consider an Abelian $\mathcal{N}=4$ super Yang–Mills theory coupled to background $\mathcal{N}=4$ conformal supergravity fields. At the classical level, this coupling is invariant under global $SU(1,1)$ transformation of the complex ("dilaton–axion") supergravity scalar combined with an on-shell $\mathcal{N}=4$ vector–vector duality. We compute the divergent part of the corresponding quantum effective action found by integrating over the super Yang–Mills fields and demonstrate its $SU(1,1)$ invariance. This divergent part related to the conformal anomaly is one-loop exact and should be given by the $\mathcal{N}=4$ conformal supergravity action containing the Weyl tensor squared term. This allows us to determine the full non-linear form of the bosonic part of the $\mathcal{N}=4$ conformal supergravity action which has manifest $SU(1,1)$ invariance.

1. Introduction

The $\mathcal{N}=4$ conformal supergravity (CSG) as formulated in [1] should have global $SU(1,1)$ or $SL(2,R)$ symmetry acting on the singlet complex scalar (described by a 4-derivative analog of the $SU(1,1)/U(1)$ coset sigma model).[2] While the complete $\mathcal{N}=4$ superconformal transformation laws were written down in [1], the full non-linear action of such $\mathcal{N}=4$ conformal supergravity was not explicitly constructed so far. The aim of this Letter is to find the full bosonic part of such action.

This manifest $SU(1,1)$ symmetry is in general broken if one couples the $\mathcal{N}=4$ CSG to $\mathcal{N}=4$ super Yang–Mills (SYM) theory [2,3]. It is, however, preserved in a weaker "on-shell" form in the case when the $\mathcal{N}=4$ SYM theory is *Abelian*: the resulting *equations of motion* are invariant under the $SU(1,1)$ acting not only on the complex scalar but also on the Abelian SYM vector via vector–vector duality transformation.[3] This symmetry is then inherited by the equations of motion of the $\mathcal{N}=4$ Poincaré supergravity [5] as it can be obtained [2] from a system of 6 Abelian vector multiplets coupled to the $\mathcal{N}=4$ conformal supergravity multiplet.[4]

As was found in [7,8], the $SU(1,1)$ invariant $\mathcal{N}=4$ CSG of [1] has non-zero beta-function or conformal anomaly and is thus inconsistent at the quantum level unless it is coupled to *four* $\mathcal{N}=4$ vector multiplets (see [9] for a review). This conclusion was confirmed in [10] on the basis of analysis of the local $SU(4)$ chiral anomaly (which is in the same multiplet with trace anomaly).

At the same time, it was suggested in [7,8] that there might exist an alternative version of $\mathcal{N}=4$ CSG without the $SU(1,1)$ invariance in which a non-minimal coupling of the singlet scalar to the square of the Weyl tensor may be present. For a particular value of such coupling the resulting "non-minimal" $\mathcal{N}=4$ CSG can be made UV finite by itself, i.e. without adding extra $\mathcal{N}=4$ vector multiplets [7].[5] Curiously, a similar type of "non-minimal" $\mathcal{N}=4$ conformal supergravity seems to emerge [11] in the twistor-string [12] context.

The coupling between $\mathcal{N}=4$ SYM and $\mathcal{N}=4$ CSG multiplets appears also in the context of the AdS/CFT correspondence

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2 To make this symmetry linearly realized one may introduce also a spurious local $U(1)$ symmetry.

3 This on-shell symmetry can be promoted to a manifest symmetry of the action (at the expense of manifest Lorentz symmetry) if one uses a phase-space type formulation where one doubles the number of vectors, see, e.g., [4].

4 This can be done by partial gauge fixing and solving for some of the CSG fields that in the absence of the pure CSG action play a role of auxiliary fields [2,3]. Potential importance of superconformal formulation of $\mathcal{N}=4$ Poincaré supergravity was recently emphasised in [6].

5 It is not clear, however, how this conjecture can be reconciled with the $SU(4)$ anomaly cancellation study [10] which does not seem to be sensitive to such non-minimal terms. That suggests a potential problem with realization of supersymmetry which should be requiring that all superconformal anomalies should belong to one supermultiplet.

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[13–15]: the $\mathcal{N}=4$ SYM path integral with the CSG fields as external "sources" may be interpreted as a generating functional for correlators of particular 1/2 BPS operators (dimension 2 chiral primary operator and its supersymmetry descendants, i.e. the fields of the stress tensor multiplet dual to $\mathcal{N}=8$, $d=5$ supergravity fields). After integrating over the quantum SYM fields, the conformal supergravity action should then be the coefficient of the logarithmic divergence in the resulting effective action. In that limited sense the $\mathcal{N}=4$ CSG may be interpreted as an "*induced*" theory.[6]

Since the superconformal anomaly should be 1-loop exact, the result for the logarithmic divergence should be given just by the 1-loop contribution.[7] This also means that the divergent term is not sensitive to the non-Abelian structure of the SYM theory, i.e. it is sufficient to consider just *one* Abelian $\mathcal{N}=4$ vector multiplet coupled to the external $\mathcal{N}=4$ CSG multiplet and do the Gaussian integral over the $\mathcal{N}=4$ vector multiplet fields.

As the full non-linear form of the coupling between the $\mathcal{N}=4$ SYM and CSG multiplets is known [2,3], and since the one-loop logarithmic divergence of the $\mathcal{N}=4$ vector multiplet fields is determined by a relevant Seeley coefficient of the corresponding 2nd-order matrix differential operator (with coefficients depending on the external CSG fields) it should thus be straightforward to reconstruct the full non-linear form of the resulting $\mathcal{N}=4$ CSG action using the standard algorithm [16], i.e. one should get [14]

$$\Gamma_\infty = -(\ln Z_{\mathcal{N}=4\,\mathrm{SYM}})_\infty = k I_{\mathcal{N}=4\,\mathrm{CSG}}, \qquad k = -\frac{N^2}{4(4\pi)^2}\ln\Lambda, \tag{1.1}$$

$$I_{\mathcal{N}=4\,\mathrm{CSG}} = \int d^4x\,\sqrt{g}\mathcal{L}_{\mathcal{N}=4\,\mathrm{CSG}} = \int d^4x\,\sqrt{g}\big(C^2+\cdots\big), \tag{1.2}$$

where N is the number of $\mathcal{N}=4$ vector multiplets, Λ is a UV cutoff. Here $I_{\mathcal{N}=4\,\mathrm{CSG}}$ should be the CSG action as it starts with the Weyl tensor squared C^2 term (up to total derivative Euler density term): since $I_{\mathcal{N}=4\,\mathrm{CSG}}$ should inherit all the symmetries of $\mathcal{N}=4$ conformal supergravity by construction[8] and contains the C^2 term it must represent the complete non-linear action of $\mathcal{N}=4$ conformal supergravity.

In particular, since the coupling between an Abelian $\mathcal{N}=4$ SYM and $\mathcal{N}=4$ CSG multiplets preserves the scalar $SU(1,1)$ symmetry combined with a duality rotation of the $\mathcal{N}=4$ SYM vector [2] and since the latter is integrated over in the path integral, the resulting "induced" CSG action should have manifest (off-shell) $SU(1,1)$ symmetry.[9] This was already demonstrated in [19] in the subsector of the standard $SL(2,R)$ invariant scalar–vector coupling ($e^{-\sigma}F_{mn}F_{mn} - i\mathcal{C}F_{mn}F^\star_{mn}$). Here we will demonstrate this for the full $\mathcal{N}=4$ vector–CSG coupling case, thus determining the full $SU(1,1)$ invariant form of the bosonic part of the $\mathcal{N}=4$ CSG action.

This computation is of interest as the complete non-linear form of the $\mathcal{N}=4$ CSG action was not explicitly given before. The terms in the CSG action which are quadratic in the non-metric fields (but non-linear in the metric) can be reconstructed [7,9] by requiring the Weyl symmetry and reparametrization invariance, but higher-order terms are hard to determine directly.[10] The non-linear terms of $\mathcal{N}=4$ CSG action should of course reduce to the corresponding terms in the full $\mathcal{N}=2$ CSG action which was found in [1]; this provides a non-trivial check.

As the "induced" CSG action we find below is manifestly $SU(1,1)$ invariant, an apparent absence of an alternative to the $SU(1,1)$ invariant coupling [2] between the Abelian $\mathcal{N}=4$ SYM and $\mathcal{N}=4$ CSG multiplets appears to rule out the possibility of some $SU(1,1)$ non-invariant "non-minimal" conformal supergravity model.

We shall start in Section 2 with a review of the Lagrangian of an Abelian $\mathcal{N}=4$ vector multiplet coupled to (bosonic part of) $\mathcal{N}=4$ conformal supergravity background. In Section 3 we shall compute the UV divergent part of the effective action found by integrating over the vector multiplet fields and show that the resulting $SU(1,1)$ invariant expression has the expected structure of the $\mathcal{N}=4$ CSG action. A short summary will be given in Section 4.

2. $\mathcal{N}=4$ Abelian vector multiplet coupled to external $\mathcal{N}=4$ conformal supergravity

Let us start with a review of the action [2] for an Abelian $\mathcal{N}=4$ vector multiplet in a background of $\mathcal{N}=4$ conformal supergravity. We shall denote the vector multiplet fields as $\mathcal{A}=\{A_m,\varphi_{ij},\psi_i\}$. In what follows $m,n,r,s=1,2,3,4$ are space–time indices and $i,j,k,l=1,2,3,4$ are $SU(4)$ indices. The scalar fields satisfy the conditions

$$\varphi_{ij} = -\varphi_{ji} = -\frac{1}{2}\varepsilon_{ijkl}\varphi^{kl}, \qquad \varphi^{ij} = (\varphi_{ij})^*. \tag{2.1}$$

For the fermions $\psi^i = P_+\psi^i$ transforms as **4** of $SU(4)$, and $\psi_i \equiv P_-\psi^i = (\psi^i)^*$, $\bar\psi^i \equiv \bar\psi^i P_+$, $\bar\psi_i \equiv \bar\psi^i P_-$, where $P_\pm$ are chiral projectors.

The bosonic CSG fields [1] are $\mathcal{G}=\{e^a_m, V^i{}_{jm}, T^{-ij}_{mn}, \zeta, E_{ij}, D^{ij}{}_{kl}\}$, while the fermionic fields are $\{\psi^i_m, \Lambda_i, \chi^{ij}{}_k\}$. In what follows we shall consider only the bosonic CSG background.

Here e^a_m is the vierbein, $V^i{}_{jm}$ is $SU(4)$ gauge field potential, T^{-ij}_{mn} are complex antisymmetric anti-self-dual tensors of dimension 1 transforming in **6** of $SU(4)$ ($T^{-ij}_{mn} = -\frac{1}{2}\varepsilon_{mn}{}^{pq}T^{-ij}_{pq}$) while $(\zeta, E_{ij}, D^{ij}{}_{kl})$ are Lorentz scalars of dimensions 0, 1 and 2 respectively (i.e. they have 4, 2 and 0 derivatives in their kinetic term in CSG action [1,9]). The complex scalars $E_{ij}=E_{ji}$ are in representation **10** of $SU(4)$, while $D^{ij}{}_{kl}$ are in real representation **20** ($D^{ij}{}_{kl} = D_{kl}{}^{ij} = (D^{ij}{}_{kl})^* = \frac{1}{4}\varepsilon^{iji'j'}\varepsilon_{klk'l'}D^{k'l'}{}_{i'j'}$).

In [1] the physical complex scalar ζ is replaced by a doublet of complex scalars ϕ_α with

$$\phi^\alpha\phi_\alpha = \phi_1\phi_1^* - \phi_2\phi_2^* = 1, \qquad \phi^1 = (\phi_1)^*, \qquad \phi^2 = -(\phi_2)^*, \tag{2.2}$$

by adding a local $U(1)$ gauge symmetry. Then ϕ_α transforms under global $SU(1,1)$ as well as local $U(1)$, $\phi'_\alpha = e^{-i\gamma(x)}U_\alpha^\beta\phi_\beta$, i.e. has the

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[6] The full SYM effective action in CSG background contains of course also a finite non-local part, see [14]. While the divergent part will preserve all the classical superconformal symmetries, the finite non-local part will contain non-invariant anomalous terms.

[7] It is thus the same at weak and at strong SYM coupling and can be also found by evaluating the $d=5$ supergravity action on the solution of the corresponding Dirichlet problem (from the cutoff-dependent part of the resulting expression [14]).

[8] The invariance of the divergent part can be seen explicitly if one uses, e.g., dimensional regularization. Let $\Gamma_{\mathrm{reg}} = \frac{1}{n-4}\Gamma_{\mathrm{div}} + \Gamma_{\mathrm{fin}}$ be the regularized effective action. Then under a superconformal transformation $\delta\Gamma_{\mathrm{reg}} = (n-4)\mathcal{A}$, so that $\delta\Gamma_{\mathrm{div}} = 0$ and $\delta\Gamma_{\mathrm{fin}} = \mathcal{A}$ (see, e.g., [17] for details).

[9] This follows, e.g., from the fact that the vector–vector duality may be performed as a change of variables in the path integral (in full analogy with 2d scalar–scalar or T-duality). More precisely, while the logarithmically divergent part of the path integral should be invariant its finite part may contain a local term not invariant under the $SU(1,1)$, similarly to what happens in the 2d case where the dilaton shifts under the T-duality (see [18] and references there).

[10] In principle, they can be reconstructed using the Noether procedure given that the full non-linear supersymmetry transformation rules are known (and close off-shell on CSG fields) [1].

$U(1)$ chiral weight -1.[11] Then only ϕ_α transforms under $SU(1,1)$ but other fields with non-zero chiral weights transform under local $U(1)$, i.e. all fields with derivative couplings and non-zero chiral weights couple to the scalar $U(1)$ connection through the covariant derivative (Ω is the chiral weight)

$$\mathrm{D}_m = \partial_m - i\Omega a_m, \quad a_m = i\phi^\alpha \partial_m \phi_\alpha. \tag{2.3}$$

The scalar connection a_m is invariant under the $SU(1,1)$ and transforms by a gradient under the $U(1)$.

The general form [2] of the $\mathcal{N}=4$ vector multiplet Lagrangian (before $U(1)$ gauge fixing) may be written as [2] $\mathcal{L} = \mathcal{L}_B + \mathcal{L}_F$, with the bosonic part[12]

$$\begin{aligned}\mathcal{L}_B &= \frac{1}{4} i\tau(\phi) F^+_{mn} F^+_{mn} - \frac{1}{4} i\bar\tau(\phi) F^-_{mn} F^-_{mn} \\ &\quad - \left(\frac{1}{\Phi} T^+_{mnij} F^+_{mn} \varphi^{ij} + \frac{1}{\Phi^*} T^{-ij}_{mn} F^-_{mn} \varphi_{ij}\right) \\ &\quad - \frac{1}{2}\left(\frac{\Phi^*}{\Phi} T^+_{mnij} T^+_{mnkl} \varphi^{ij}\varphi^{kl} + \frac{\Phi}{\Phi^*} T^{-ij}_{mn} T^{-kl}_{mn} \varphi_{ij}\varphi_{kl}\right) \\ &\quad - \frac{1}{2} \mathrm{D}_m \varphi^{ij} \mathrm{D}_m \varphi_{ij} \\ &\quad - \frac{1}{12}\left(R + \frac{1}{2} E^{kl} E_{kl} + 2\mathrm{D}_m \phi^\alpha \mathrm{D}_m \phi_\alpha\right) \varphi^{ij}\varphi_{ij} \\ &\quad + \frac{1}{4} D_{ij}{}^{kl} \varphi_{kl} \varphi^{ij},\end{aligned} \tag{2.4}$$

$$\begin{aligned} i\tau(\phi) &\equiv -\frac{\phi_1^* + \phi_2^*}{\phi_1^* - \phi_2^*}, \qquad i\bar\tau(\phi) = \frac{\phi_1 + \phi_2}{\phi_1 - \phi_2}, \\ \Phi(\phi) &\equiv \phi_1^* - \phi_2^*, \qquad \Phi^* = \phi_1 - \phi_2, \end{aligned} \tag{2.5}$$

and the fermionic part

$$\begin{aligned}\mathcal{L}_F &= -\frac{1}{2} \bar\psi^i \not{D} \psi_i - \frac{1}{2} \bar\psi_i \not{D} \psi^i - \frac{1}{4} E_{ij} \bar\psi^i \psi^j - \frac{1}{4} E^{ij} \bar\psi_i \psi_j \\ &\quad + \frac{1}{4} \varepsilon_{iklj} \bar\psi^i \sigma_{mn} T^{-kl}_{mn} \psi^j + \frac{1}{4} \varepsilon^{iklj} \bar\psi_i \sigma_{mn} T^+_{mnkl} \psi_j.\end{aligned} \tag{2.6}$$

In general, the derivative D_m contains the gravitational ∇_m part as well as the $SU(4)$ gauge potential (V_m), in addition to the $U(1)$ term (a_m) in (2.3) (note that the bosonic vector multiplet fields have zero chiral weights while ψ_i has weight $-1/2$).

While the $F_{mn}(A)$ dependent part of the action (2.4) is not invariant under $SU(1,1)$ acting on ϕ_α, it was shown in [2] that the corresponding equations of motion (written in first-order form) are invariant provided one also "duality-rotates" the vector field strength as in the closely related case of the Poincaré supergravity [5].

Our aim will be to integrate over the vector multiplet fields $\{A_m, \varphi_{ij}, \psi_i\}$ in (2.4), (2.6) and compute the divergent part of the resulting effective action. For this we do not need to fix the local $U(1)$ symmetry and may treat the scalar functions $\tau(\phi)$, $\Phi(\phi)$ and a_m as arbitrary background fields. Equivalently, we may choose to fix the spurious local $U(1)$ by a "physical" gauge, e.g., $\phi_1 = \phi_1^*$ [1,2]

$$\phi_1 = \left(1 - \zeta\zeta^*\right)^{-1/2}, \qquad \phi_2 = \zeta\left(1 - \zeta\zeta^*\right)^{-1/2}, \tag{2.7}$$

where the complex scalar ζ (taking values in the disc $|\zeta| \leqslant 1$) is an independent degree of freedom. Then a_m is no longer an invariant of a redefined $SU(1,1)$ acting on ζ (that preserves the gauge condition) but it changes only by a gradient. Explicitly,[13]

$$\begin{aligned} a_m &= i\frac{\zeta\partial_m\zeta^* - \zeta^*\partial_m\zeta}{2(1-\zeta\zeta^*)}, \\ F_{mn}(a) &\equiv \partial_{[m} a_{n]} = i\frac{\partial_{[m}\zeta\partial_{n]}\zeta^*}{(1-\zeta\zeta^*)^2}. \end{aligned} \tag{2.8}$$

Instead of ζ it is useful to use the complex scalar which is directly equal to the scalar–vector coupling $\tau(\phi)$ in (2.4)

$$\tau \equiv \mathcal{C} + ie^{-\sigma} = i\frac{\phi_1^* + \phi_2^*}{\phi_1^* - \phi_2^*} = i\frac{1+\zeta^*}{1-\zeta^*}, \tag{2.9}$$

$$\begin{aligned} a_m &= i\frac{\partial_m(\tau+\bar\tau)}{4\,\mathrm{Im}\,\tau} + \frac{1}{2}\partial_m \ln\frac{\tau+i}{\bar\tau - i}, \\ F_{mn}(a) &= i\partial_{[m}\phi^\alpha\partial_{n]}\phi_\alpha = i\frac{\partial_{[m}\tau\partial_{n]}\bar\tau}{4(\mathrm{Im}\,\tau)^2}. \end{aligned} \tag{2.10}$$

The transformation from ζ to τ in (2.9) maps a unit disc into half-plane, so that τ transforms as $\tau \to \frac{a\tau+b}{c\tau+d}$ under the corresponding $SL(2,R)$ equivalent to original $SU(1,1)$ (see, e.g., [20]). One has in (2.5)

$$i\bar\tau = g^{-2} + i\mathcal{C}, \qquad \Phi\Phi^* = g^2 = (\mathrm{Im}\,\tau)^{-1}, \qquad g \equiv e^{\sigma/2}. \tag{2.11}$$

Note also that[14]

$$\begin{aligned} -4\mathrm{D}_m\partial^\alpha\mathrm{D}_m\partial_\alpha &= 4\frac{\partial_m\zeta\partial_m\zeta^*}{(1-\zeta\zeta^*)^2} \\ &= \frac{\partial_m\tau\partial_m\bar\tau}{(\mathrm{Im}\,\tau)^2} = (\partial_m\sigma)^2 + e^{2\sigma}(\partial_m\mathcal{C})^2. \end{aligned} \tag{2.12}$$

3. Divergent part of $\mathcal{N}=4$ SYM effective action in conformal supergravity background

The UV divergent part of the SYM effective action in the CSG background is related to conformal anomaly and thus should be given to all orders by the 1-loop logarithmically divergent term. To determine the latter one may just consider a single Abelian vector multiplet action (2.4), (2.6) quadratic in $\mathcal{A} = \{A_m, \varphi_{ij}, \psi_i\}$ but keeping full dependence on the (bosonic) background fields $\mathcal{G} = \{e^a_m, V^i_{jm}, T^{-ij}_{mn}, \zeta, E_{ij}, D^{ij}{}_{kl}\}$. As already mentioned, while it is not necessary to fix the $U(1)$ gauge for concreteness we will be expressing all the scalar functions in terms of the complex scalar τ in (2.9)–(2.12).

The 1-loop effective action is given by the contribution of the mixed vector–scalar sector, the vector ghosts and the fermions

$$\Gamma = \frac{1}{2}\ln\mathrm{Det}\,\mathcal{H}_{1,0} - \ln\mathrm{Det}\,\mathcal{H}_{gh} - \frac{1}{2}\ln\mathrm{Det}\,\mathcal{H}_{1/2}, \tag{3.1}$$

where $\mathcal{H}$ are second-order matrix differential operators, depending on the background fields $\mathcal{G}$. Then

$$\Gamma_\infty = -\frac{1}{(4\pi)^2}\ln\Lambda\int d^4x\sqrt{g}\,(a_2)_{\mathcal{N}=4\,\mathrm{tot}}, \tag{3.2}$$

where the diagonal DeWitt–Seeley coefficient a_2 of the generic operator

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[11] Other CSG fields having non-zero chiral weights are: ${T^{-ij}}_{mn}$ (−1); $E_{(ij)}$ (-1); Λ_i ($-\frac{3}{2}$); $\chi_k^{[ij]}$ ($-\frac{1}{2}$); ψ^i_μ ($-\frac{1}{2}$). The Q-susy parameter ϵ_i has weight 1/2.

[12] We use Euclidean signature with imaginary time (fourth) component, with $\varepsilon^{1234} = 1$. For simplicity we shall often ignore trivial metric factors not distinguishing between coordinate and target-space indices (which are always contracted with Euclidean signature metric so we will often not raise them in the contractions). Self-dual parts of 2nd rank tensors are defined as $F^\pm_{mn} = \frac{1}{2}(F_{mn} \pm F^\star_{mn})$, $F^+_{mn} = (F^-_{mn})^*$, $F^\star_{mn} = \frac{1}{2}\varepsilon_{mnpq}F^{pq}$.

[13] In our notation here $A_{[n}B_{m]} = A_nB_m - A_mB_n$.

[14] Here $\mathrm{D}_m\phi_\alpha = (\partial_m + ia_m)\phi_\alpha$, see (2.3). $\mathrm{D}_m\partial^\alpha\mathrm{D}_m\partial_\alpha$ is manifestly $SU(1,1)$ invariant, and thus invariant under the $SL(2,R)$ acting on τ, with $\mathrm{Im}\,\tau \to \frac{1}{(c\tau+d)(c\bar\tau+d)}\mathrm{Im}\,\tau$, $\partial_m\tau \to \frac{1}{(c\tau+d)^2}\partial_m\tau$.

$$\mathcal{H}_{AB} = -1_{AB}\hat{\nabla}^2 + 2h^m_{AB}\hat{\nabla}_m + \Pi_{AB} \tag{3.3}$$

has the following form [16]

$$a_2 = \mathrm{tr}\left[\frac{1}{180}\left(R_{mnrs}R^{mnrs} - R_{mn}R^{mn} + \nabla^2 R\right) + \frac{1}{6}\nabla^2\hat{P} + \frac{1}{2}\hat{P}\cdot\hat{P} + \frac{1}{12}\hat{\mathcal{F}}_{mn}\hat{\mathcal{F}}^{mn}\right],$$

$$\hat{P}_{AB} = \Pi_{AB} - \frac{1}{6}R1_{AB} - \hat{\nabla}_m h^m_{AB} + h_{mAC}h^m_{CB},$$

$$\hat{\mathcal{F}}_{mnAB} = [\hat{\nabla}_m, \hat{\nabla}_n]_{AB} - \hat{\nabla}_{[m}h_{n]AB} + h_{[mAC}h_{n]CB}. \tag{3.4}$$

Here $\hat{\nabla}_m$ is given by the gravitational covariant derivative ∇_m plus possible extra gauge ($SU(4)$ and $U(1)$) field potentials for unmixed fields, while h^m_{AB} accounts for the mixing between different types of fields.

The vector–scalar operator originating from (2.4) may be written as

$$\mathcal{H}_{1,0} = \begin{pmatrix} \mathcal{H}_1 & -2g\overrightarrow{\mathrm{D}_m}\frac{1}{\Phi^*}T^{-kl}_{mn} & -2g\overrightarrow{\mathrm{D}_m}\frac{1}{\Phi}T^{+}_{mnkl} \\ 2T^{+}_{ijnm}\frac{1}{\Phi}\overrightarrow{\mathrm{D}_n}g & \mathcal{H}_0 & \frac{\Phi^*}{\Phi}T^{+}_{ij}\cdot T^{+}_{kl} \\ 2T^{-ij}_{nm}\frac{1}{\Phi^*}\overrightarrow{\mathrm{D}_n}g & \frac{\Phi}{\Phi^*}T^{-ij}\cdot T^{-kl} & \mathcal{H}_0 \end{pmatrix}, \tag{3.5}$$

where $g = e^{\sigma/2}$ is a coupling function (see (2.11)), $\mathrm{D}_m = \nabla_m + ia_m - V_m$ and

$$(\mathcal{H}_0)^{kl}_{ij} = \left(-\mathrm{D}^2 + \frac{1}{6}R + \frac{1}{12}M\right)1^{kl}_{ij} - \frac{1}{2}D_{ij}{}^{kl},$$

$$M = E^{kl}E_{kl} + 4\mathrm{D}_m\phi^\alpha\mathrm{D}_m\phi_\alpha. \tag{3.6}$$

The fermionic operator can be found by squaring the first-order operator in (2.6)

$$-\frac{1}{2}\begin{pmatrix}\bar{\psi}^i & \bar{\psi}_i\end{pmatrix}\begin{pmatrix} \not{D}\delta^j_i P_- & (\frac{1}{2}E_{ij} + \sigma\cdot T^-_{ij})P_+ \\ (\frac{1}{2}E^{ij} + \sigma\cdot T^{+ij})P_- & \not{D}\delta^i_j P_+ \end{pmatrix} \times \begin{pmatrix}\psi_j \\ \psi^j\end{pmatrix}. \tag{3.7}$$

Here $\mathrm{D}_m = \partial_m + \frac{1}{2}\sigma_{ab}\omega^{ab}_m + \frac{i}{2}a_m - V_m$ and $P_\pm$ are chiral projectors.

3.1. Vector–scalar sector

Let us start with the contribution of the vector–scalar sector (in which we will include also the ghost contribution). Ignoring first the vector–scalar mixing due to the T^{-ij}_{mn} background in (2.4) one is to account for the presence of a non-trivial scalar background-dependent factor in the vector kinetic operator $\mathcal{H}_1$. This issue was dealt with already in [19] in the case of a simple vector coupling in the first line of (2.4) and we will follow the same approach here.

Choosing the gauge fixing term as $g^2[\nabla_m(\frac{1}{g^2}A_m)]^2$ where $g = e^{\sigma/2}$ and redefining $A_m \to gA_m$ the vector operator $\mathcal{H}_1$ may be written as (here $\mathcal{C}$ is the real part of τ in (2.9))

$$\mathcal{H}_{1mn} = g_{mn}\left(-\tilde{\nabla}^2 + \Pi\right) + \Pi_{mn}, \tag{3.8}$$

$$\Pi_{mn} = R_{mn} + g^4\nabla_m\frac{1}{g^2}\nabla_n\frac{1}{g^2} - g^2\nabla_m\nabla_n\frac{1}{g^2} + \frac{1}{2}g^4(g_{mn}\nabla_r\mathcal{C}\nabla_r\mathcal{C} - \nabla_m\mathcal{C}\nabla_n\mathcal{C}),$$

$$\Pi = \frac{1}{2}g^2\nabla^2\frac{1}{g^2} - \frac{1}{4}g^4\nabla_m\frac{1}{g^2}\nabla_m\frac{1}{g^2},$$

$$\tilde{\nabla}_m A_n \equiv \nabla_m A_n - \frac{i}{2}g^2\varepsilon_{mn}{}^{rs}\nabla_r\mathcal{C}A_s. \tag{3.9}$$

The corresponding ghost operator is

$$\mathcal{H}_{gh} = -\nabla^2 + \Pi. \tag{3.10}$$

Then in addition to the standard single-vector gravitational contribution to a_2 [21][15]

$$(a_2)_{1\,\mathrm{grav}} = \frac{1}{10}C^2 - \frac{31}{180}E, \tag{3.11}$$

$$C^2 = R^{mnpq}R_{mnpq} - 2R^{mn}R_{mn} + \frac{1}{3}R^2,$$

$$E \equiv R^\star R^\star = R^{mnpq}R_{mnpq} - 4R^{mn}R_{mn} + R^2,$$

$$C^2 - E = 2\left(R^2_{mn} - \frac{1}{3}R^2\right), \tag{3.12}$$

there is also a non-trivial scalar background contribution [19] ($\nabla_m\tau = \partial_m\tau$)

$$\mathrm{S}(\tau) = \frac{1}{4(\mathrm{Im}\,\tau)^2}\left[\mathcal{D}^2\tau\mathcal{D}^2\bar{\tau} - 2\left(R_{mn} - \frac{1}{3}R\right)\nabla_m\tau\nabla_n\bar{\tau}\right] + \frac{1}{48(\mathrm{Im}\,\tau)^4}(\nabla_m\tau\nabla_m\tau\nabla_n\bar{\tau}\nabla_n\bar{\tau} + 2\nabla_m\tau\nabla_m\bar{\tau}\nabla_n\tau\nabla_n\bar{\tau}),$$

$$\mathcal{D}^2\tau \equiv \nabla^2\tau + \frac{i}{\mathrm{Im}\,\tau}\nabla_m\tau\nabla_m\tau,$$

$$\mathcal{D}^2\bar{\tau} \equiv \nabla^2\bar{\tau} - \frac{i}{\mathrm{Im}\,\tau}\nabla_m\bar{\tau}\nabla_m\bar{\tau}. \tag{3.13}$$

The quadratic part of this 4-derivative action is the same as found for the singlet scalar kinetic term in the CSG action [9]. The full non-linear expression (3.13) is invariant under the $SL(2,R)$ acting on the local scalar coupling $\tau = \mathcal{C} + ig^{-2}$ [19] (note, e.g., that $\frac{1}{\mathrm{Im}\,\tau}\mathcal{D}^2\tau \to \frac{c\bar{\tau}+d}{c\tau+d}\frac{1}{\mathrm{Im}\,\tau}\mathcal{D}^2\tau$).

To compute the scalar contribution we need to account for the reality constraints (2.1): we may solve them explicitly[16] or formally do the summation over i, j in (3.5), adding extra 1/2 factor in the final result.

The operator (3.5) has the form (3.3) where

$$1_{AB} = \begin{pmatrix} g_{mn} & 0 & 0 \\ 0 & 1^{kl}_{ij} & 0 \\ 0 & 0 & 1^{ij}_{kl} \end{pmatrix}, \qquad \hat{\nabla}_{mAB} = \begin{pmatrix} \tilde{\nabla}_m & 0 & 0 \\ 0 & \mathrm{D}_m & 0 \\ 0 & 0 & \mathrm{D}_m \end{pmatrix},$$

$$h_{mAB} = \begin{pmatrix} 0 & T^{-kl}_{nm}\frac{g}{\Phi^*} & T^{+}_{nmkl}\frac{g}{\Phi} \\ T^{+}_{ijmr}\frac{g}{\Phi} & 0 & 0 \\ T^{-ij}_{mr}\frac{g}{\Phi^*} & 0 & 0 \end{pmatrix},$$

$$\Pi_{AB} - \frac{1}{6}R1_{AB} = \begin{pmatrix} \Pi_{mn} + g_{mn}(\Pi - \frac{1}{6}R) & -2g\mathrm{D}_r(\frac{1}{\Phi^*}T^{-kl}_{rn}) & -2g\mathrm{D}_r(\frac{1}{\Phi}T^{+}_{rnkl}) \\ 2T^{+}_{ijrm}\frac{1}{\Phi}\nabla_r g & -\frac{1}{2}D_{ij}{}^{kl} + \frac{1}{12}1^{kl}_{ij}M & \frac{\Phi^*}{\Phi}T^{+}_{ij}\cdot T^{+}_{kl} \\ 2T^{-ij}_{rm}\frac{1}{\Phi^*}\nabla_r g & \frac{\Phi}{\Phi^*}T^{-ij}\cdot T^{-kl} & -\frac{1}{2}D^{ij}{}_{kl} + \frac{1}{12}1^{ij}_{kl}M \end{pmatrix}. \tag{3.14}$$

[15] We include the ghost contribution and ignore the scheme-dependent total derivative term $\nabla^2 R$.

[16] A solution to these constraints may be chosen as

$$\varphi_{ij} = \begin{pmatrix} 0 & \varphi_{12} & \varphi_{13} & \varphi_{14} \\ -\varphi_{12} & 0 & -\varphi^*_{14} & \varphi^*_{13} \\ -\varphi_{13} & \varphi^*_{14} & 0 & -\varphi^*_{12} \\ -\varphi_{14} & -\varphi^*_{13} & \varphi^*_{12} & 0 \end{pmatrix},$$

$\partial_m\varphi^{ij}\partial_m\varphi_{ij} = 4(\partial_m\varphi^*_{12}\partial_m\varphi_{12} + \partial_m\varphi^*_{13}\partial_m\varphi_{13} + \partial_m\varphi^*_{14}\partial_m\varphi_{14})$.

Also,

$$\hat{\mathcal{F}}_{rs} = [\hat{\nabla}_r, \hat{\nabla}_s] - \hat{\nabla}_{[r}h_{s]} + h_{[r}h_{s]}$$
$$= \begin{pmatrix} -R_{nmrs} + T^{-kl}_{[n[r}T^{+}_{s]m]kl} & -\tilde{\nabla}_{[r}(T^{-kl}_{ns]}\frac{g}{\Phi^*}) & -\tilde{\nabla}_{[r}(T^{+}_{ns]kl}\frac{g}{\Phi}) \\ -\mathrm{D}_{[r}(T^{+}_{ijs]m}\frac{g}{\Phi}) & F_{rs}(V) & T^{+}_{ij[rt}T^{+}_{klts]}\frac{\Phi^*}{\Phi} \\ -\mathrm{D}_{[r}(T^{-ij}_{s]m}\frac{g}{\Phi^*}) & T^{-ij}_{[rt}T^{-kl}_{ts]}\frac{\Phi}{\Phi^*} & F_{rs}(V) \end{pmatrix}. \tag{3.15}$$

Applying the algorithm in (3.4) to this operator we find the total vector–scalar sector (1 vector, 6 real scalars) contribution to the logarithmic divergence coefficient

$$\begin{aligned}(a_2)_{1,0} = {}& \left(\frac{1}{10} + \frac{6}{120}\right)C^2 - \left(\frac{31}{180} + \frac{6}{360}\right)E + \mathrm{S}(\tau) \\ & + \frac{1}{6}F^2_{mn}(V) + \frac{1}{48}M^2 + \frac{1}{8}D_{ij}{}^{kl}D_{kl}{}^{ij} \\ & + \left(\frac{2}{3} + 2\right)\mathrm{D}_r T^{-kl}_{rm}\mathrm{D}_s T^{+}_{smkl} \\ & + \left(\frac{2}{3} + 1\right)R_{mn}T^{-kl}_{mr}T^{+}_{rnkl} - \frac{\nabla_n\tau\nabla_m\bar{\tau}}{(\mathrm{Im}\,\tau)^2}T^{-ij}_{nr}T^{+}_{rmij} \\ & + T^{-ij}_{ma}T^{+}_{anij}T^{-kl}_{mb}T^{+}_{bnkl} + \frac{2}{3}T^{-ij}_{ma}T^{+}_{anij}T^{-kl}_{mb}T^{+}_{bnkl} \\ & - \frac{1}{3}T^{-ij}_{mn}T^{+}_{abij}T^{-kl}_{mn}T^{+}_{abkl}.\end{aligned} \tag{3.16}$$

Here M and S were defined in (3.6), (3.13).

3.2. Fermionic sector

Let us now determine the fermionic contribution to (3.2). Squaring the operator in (3.7) and putting it into the form (3.3) gives

$$\begin{aligned}\mathcal{H}_{1/2} = {}& -\begin{pmatrix} \delta^k_i P_+ & 0 \\ 0 & \delta^i_k P_- \end{pmatrix}\mathrm{D}^2 \\ & + \begin{pmatrix} \mathcal{R}^k_i + e_{ij}e^{jk} & (\not{\mathrm{D}}e_{ik}) \\ (\not{\mathrm{D}}e^{ik}) & \mathcal{R}^i_k + e^{ij}e_{jk} \end{pmatrix}\begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix} \\ & + 2\begin{pmatrix} 0 & T^{-}_{ikmr}\gamma_r \\ T^{+ik}_{mr}\gamma_r & 0 \end{pmatrix}\begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix}\mathrm{D}_m, \\ \mathcal{R}^k_i \equiv {}& \frac{1}{4}R\delta^k_i - \sigma_{rs}F^{k}_{i\,rs}(V) + \frac{1}{2}\delta^k_i\sigma_{rs}F_{rs}(a), \\ e_{ij} \equiv {}& \frac{1}{2}E_{ij} + \sigma\cdot T^{-}_{ij}.\end{aligned} \tag{3.17}$$

The corresponding matrices $\hat{P}$ and $\hat{\mathcal{F}}$ in (3.4) are

$$\begin{aligned}\hat{P} = {}& \begin{pmatrix} Y^k_i & (\not{\mathrm{D}}e_{ik}) - \mathrm{D}_m T^{-}_{ikmn}\gamma_n \\ (\not{\mathrm{D}}e^{ik}) - \mathrm{D}_m T^{+ik}_{mn}\gamma_n & Y^i_k \end{pmatrix}\begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix}, \\ Y^k_i \equiv {}& \frac{1}{12}R\delta^k_i - \sigma_{rs}F^{k}_{i\,rs}(V) \\ & + \frac{1}{2}\delta^k_i\sigma_{rs}F_{rs}(a) + e_{ij}e^{jk} + T^{-}_{ijrm}T^{+jk}_{ms}\gamma_r\gamma_s,\end{aligned} \tag{3.18}$$

$$\begin{aligned}\hat{\mathcal{F}}_{sr} = {}& \begin{pmatrix} Z^{j}_{i\,sr} & -\mathrm{D}_{[s}T^{-}_{ikr]m}\gamma_m \\ -\mathrm{D}_{[s}T^{+ik}_{r]m}\gamma_m & Z^{j}_{i\,sr} \end{pmatrix}, \\ Z^{j}_{i\,sr} \equiv {}& \frac{1}{2}R_{sr}{}^{mn}\sigma_{mn}\delta^j_i + F^{j}_{i\,sr}(V) \\ & - \frac{1}{2}\delta^j_i F_{sr}(a) + T^{-}_{ik[sm}T^{+kj}_{r]n}\gamma_m\gamma_n.\end{aligned} \tag{3.19}$$

This gives (for the number $n_F = \delta^i_i$ of Weyl fermions)[17]

$$\begin{aligned}\frac{1}{2}\,\mathrm{tr}\,\hat{P}^2 = {}& n_F\left[\frac{1}{72}R^2 - \frac{1}{4}F^2_{mn}(a)\right] - F^2_{mn}(V) \\ & + \frac{1}{12}RE_{ij}E^{ij} + \frac{1}{8}E_{ij}E^{jk}E_{kl}E^{li} \\ & - 2\mathrm{D}_m T^{-}_{klmr}\mathrm{D}_n T^{+kl}_{nr} + \frac{1}{2}\mathrm{D}_r E_{kl}\mathrm{D}_r E^{kl},\end{aligned} \tag{3.20}$$

$$\begin{aligned}\frac{1}{12}\,\mathrm{tr}\,\hat{\mathcal{F}}_{mn}\hat{\mathcal{F}}_{mn} = {}& \frac{1}{12}\Big[n_F F^2_{mn}(a) + 4F^2_{mn}(V) \\ & - \frac{1}{2}n_F R_{srmn}R^{srmn} + 8R_{sr}{}^{mn}T^{-}_{klsm}T^{+kl}_{rn} \\ & + 8\big(2T^{-}_{mrik}T^{+kj}_{rn}T^{-}_{msjl}T^{+li}_{sn} - T^{-}_{mnik}T^{+kj}_{rs}T^{-}_{mnjl}T^{+li}_{rs}\big) \\ & - 8\mathrm{D}_s T^{+ij}_{sm}\mathrm{D}_r T^{-}_{rmij}\Big].\end{aligned} \tag{3.21}$$

Then finally we get for the corresponding a_2 coefficient in (3.4) (here $n_F = 4$ and we include the minus sign in front of the fermionic contribution in (3.1))

$$\begin{aligned}(a_2)_{1/2} = {}& \frac{1}{10}C^2 - \frac{11}{180}E + \frac{1}{3}F^2_{mn}(V) + \frac{1}{3}F^2_{mn}(a) \\ & - \frac{1}{4}\left(\mathrm{D}_m E_{ij}\mathrm{D}_m E^{ij} + \frac{1}{6}RE_{ij}E^{ij}\right) - \frac{1}{16}E_{ij}E^{jk}E_{kl}E^{li} \\ & + \frac{4}{3}\mathrm{D}_m T^{+}_{ijmr}\mathrm{D}_n T^{-ij}_{nr} + \frac{1}{3}R_{mn}T^{-kl}_{mr}T^{+}_{rnkl} \\ & + \frac{1}{3}\big(2T^{-ik}_{mr}T^{+}_{rnkj}T^{-jl}_{mr}T^{+}_{rnli} - T^{-ik}_{mn}T^{+}_{rskj}T^{-jl}_{mn}T^{+}_{rsli}\big).\end{aligned} \tag{3.22}$$

This expression is obviously $SU(1,1)$ invariant.

3.3. Final result

The total $\mathcal{N}=4$ vector multiplet contribution $(a_2)_{\mathcal{N}=4\,\mathrm{tot}}$ is given by the sum of (3.16) and (3.22). It thus starts with $(a_2)_{1,0} + (a_2)_{1/2} = \frac{1}{4}(C^2 - E) + \cdots = \frac{1}{2}(R^2_{mn} - \frac{1}{3}R^2) + \cdots$. The complete expression may be written as

$$(a_2)_{\mathcal{N}=4\,\mathrm{tot}} = \frac{1}{4}\mathcal{L}_{\mathcal{N}=4\,\mathrm{CSG}}, \tag{3.23}$$

$$\begin{aligned}\mathcal{L}_{\mathcal{N}=4\,\mathrm{CSG}} = {}& 2\left[R_{mn} - \frac{1}{4}\frac{\nabla_{(m}\tau\nabla_{n)}\bar{\tau}}{(\mathrm{Im}\,\tau)^2} + 2T^{-ij}_{mr}T^{+}_{rnij}\right]^2 \\ & - \frac{2}{3}\left[R - \frac{\nabla_m\tau\nabla_m\bar{\tau}}{2(\mathrm{Im}\,\tau)^2}\right]^2 + 2F^{i}_{jmn}(V)F^{j}_{i\,mn}(V) \\ & + \frac{1}{(\mathrm{Im}\,\tau)^2}\left|\nabla^2\tau + \frac{i}{\mathrm{Im}\,\tau}\nabla_m\tau\nabla_m\tau\right|^2 \\ & + 16\mathrm{D}_r T^{-ij}_{rm}\mathrm{D}_s T^{+}_{smij} \\ & + \frac{4}{3}\big(2T^{-ik}_{mr}T^{+}_{rnkj}T^{-jl}_{ms}T^{+}_{snli} - T^{-ij}_{mr}T^{+}_{rnij}T^{-kl}_{ms}T^{+}_{snkl}\big) \\ & - \mathrm{D}_r E_{ij}\mathrm{D}_r E^{ij} - \frac{1}{6}\left(R - \frac{\nabla_m\tau\nabla_m\bar{\tau}}{2(\mathrm{Im}\,\tau)^2}\right)E_{ij}E^{ij} \\ & - \frac{1}{6}E_{ij}E^{jk}E_{kl}E^{li} + \frac{1}{2}D_{ij}{}^{kl}D_{kl}{}^{ij}.\end{aligned} \tag{3.24}$$

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[17] Note the following identities

$T^{-ik}_{mn}T^{+}_{kjmn} + T^{-}_{jkmn}T^{+ki}_{mn} = -\frac{1}{2}\delta^i_j T^{-kl}_{mn}T^{+}_{klmn}$, $\quad T^{-}_{ms}T^{+}_{sn} = T^{-}_{ns}T^{+}_{sm}$,

$R_{mnsr}T^{-}_{ms}T^{+}_{nr} = -R_{mn}T^{-}_{ms}T^{+}_{sn}$.

This should represent (up to an overall factor of 1/4, cf. (1.1), (3.2)) the bosonic part of the full $\mathcal{N}=4$ conformal supergravity Lagrangian.

This expression passes several checks. The resulting action (1.2) is Weyl-invariant; in particular, all the fields have the expected Weyl-invariant kinetic terms. Also, the truncation to $\mathcal{N}=2$ theory (when $i, j=1,2$) is consistent with the known non-linear action of $\mathcal{N}=2$ supergravity [1].

The resulting CSG Lagrangian is invariant under the global $SU(1,1)$, supporting the proposal [1] about the existence of the full non-linear $\mathcal{N}=4$ CSG action with such symmetry.

The final expression in (3.24) may be rewritten in the manifestly $SU(1,1)$ invariant form with local $U(1)$ invariance by replacing the $SL(2,R)$ invariants built out of derivatives of τ by the corresponding combinations involving ϕ_α as in (2.10), (2.12), or by using the direct relation between τ and ϕ_α in (2.9) in the gauge (2.7). In particular, for the double-derivative term in (3.13), (3.24) one has $\frac{\mathcal{D}^2\tau\mathcal{D}^2\bar{\tau}}{4(\mathrm{Im}\,\tau)^2}=(\varepsilon^{\alpha\beta}\phi_\alpha \mathrm{D}^2\phi_\beta)(\varepsilon_{\gamma\delta}\phi^\gamma \mathrm{D}^2\phi^\delta)$.

4. Summary

The above computation of divergent term in the $\mathcal{N}=4$ SYM effective action in conformal supergravity background allowed us to find the complete $SU(1,1)$ symmetric action of $\mathcal{N}=4$ conformal supergravity in the bosonic sector. We used that the divergent part of the effective action is local, preserves all the symmetries of the underlying classically superconformal theory and starts with the Weyl tensor squared term.

The fermionic part of the $\mathcal{N}=4$ conformal supergravity action can be found by the same method. Indeed, the $\mathcal{N}=4$ SYM–CSG coupling given in [2] contains all the required fermionic terms. This is still straightforward but technically more involved.

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