Phase and group velocities of bulk optic and acoustic waves in crystals and periodically structured media

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Abstract

We examine propagation and reflection of bulk optic and acoustic waves in media possessing strong anisotropy of physical properties. We also consider unusual wave phenomena taking place in artificial optic and acoustic “double negative” materials as well as in natural crystals demonstrating large birefringence and strong elastic anisotropy. It is shown that in the media, the Poynting vector and the wave vector may be separated by the extremely wide walkoff angles. In modern optic an acoustic crystals, the angles may correspondingly be as wide as 25° and 74°, while in “metamaterials”, the two vectors are antiparallel. Using acousto-optic methods, we observed and examined a few unusual effects of the wave propagation and reflection in the crystals.

Keywords: optic and acoustic waves; crystals; metamaterials; phase and group velocity; walkoff angle

1. Introduction

Recently much interest of scientists has been paid to wave phenomena taking place in media having unusual physical properties. Natural optic and acoustic crystals demonstrating extremely strong anisotropy of their physical properties may be attributed to these media. The media also include the so-called “left-handed” or “metamaterials” predicted by Veselago [1] and then examined by Pendry [2], Burov et al. [3], Veselago [4] and Burov et al. [5].

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According to Veselago [1], in an artificial left-handed electromagnetic material, the dielectric permittivity $\varepsilon$ as well as the magnetic permeability $\mu$ is simultaneously negative. Experimental verification of the prediction carried out by Pendry [2] stimulated research on the “double-negative” media, i.e., artificial structures with characteristic sizes of elements that shorter than the wavelength of electromagnetic radiation.

Maybe one of the most unusual features of plane wave propagation in these artificial media is that the Poynting vector is antiparallel to the wave vector. As proved by Burov et al. [3] and later by Burov et al. [5], this feature of the wave propagation is of general character and typical not only of electromagnetic metamaterials but also of double-negative acoustic media. Since the phase and group velocities of bulk optic and acoustic beams in the artificial media appeared antiparallel, Voloshinov and Polikarpova [6] and also Voloshinov et al. [7] examined other possible mutual orientations of these vectors. In the cited references, the authors evaluated the walkoff angles in the natural strongly anisotropic optic and acoustic crystals such as tellurium dioxide, calomel, mercury bromide, etc. As known, in a propagating bulk and plane wave, the walkoff angles separate in space the two velocity vectors as well as the wave vector and the Poynting vector. The extremely wide walkoff angles, e.g., up to 25°, in optics and wider than 70°, in acoustics, provide in the crystals observation of unusual wave effects. The papers by Voloshinov et al. [8] and Djakonov et al. [9] describe a peculiar back reflection of acoustic waves following glancing incidence on a free boundary separating a crystal and the vacuum. Application by Voloshinov [10] and Voloshinov et al. [11] of acousto-optic methods provided registration of the new effects and experimental confirmation of theoretical predictions. In this paper, we briefly review a few of the new wave phenomena and describe the unusual effects originating from the anisotropy of natural optic and acoustic crystals. Finally, we mention possible applications of the examined effects in acousto-optic and acousto-electronic devices.

2. Double-negative media in electrodynamics and acoustics

In electrodynamical media, according to prediction of Veselago [1], the vectors of electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields are directed with respect to the wave vector $\mathbf{k}$ and the Poynting vector $\mathbf{S}$ in the manner shown in Fig. 1. The left drawing illustrates a natural medium with the positive magnitudes of the dielectric permittivity $\varepsilon$ and the magnetic permeability $\mu$, while the right picture corresponds to a metamaterial with the negative parameters. The directions of the Pointing vector $\mathbf{S} = (c/4\pi) [\mathbf{EH}]$ and the wave vector $\mathbf{k}$ are parallel in the natural medium, however, the vectors are directed opposite in the metamaterial. As a result, optic waves pass a boundary separating two media with the indexes of refraction $n_1$ and $n_2$ in the way shown in Fig. 2. In Fig. 2a, the indexes of refraction $n_1$ and $n_2$ are positive, therefore the angles of light incidence $\alpha$ and refraction $\beta$ are also positive. As for Fig. 2b, the index of refraction is negative $n_2<0$ making negative the angle $\beta$. The unusual direction of the refracted optic beam in Fig. 2b is typical of an artificial metamaterial. It is known that the unusual refraction of light is not the only peculiar effect existing in the metamaterials. According to Veselago [1], Pendry [2] and Burov et al. [3], a few other peculiar effects such as “perfect focusing of light”, “cloaking of objects”, etc., may be observed in the artificial double-negative optic and acoustic media. In the artificial acoustic medium, dynamic density and compliance are negative.

Fig. 1. (a) mutual directions of vectors in natural media; (b) vectors in double-negative media.
3. Bulk waves in natural media characterized by strong optic and acoustic anisotropy

As proved by Voloshinov and Polikarpova [6], Voloshinov et al. [7] and also by Voloshinov [10], the walkoff angles $\psi$ in optics and acoustics may be as wide as dozens of degrees. Data in Fig. 3a illustrate dependence of the optical walkoff angle $\psi$ on relative birefringence $\Delta n/n$ in various optical media. It is seen that the walkoff angles in the single crystals tellurium, mercury chloride and mercury bromide are as wide as $\psi = 15^0 - 20^0$. Moreover, there are natural optic crystals in which $\Delta n/n > 0.5$. It results in even wider optic walkoff angles $\psi > 20^0$. That is why, optic beams propagate in samples fabricated of the crystals so as shown in Figs. 3b and 3c. It should be pointed out that the phase velocities of the incident optic beam (shown in Fig. 3b by solid curve) and the refracted beam (shown by dash line) are directed according to the Snell law. The unusual propagation of the refracted beam originates from the optic anisotropy and the oblique orientation of the group velocity of light. The picture proves that the energy flow propagates at the walkoff angle $\psi > \theta$ relatively to the optic phase velocity. As for data in Fig 3c, the phase velocity of the optic beam (dash line) is directed with respect to the bottom facet and its normal $\mathbf{m}$ at the angle $\theta$ exceeding $90^0$, while the energy flow (solid line) propagates to the crystal bottom due to the walkoff effect.

In acoustics, the anisotropy manifests itself even in a more sounding manner so that the acoustic walkoff angles exceed the optic angles to a factor of 2.5. As an example, data in Fig. 4 illustrate cross-section of acoustic slowness surface in (001) plane of a tellurium dioxide crystal. It is seen that the material is extremely anisotropic in case of the slow shear acoustic mode because the cross-section of the surface looks like a flower with long “petals”. As found, phase velocity of the slow shear acoustic wave in paratellurite varies with direction of propagation from $V = 616$ m/s to $V = 3050$ m/s. It means that this acoustic mode is characterized by the extremely wide angles separating the phase and group velocities. Calculations and experiments prove that the maximal walkoff angle in the (001) plane of the crystal is equal to $\psi = 74^0$. Therefore, it is reasonable to expect existence, in this natural the medium, of new and peculiar wave effects difficult for observation in optical media.
4. Unusual wave phenomena in natural media with strong elastic anisotropy

One of the unusual acoustic wave effects is shown in Fig. 4b. The effect was observed in tellurium dioxide crystal and later defined as “back reflection” of acoustic wave at glancing incidence on a free boundary separating a crystal and the vacuum. The incident wave in Fig. 4b is marked as wave (1), while the two reflected waves we number (2) and (3). The corresponding walkoff angles ψ₁, ψ₂, and ψ₃, the phase velocities V₁, V₂, and V₃, as well as the group velocities Vₙ₁, Vₙ₂, and Vₙ₃, may be seen in the picture. In the figure, the reflected wave (3) propagates practically backwards relatively to the incident wave (1). The research carried out by Voloshinov et al. (2009) proved that the angle Δθ separating in space the energy flows of the incident (1) and the back reflected wave (3) is very narrow, e.g., about Δθ = 6°. Moreover, the “boomerang” reflection of the wave has one more unique characteristic feature. The reflection coefficient of the wave (3) may be as high as 100%. It means that the wave (2), reflected according to expectations, is practically absent. There are a few other wave effects originating from the strong elastic anisotropy of natural crystals. Acoustic reflections at two Brewster angles and at a total reflection angle, propagation of energy flows, after glancing incidence, of longitudinal and shear waves along one and the same direction orthogonal to a reflecting boundary, compression of acoustic beams, etc., may be mentioned in this context. As proved by Molchanov et al. [12] and Voloshinov et al. [13], a few unusual acoustic wave effects have been successfully applied in development of novel acousto-optic and acousto-electronic instruments.

5. Possible directions of phase and group velocity vectors in crystals, metamaterials and periodically structured media

Based on the carried analysis of bulk wave propagation in crystals, metamaterials and periodically structured media, we propose the following description of orientation of group velocity vector with respect to corresponding phase velocity vector. Figure 5 presents a diagram showing the directions of phase and group velocity vectors in optic and acoustic crystals, as well as in metamaterials and periodically structured media. In the diagram, we plotted the phase velocity vector $V_{ph}$ along the arbitrary axis $X$, while directions of the group velocity vectors with respect to the phase velocity vector are shown by arrows directed clock-wise and counter clock-wise. Angles of inclination of the arrows correspond to the walkoff angles ψ that may be registered in the examined natural and artificial media.

As mentioned in the first section of the paper, in optics, the maximal walkoff angle between the wave vector and the Poynting vector in the single crystal calomel (Hg₂Cl₂) is equal to $ψ = 16°$. The walkoff angle in mercury bromide (Hg₂Br₂) is even wider $ψ = 20°$. However, one of the widest walkoff angles so far registered in a birefringent crystal is as wide as $ψ = 25°$. This angle was measured in the double-axis optic material stibium-sulur-iodide (SbSI). These walkoff angles, as well as walkoff angles in all known so far natural optic crystals are included in the limits $-25° ≤ ψ ≤ +25°$ schematically shown in Fig. 5.
The maximum acoustic walkoff angle $\psi = 74^\circ$ was registered in the single crystal tellurium dioxide. Therefore, all known so far acoustic crystals may be represented in the diagram by the dashed area limited by the angles $-74^0 \leq \psi \leq +74^0$. As for the metamaterials predicted by Veselago [1], they are described in the diagram by the angle $\psi = 180^0$. The directions at $\psi = \pm 90^0$ seem to be restricted for bulk plane waves. This conclusion is confirmed by the relation between the phase and group velocities of waves $V_{ph} = V_{gr} \cos \psi$. It is evident that at $\psi = \pm 90^0$, either $V_{ph} = 0$ or $V_{gr} = \infty$ thus indicating that most likely these bulk plane waves do not exist in nature. Finally, the dashed square included in the limits $90^0 < \psi < 270^0$ may be attributed to the artificial metamaterials demonstrating energy walkoff.

6. Conclusion

The carried our research shows that the left-handed electrodynamic media have negative dielectric permittivity and magnetic permeability while the left-handed acoustic media possess negative dynamic density and compliance. Strong anisotropy of optic and elastic properties in natural crystals may result in unusual propagation and reflection in them of bulk and plane waves. In general, phase and group velocities of waves in crystals, metamaterials and structured media may be separated in space by arbitrary walkoff angles, except the angles $+90^0$ and $-90^0$.

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