FrFT-CSWSF: Estimating cross-range velocities of ground moving targets using multistatic synthetic aperture radar

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Abstract Estimating cross-range velocity is a challenging task for space-borne synthetic aperture radar (SAR), which is important for ground moving target indication (GMTI). Because the velocity of a target is very small compared with that of the satellite, it is difficult to correctly estimate it using a conventional monostatic platform algorithm. To overcome this problem, a novel method employing multistatic SAR is presented in this letter. The proposed hybrid method, which is based on an extended space-time model (ESTIM) of the azimuth signal, has two steps: first, a set of finite impulse response (FIR) filter banks based on a fractional Fourier transform (FrFT) is used to separate multiple targets within a range gate; second, a cross-correlation spectrum weighted subspace fitting (CSWSF) algorithm is applied to each of the separated signals in order to estimate their respective parameters. As verified through computer simulation with the constellations of Cartwheel, Pendulum and Helix, this proposed time-frequency-subspace method effectively improves the estimation precision of the cross-range velocities of multiple targets.

1. Introduction
Space-borne multistatic synthetic aperture radar (SAR) consists of several small, cooperating satellites (a “constellation”) in which electromagnetic waves transmitted by one satellite are received by the others as coherent echoes. Three typical multistatic SAR constellation types are the Cartwheel, the Pendulum and the Helix, all of which can have along-track, across-track, and/or vertical baselines that are not limited to linear arrays. Multistatic SAR is more powerful than monostatic SAR in a wide range of applications including high resolution wide-swath and 3/4/5-D imaging as well as ground moving target indication (GMTI). In particular, multistatic SAR is excellent at estimating the motion parameters of moving targets—an important step in GMTI. One such motion parameter is velocity, which is usually represented using two orthogonal projections: range/radial velocity and cross-range velocity (also called along-track velocity). Some of the GMTI literature has focused more closely on obtaining the range/radial velocity of a moving target, as disregarding cross-range velocity has...
no impact on extracting a moving target from its topographic background. However, this results in a defocusing of the target within the image, as there is a difference between the Doppler chirp rate of a moving target and that of static scatter. In addition, indicating the cross-range velocity with as much accuracy as the other motion parameters more fully completes the GMTI process (i.e. on an SAR image, there is no reason to indicate only the range/radial velocity while ignoring the cross-range velocity). Thus, the cross-range velocity is also computed in other GMTI work, although this is a challenging task. For example, Baumgartner and Krieger obtained the cross-range velocity by searching for a matching Doppler slope that maximized the output for the target via matched filtering, while Dragosevic et al. attempted to estimate the cross-range velocity by means of a fractional Fourier transform (FrFT), although doing so with sufficient precision proved untenable, as the true echo is a finite length digital signal, which limits parameter resolution. Further efforts to develop conventionally effective methods using single-channel and multi-channel airborne SAR have also failed to achieve the required precision, as the speed of a space-borne platform is much higher than that of an aircraft. In light of these only partially successful efforts, it is necessary to attempt new approaches for improving precision.

Based on the suggestion of Krim and Viberg, we propose a multi-step procedure for developing a time-frequency-subspace method. The paper is organized as follows: First, in Section 2, we derive a novel signal form of the SAR echo for space-borne multistatic platforms in the form of an extended space-time model (ESTIM) by exploiting the space-time properties of the azimuth signal. Then we in Section 3.1 design a set of FrFT-FIR filter banks for separating multiple moving targets within a range gate by utilizing the advantages of the FrFT, which has perfect time-frequency aggregation for signals with a second-order polynomial phase (such as those used in ESTIM). Next, in Section 3.2, we develop a cross-correlation-spectrum weighted subspace fitting (CSWSF) algorithm to estimate the cross-range velocities of respective separated targets. Finally in Section 4, we assess the effectiveness of the proposed method through computer simulation with the constellation of Cartwheel, Pendulum and Helix, and demonstrate that it performs better than conventional FrFT methods.

2. Multistatic SAR moving target formula

The scene of spaceborne multistatic SAR surveying ground moving targets is shown in Fig. 1, where the corresponding scales of satellites and moving targets are much larger than the real for description convenience. The x, y, and z axes represent the along-track, cross-track, and vertical directions, respectively, and constitute a left-handed coordinate system. The multistatic SAR system described here consists of a transmitter satellite denoted as Sat0 (in side-looking stripmap mode) and N receiver satellites (denoted as Satn, n = 0, 1, . . . , N) flying along the x axis at speed V. At the acquisition time of the mth pulse tm = m/PRF (PRF is the pulse repetition frequency, m is an integer, m = 1, 2, . . . , M), the coordinates of the nth radar transceiver are (Bx,n + Vtm, By,n, H + Bz,n), where H denotes the flight altitude of Sat0, and Bx,n, By,n, and Bz,n are, respectively, the along-track (also azimuth direction), cross-track (also range direction), and vertical baselines of the nth satellite. In particular, for the transmitter, Bx,0 = By,0 = H = 0. During the duration of the survey, there is a moving target on flat ground with velocity v = [vx, vy], where vx represents cross-range velocity and vy denotes range velocity, an initial location at t0 = 0 of (x0, y0, 0), and a shortest range gate slant range to the transmitter of R0. In Fig. 1, θ and φ are respectively the elevation and azimuth angles of the moving target, that θ = arctan Vy / Vx, φ = arctan y0 / x0.

2.1. Extended space-time model

The azimuth signal of a SAR is usually approximated with a linear frequency modulation (LFM) signal; this method, however, is not optimal for estimating multistatic SAR parameters, and we will correspondingly derive the azimuth signal in an extended form. The azimuth signal of a moving target at the nth receiver satellite is

\[ s_a(n, t_m) = \exp \left[-\frac{4\pi R_a(t_m)}{\lambda} \right] \]

\[ = \exp \left[-\frac{2\pi}{\lambda} \left( r_0(n) + r_a(t_m) \right) \right] \]

where r_a(t_m) is the range between the target and Sat0, r_a(t_m) is the range between the target and Satw, R_a(t_m) = \frac{1}{2}[r_0(t_m) + r_a(t_m)], and \lambda is the wave length. The scattering coefficient is neglected for simplicity, and

\[ r_a(t_m) = \left( V_x t_m + V_y t_m - B_x n - V t_m \right)^2 + \left( V_y t_m - B_y n \right)^2 + \left( H + B_z n \right)^2 \right]^{\frac{1}{2}} \]

\[ = \left| R_0 - 2 \left( V_x t_m - v_x t_m + B_x n \right) R_0 \cos \theta \cos \phi \right| \]

\[ + \left( V_y t_m - v_y t_m + B_y n \right)^2 + \left( V_x t_m - B_x n \right)^2 \]

\[ + 2 \left( v_x t_m - B_x n \right) R_0 \cos \phi \sin \phi + 2 \left( B_y n R_0 \sin \theta + B_z n \right) \]
The right side of Eq. (2) is expanded as
\[
\begin{align*}
  r_n(t_m) &\approx R_0 - (V_{lm} - v_y t_m + B_{a,n}) \cos \theta \cos \varphi \\
  &\quad + \frac{1}{2R_0} (V_{lm} - v_y t_m + B_{a,n})^2 + (v_y t_m - B_{a,n}) \cos \theta \sin \varphi \\
  &\quad + \frac{1}{2R_0} (v_y t_m - B_{a,n})^2 + B_{a,n} \sin \theta + \frac{B_{az,n}^2}{2R_0}.
\end{align*}
\]

(3)

Using the four-element second-order Taylor expansion of Eq. (3) at \((B_{a,n}, B_{2,n}, B_{3,n}, t_m) = (0, 0, 0, 0)\), Eq. (1) becomes
\[
s_n(n, t_m) = \exp \left( -j \frac{\pi f_s}{R_0} n \right) \\
\times \exp \left[ j \pi \left( f_{sp} B_{a,n} + f_{sp} B_{1,n} + f_{sp} B_{2,n} - \frac{B_{a,n}^2}{2R_0} \right) \right] \\
\times \exp \left( -j \pi \left( \frac{k_0^2}{2} - f_{sp} t_m \right) \right) \exp \left( -j \pi \frac{\pi}{R_0} \right)
\]
\[
(4)
\]

where \(B_0 = \sqrt{B_{a,n}^2 + B_{1,n}^2 + B_{2,n}^2}\), the semi-chirp rate is \(k = \frac{1}{2R_0} \left( (V - v_y)^2 + v_y^2 \right)\); the azimuth, range, and elevation space frequencies are respectively
\[
\begin{align*}
  f_{sp} &= \frac{2 \nu_0}{\lambda} = \frac{2 \nu_0}{\lambda} \cos \theta \cos \varphi \\
  f_{sp} &= \frac{2 \nu_0}{\lambda} = \frac{2 \nu_0}{\lambda} \cos \theta \sin \varphi \\
  f_{sp} &= -\frac{2H}{\lambda} = -\frac{2H}{\lambda} \sin \theta
\end{align*}
\]
\[
(5)
\]

The central Doppler centre frequency is
\[
\begin{align*}
  f_{dc} &= \frac{2}{\lambda} \left( (V - v_y) \nu_0 - v_y \frac{\nu_0}{\lambda} \right) \\
  &= \frac{2}{\lambda} \left[ (V - v_y) \cos \theta \cos \varphi - v_y \cos \theta \sin \varphi \right]
\end{align*}
\]
\[
(6)
\]

the space-time coupling frequency is
\[
\begin{align*}
  f_{st} &= \frac{1}{2R_0} \left( B_{a,n} (V - v_y) - B_{r,n} v_y \right)
\end{align*}
\]
\[
(7)
\]

Since there are time and space frequencies in Eq. (4), and when \(B_{a,n} = B_{2,n} = B_{3,n} = 0\), Eq. (4) degenerates to conventional azimuth signal form of SAR, we call Eq. (4) the ESTIM.

2.2. Error analysis

Since the residual of the second Taylor expansion in the foregoing derivation is equal to zero, the error in the ESTIM totally arises from the expansion residual (denoted as \(\delta R\)) in Eq. (3). It is obvious that the larger the weight of \(R_0\) is in Eq. (2), the smaller \(\delta R\) is. It is useful to distinguish among the following cases.

(1) The constellation is a linear array, where \(B_{r,n} = B_{a,n} = 0\), \(B_{2,n} \neq 0\). In this case, Eq. (2) becomes
\[
r_n(t_m) = \sqrt{\left( x_0 + v_t t_m - B_{2,n} - V_{lm} \right)^2 + \left( y_0 + v_t t_m \right)^2 + H^2} \\
  = R_0 \sqrt{1 + \zeta}
\]
\[
(8)
\]

where
\[
\zeta = \frac{2}{R_0} \left[ v_y t_m \cos \theta \sin \varphi - (V_{lm} - v_y t_m + B_{a,n}) \cos \theta \cos \varphi \right] \\
+ \frac{(V_{lm} - v_y t_m + B_{a,n})^2 + (v_y t_m)^2}{R_0}
\]
\[
(9)
\]

In spaceborne SAR, \(V_{lm}, v_y t_m, B_{a,n}, v_y t_m \ll R_0\), (particularly, even in the large along-track baseline SAR system where \(B_{a,n}\) is in the order of 20 km, it is still far smaller than \(R_0\) which is in the order of 500 km) therefore
\[
\zeta \approx \frac{2}{R_0} \left[ v_y t_m \cos \theta \sin \varphi - (V_{lm} - v_y t_m + B_{a,n}) \cos \theta \cos \varphi \right]
\]
\[
(10)
\]

Consequently
\[
\delta R = \left[ \left( 1 \right)^n - \frac{1}{2^n} \sum_{n=0}^{n} \frac{2^n}{2^n} + \frac{1}{2^n} \left( 1 + \rho^2 \right)^{n+1} \right] R_0 \left| n=1 \right|
\]
\[
= -\frac{1}{8} \frac{\zeta^2 R_0}{1 + \rho^2}
\]
\[
(11)
\]

where \(0 < \rho < 1\). Since \(|\zeta| \ll 1\), we have
\[
\delta R > -\frac{1}{8} \frac{\zeta^2 R_0}{1 + \rho^2}
\]
\[
\approx -\frac{1}{2R_0} \left[ v_y t_m \cos \theta \sin \varphi \right] \\
- (V_{lm} - v_y t_m + B_{a,n}) \cos \theta \cos \varphi)^2 \\
\approx \left[ O(10^{-3}) \right] m, O(10^{-2}) m
\]
\[
(12)
\]

Therefore \(\delta R \approx O(10^{-3}) m-O(10^{-2}) m \approx 0\) for any appropriate value of \(B_{a,n}\) (as long as the echoes of the satellites keep coherent and the satellites will not collide each other, i.e. 100 m < \(B_{a,n} < B_{2,n}\), where \(B_2\) is the critical baseline). As a result, the error in the ESTIM is negligibly.

(2) The constellation is a planar array, where \(B_{a,n} \neq 0, B_{r,n} \neq 0, B_{2,n} = 0\). In this case, Eq. (2) becomes
\[
r_n(t_m) = \sqrt{\left( x_0 + v_t t_m - B_{a,n} - V_{lm} \right)^2 + \left( y_0 + v_t t_m - B_{r,n} \right)^2 + H^2} \\
  = R_0 \sqrt{1 + \zeta}
\]
\[
(13)
\]

where
\[
\zeta = \frac{(V_{lm} - v_y t_m + B_{a,n})^2 + (v_y t_m - B_{r,n})^2}{R_0^2} \\
+ \frac{2}{R_0} \left[ v_y t_m \cos \theta \sin \varphi - (V_{lm} - v_y t_m + B_{a,n}) \cos \theta \cos \varphi \right] \\
- (V_{lm} - v_y t_m + B_{a,n}) \cos \theta \cos \varphi \\
\approx 2 \left[ \left( v_y t_m - B_{a,n} \right) \cos \theta \cos \varphi \right]
\]
\[
(14)
\]

From (1), we get \(-\frac{1}{2} \zeta^2 R_0 \ll \delta R \ll \frac{1}{4} \zeta^2 R_0\). Herein \(\zeta^2 R_0 = O(10^{-2}) m-O(10^{-3}) m\), then \(\delta R \approx O(10^{-3}) m-O(10^{-2}) m\). In practice, \(B_{a,n}\) is in the order of 200–1000 m, therefore the error in the ESTIM is very small in most of conditions in this case.

(3) The constellation is a 3D array, where none of the baselines are equal to zero. In this case,
\[ \xi = \frac{(V_{fm} - v_c t_m + B_{sa})^2 + (v_{f1} t_m - B_{sa})^2 + B_{sa}^2}{R_0^2} \]
\[ + \frac{2}{R_0^2} \left[(v_{f1} t_m - B_{sa}) \cos \theta \sin \varphi \right. \]
\[ - (V_{fm} - v_c t_m + B_{sa}) \cos \theta \cos \varphi + B_{sa} \sin \theta \right] \]
\[ \approx \frac{2}{R_0^2} \left[(v_{f1} t_m - B_{sa}) \cos \theta \sin \varphi \right. \]
\[ - (V_{fm} - v_c t_m + B_{sa}) \cos \theta \cos \varphi + B_{sa} \sin \theta \right] \quad (13) \]

In practice, \( B_{sa} \) is in the order of 10–1000 m, thus \( \xi^2 \approx O(10^{-2}) \) m–O(2 \times 10^3) m, then \( \delta R \approx O(10^{-2}) \) m–O(10^3) m. It is obvious that for many conditions in this case, \( \delta R \) cannot be neglected, therefore the error in the ESTIM should be controlled. The constellation geometry of the multistatic SAR slowly changes during the survey, thus the baselines can be regarded as time-invariant within this duration. Furthermore, the minimum of the error of ESTIM should be taken into consideration during the design of the satellite orbits for the constellation. The examples of the constellations are given in Section 4.

3. Parameter estimation via FrFT-CSWSF

3.1. FrFT-FIR filter banks

Since the ESTIM is an extended form of an LFM signal, it has a second-order polynomial phase, as represented by the third and fourth exponential terms in Eq. (4). For multiple targets in a range gate, the azimuth signal is a second-order multicomponent signal, which degrades the performance of the dechirp method (see the next subsection) by producing cross terms. This problem motivates us to introduce a method for separating the multiple components within the ESTIM-form azimuth signal.

As the azimuth signal has a large time-bandwidth product, multiple components are aliased within both the time and frequency domains, which makes them almost impossible to separate. One feasible solution is to transform the signals into the fractional Fourier domain, in which the multiple components can be aggregated into delta or sinc functions whose peaks are rarely aliased.

Although a filter that could separate these peaks without changing the phase would be ideal, a set of FIR filter banks with linear phases are also acceptable for such processing, as each component can be transformed back into the time domain with only a fixed time delay in its phase using an inverse FrFT.

3.1.1. Implementation of FrFT

We rewrite Eq. (4) as follows:

\[ s_a(n, t_m) = \exp \left\{ -j \pi \left[ 2 \left( f_{0a} - f_{0b} \right) t_m \right. \right. \]
\[ \left. + \left. \frac{4 R_0}{\lambda} \right] \right\} \quad (14) \]

For each satellite, this form is an LFM signal. According to work in Ref. 11, the FrFT of \( s_a(n, t_m) \) with angle \( \alpha \) is defined by

\[ S_a(n, u) = \left\{ \begin{array}{ll}
\frac{1}{2 \pi} \exp \left( \frac{j \pi}{2} \cot \alpha \right) \\
\times \int_{-\infty}^{\infty} s_a(n, t_m) \exp \left( \frac{j \pi}{2} \cot \alpha \right) e^{j \nu t_m \cos \varphi + j \mu t_m \sin \varphi} dt_m
\end{array} \right. \]
\[ \neq k \pi \]
\[ s_a(n, t_m) \]
\[ s_a(n, -t_m) \]
\[ \alpha = \pm 1, \pm 2, \ldots \]

If we choose the transformation kernel \( K_a(t_m, u) \) as

\[ K_a(t_m, u) = \left\{ \begin{array}{ll}
\frac{1}{2 \pi} \exp \left( \frac{j \pi}{2} \cot \alpha \right) \\
\times \int_{-\infty}^{\infty} s_a(n, t_m) \exp \left( \frac{j \pi}{2} \cot \alpha \right) e^{j \nu t_m \cos \varphi + j \mu t_m \sin \varphi} dt_m
\end{array} \right. \]
\[ \neq k \pi \]
\[ \alpha = \pm 1, \pm 2, \ldots \]

then we have \( S_a(n, u) = \int_{-\infty}^{\infty} s_a(n, t_m) K_a(t_m, u) dt_m \).

The value of \( S_a(n, u) \) is

\[ S_a(n, u) = e^{s_a(n, u) T \sqrt{1 - j \cot \alpha} \exp \left( j \frac{\pi}{2} \csc \alpha \cot \alpha \right) \cdot \text{sinc} \left( T \csc \alpha \right)} \]
\[ + f_{0a} - f_{0b} \]
\[ \frac{1}{M} \sum_{m=1/2}^{M/2} \left\{ \text{sinc} \left( \frac{T \csc \alpha}{M} \right) \cdot \text{sinc} \left( \frac{T \csc \alpha}{M} \right) \right\} \quad (17) \]

where \( T = M/\text{PRF} \). When \( \alpha = \text{arccot}(2k/\lambda) \), \( u = (-f_{0a} - f_{0b}) \sin \alpha \), \( S_a(n, u) \) reaches the maximum value.

An important property of FrFT is linear operator. Suppose that the FrFT operator is designated by \( F(t) \), we get

\[ F[p(f(t) + q(t))] = F[p(f(t)) + q(F(f(t))) \quad (18) \]

According to Eqs. (17) and (18), multiple LFM signals can be separated in the fractional Fourier domain, and then respectively transformed back to original time domain.

In practice, the azimuth signal is in the discrete form, and the calculation of FrFT on computer is in fact the discrete FrFT, i.e. DFRFT. According to algorithm in Ref. 10, the corresponding discrete form of Eq. (17) is

\[ S_a(n, m) = \left\{ \begin{array}{ll}
\frac{1}{M} \sum_{m=1/2}^{M/2} \left\{ \text{sinc} \left( \frac{T \csc \alpha}{M} \right) \cdot \text{sinc} \left( \frac{T \csc \alpha}{M} \right) \right\} \right\}
\]
\[ (-f_{0a} + f_{0b})/\text{PRF} \]
\[ \left\{ \text{sinc} \left( \frac{T \csc \alpha}{M} \right) \cdot \text{sinc} \left( \frac{T \csc \alpha}{M} \right) \right\} \right\} \quad (19) \]

When \( \alpha = \text{arccot}(2k/\lambda/\text{PRF}^2) \), \( m = (-f_{0a} + f_{0b}) M/\text{PRF} \). \( S_a(n, m) \) reaches the peak.

As a discrete LFM signal with fixed length, the resolution of the FrFT spectrum is limited. In order to find the precise position of the peak, herein we employ a zoom DFRFT algorithm.

In order to compute the \( L \) uniformly spaced samples of the transformed function \( S_a(n, t_m) \) on the interval \( [u_1, u_2] \) for arbitrary values of the parameters \( L, u_1 \) and \( u_2 \), we substitute \( u = u_0 + l \Delta u \) in Eq. (17), where \( -L/2 \leq l \leq L/2 \), \( u_0 = (u_1 + u_2)/2 \), and \( \Delta u = (u_2 - u_1)/(L - 1) \). Then we get

\[ S_a(n, u_0 + l \Delta u) = A_{2 \text{PRF}} \exp \left( j \pi \cot \alpha (u_0 + l \Delta u) \right) \exp \left( -j \pi \frac{\csc \alpha \Delta \zeta}{2 \text{PRF}} \left( \Delta \zeta l \right) \right) \]
\[ \frac{1}{M} \sum_{m=1/2}^{M/2} \left\{ \exp \left[ j \pi \frac{\text{PRF} (l - m)^2}{2 \text{PRF}^2} \right] \right\} \]
\[ \times \left\{ \text{sinc} \left( \right) \right\} \quad (20) \]
where $A_0 = \sqrt{\frac{\lambda_{\text{los}}}{4\pi}}$. Using Eq. (20), the calculation of FrFT is fast and the precise position of the peak can be found with this “zooming-in” ability. See the work in Ref. [17] for more detail.

Since the multiple targets in a range gate can be separated in the fractional Fourier domain, the next step is to respectively return each fractional Fourier peak to the time domain. A feasible approach to achieving this is the FIR filter banks with narrow bands.

### 3.1.2. Filter design

Because a finite impulse response (FIR) filter has a linear phase, and inherent stability, it is widely used in many digital signal processing applications. Furthermore, it has far less impact on the coefficients of first and second orders than infinite impulse response (IIR) does. Therefore, we employ a set of FIR filter banks to separate multiple azimuth signals.

Generally, the design of an FIR filter using a window function method is an iterative process; that is, the parameters of the window function must be repeatedly redesigned until the impulse response is sufficiently sharpened. However, an improved design for avoiding the use of iteration proposed by Chaturvedi et al. [18] is incorporated into our sub-band FrFT-FIR filter design.

In a Kaiser window, the side lobe level can be controlled by adjusting the length of the filter. The Kaiser window function is given by

$$
\omega_k(m_k) = \begin{cases} 
I_0(\beta) & |m_k| \leq \frac{M - 1}{2} \\
0 & \text{Otherwise}
\end{cases}
$$

where $\gamma$ is an independent variable determined by Kaiser. The parameter $\beta$ is formed as

$$
\beta = \gamma \left[ 1 - \left( \frac{2m_k}{M - 1} \right)^2 \right]^{1/2}
$$

$I_0(x)$ is the modified Bessel function of the first kind:

$$
I_0(x) = 1 + \sum_{k=1}^{\infty} \left[ \frac{1}{2^k k!} \right]^2
$$

The overall filtering process for $Q$ moving targets in a range gate is shown in the flow chart in Fig. 2.

As is shown in Fig. 2, first, using zoom-DFrFT (in the order $z$), the azimuth signals of the $Q$ moving targets in time domain are simultaneously transformed into the fractional Fourier domain, where the signals become $Q$ separated peaks. Then, all the peaks pass a set of Kaiser window FIR filter banks, where each peak passes a sub-band filter with the others filtered out, and the output of each sub-band filter is $S_{q0}^{N0} (q = 1, 2, \ldots, Q)$. Finally, the outputs of the filter banks are respectively transformed back into the time domain, using the inverse FrFT which actually can be represented by zoom DFrFT in a fractional order $(-z)$ opposite to that of the foregoing zoom DFrFT. The final outputs are separated azimuth signals in time domain.

### 3.2. CSWSF

In the ESTIM, the time-space coupling term $\tilde{f}_{0a}E_0$ makes the space steering element, i.e. the final exponential term in Eq. (4), time-variant, which undermines the foundation of subspace theory. To solve this problem, we introduce a subsection dechirp algorithm in which the azimuth signal is divided into three segments of length $L$: 0–$\tau$, $\tau$–2$\tau$, and 2$\tau$–3$\tau$. These are then multiplied as follows:

$$
y_a(n, t_m) = x_a(n, t_m + \tau)x_a^\ast(n, t_m)
$$

$$
= \left[ s_a(n, t_m + \tau) + w(n, t_m + \tau) \right]
\times \left[ s_a(n, t_m) + w(n, t_m) \right]^* 
\exp \left\{ -j2\pi f_{bs1} + \left[ -j2\pi(2k t_m + \dot{k}t - f_{dc}) \right] \right\} 
+ w_j(n, t_m) 
$$

$$
z_a(n, t_m) = x_a(n, t_m + 2\tau)x_a^\ast(n, t_m + \tau)
$$

$$
= \left[ s_a(n, t_m + 2\tau) + w(n, t_m + 2\tau) \right]
\times \left[ s_a(n, t_m + \tau) + w(n, t_m + \tau) \right]^* 
\exp \left\{ -j2\pi f_{bs1} + \left[ -j2\pi(2k t_m + 3\dot{k}t - f_{dc}) \right] \right\} 
+ w_j(n, t_m)
$$

where the superscript $^\ast$ denotes the conjugation operator, and $w(n, t_m)$ represents additional white Gaussian noise. There are no time-space coupling terms in either Eq. (24) or Eq. (25). Both $w_j(n, t_m)$ and $w_j(n, t_m)$ can both be regarded as colored noise. To accomplish parameter estimation of a signal with added colored noise, we introduce a method called CSWSF, wherein “WSF” is one of the improved subspace algorithms, whose performance overweights conventional subspace algorithms [19], and “CS” implies that the proposed method is based on the cross correlation matrix:

$$
R_{yz} = E[Y(j)Z^H(j)] = \begin{pmatrix}
    r_{y_0}(0) & r_{y_1}(0) & \cdots & r_{y_0}(-N+1) \\
    r_{y_1}(1) & r_{y_1}(0) & \cdots & r_{y_1}(-N+2) \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{y_0}(N-1) & r_{y_1}(N-2) & \cdots & r_{y_0}(0)
\end{pmatrix}
$$

Fig. 2 FrFT-FIR filtering flow chart.
where \( Y_l = [y_a(1,l), y_a(2,l), \cdots, y_a(N,l)]^T, Z_l = [z_a(1,l), z_a(2,l), \cdots, z_a(N,l)]^T (l = 1, 2, \ldots, L), \) and the cross correlation function is

\[
(r_{xy})_{ij} = r_{xy}(i - k) = E[y_a(i,l)z_a^*(k,l)]
\]  

(27)

Because \( w_s(n,t_m) \) and \( w_s(n,t_m) \) are independent of each other, the expected value of their product approaches zero as the number of samples increases. Therefore, the eigenvalue decomposition (EVD) of \( R_x \) is:

\[
R_x \approx U \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} U^H = [U_S, U_N] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_S^H \\ U_N^H \end{bmatrix} 
\]  

(28)

where \( U \) is the eigen matrix, \( U_S \) and \( U_N \) are respectively the signal-subspace and noise-subspace matrices, and \( \Sigma_1 = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_Q) \) with \( \sigma_q(q = 1, 2, \ldots, Q) \) is a diagonal matrix with non-zero eigenvalues.

Finally, we obtain the objective function

\[
\hat{v}_i = \arg \max_{v_i} \frac{1}{\text{tr} \{ P_{10}^i (v_i) U_S W U_S^H \}}
\]  

(29)

where \( P_{10}^i = I - P_{10} \) is \( I - \Omega(\Omega^H\Omega)^{-1}\Omega^H \), \( W \) is the weighting matrix which gives the lowest asymptotic variance when \( W = \bar{A}^2 U_S^H \), where \( \bar{A} = U_S - \sigma I \). \( I \) is unit matrix, and \( \sigma^2 \) is any consistent estimate of the noise variance, \( \Omega(v_i) = [e^{-2\pi x_{h_1}(v_i)}, e^{-2\pi x_{h_2}(v_i)}, \ldots, e^{-2\pi x_{h_Q}(v_i)}]^T \).

To summarize the FrFT-CSWSF process described in this section: following FrFT-FIR filtration, multiple targets are separated and then dechirped, and finally, their parameters are estimated using CSWSF.

4. Computer simulation and analysis

To validate the effectiveness of the FrFT-CSWSF method, computer simulations were conducted via Matlab for a range gate containing two moving targets with respective velocities \( v_{s1} = 15 \text{ m/s} \) and \( v_{s2} = 3 \text{ m/s} \) and under observation by each of the three multistatic SAR types of Cartwheel, Pendulum and Helix. The targets had been previously separated from the ground clutter using the method in Ref.5. As most of the clutter has been suppressed, suppose the residual clutter is on \(-20 \text{ dB} \text{ level}, \text{i.e. signal to clutter ratio (SCR) is } 20 \text{ dB} \). The other system parameters were: the transmitted pulse width is 40 \( \mu \text{s} \), the transmitted bandwidth is 20 MHz, the carrier frequency is 5.3 GHz, \( \text{PRF}=1400 \text{ Hz}, \ H = 800 \text{ km}, \) and \( V = 7 \text{ km/s} \).

4.1. Configurations of the constellations

According to the relative motion equations,\(^2\) the orbit motion of a satellite can be described using 6 orbital elements, i.e. major semi-axis \( a \), eccentricity \( e \), orbit inclination \( i \), right ascension of ascending node (RAAN) \( \Omega \), argument of periuge \( \omega \), and mean anomaly \( M_e \). The orbital-element set is denoted as \( \mathbf{e}_o = [a, e, i, \Omega, \omega, M_e]^T \). Here we design the constellation formation by searching for the minimum value of the error of ESTIM, subject to the relative motion equations. The examples of the design results are given as follows.

Tables 1–3 are the design results for Cartwheel, Pendulum and Helix. With the orbit elements, the baselines of the surrounding satellites are obtained. Since the angular frequency of the satellite orbit (about 0.001 rad/s) is so small that the baselines vary extremely slightly during the survey duration, as a result, they can be regarded as time-invariant. Therefore, the baselines are represented by the time-average of them, and their values are given in Fig. 3.

From Fig. 3, it is easy to obtain the values of the baselines, which listed in Tables 4-6.

4.2. Output of the FrFT-FIR filter banks

The process of FrFT-FIR target filtering, which was undertaken by the transmitter satellite in the respective multistatic

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sat0</th>
<th>Sat1</th>
<th>Sat2</th>
<th>Sat3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a ) (m)</td>
<td>7171037</td>
<td>7171037</td>
<td>7171037</td>
<td>7171037</td>
</tr>
<tr>
<td>( e )</td>
<td>0.01</td>
<td>1.0000209 \times 10^{-2}</td>
<td>9.950729 \times 10^{-3}</td>
<td>1.0000209 \times 10^{-2}</td>
</tr>
<tr>
<td>( i ) (°)</td>
<td>97</td>
<td>97</td>
<td>97</td>
<td>97</td>
</tr>
<tr>
<td>( \Omega ) (°)</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \omega ) (°)</td>
<td>0.28363581</td>
<td>1.0000209 \times 10^{-2}</td>
<td>1.0000209 \times 10^{-2}</td>
<td>1.0000209 \times 10^{-2}</td>
</tr>
<tr>
<td>( M_e ) (°)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

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The process of FrFT-FIR target filtering, which was undertaken by the transmitter satellite in the respective multistatic

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<td>( a ) (m)</td>
<td>7171037</td>
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<td>7171037</td>
</tr>
<tr>
<td>( e )</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( i ) (°)</td>
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<td>97.00525969</td>
<td>97.00525969</td>
</tr>
<tr>
<td>( \Omega ) (°)</td>
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<td>1.89111258 \times 10^{-6}</td>
<td>1.12639011 \times 10^{-3}</td>
<td>1.12639011 \times 10^{-3}</td>
</tr>
<tr>
<td>( \omega ) (°)</td>
<td>0</td>
<td>1.28660246 \times 10^{-3}</td>
<td>2.3502260 \times 10^{-3}</td>
<td>2.3502260 \times 10^{-3}</td>
</tr>
<tr>
<td>( M_e ) (°)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
SAR configurations, is shown in Fig. 4. In Fig. 4(a)–(c), the solid lines represent the absolute values of FrFT of the both targets, while the dot lines represent the sub-band filter envelopes. We then investigate the effect of the filtering via signal phase. For each target, the output phase is close to that of the original azimuth signal and differs by only a fixed degree of shift (see Fig. 4(d)–(f)); therefore, the filter is effective. The Kaiser window function parameter $\beta = 3$.

### 4.3. Estimation results

The CSWSF spectra of the two targets (Target 1 and Target 2) are shown in Fig. 5. It is found that the respective peaks are close to their corresponding default values of cross-range velocity.

Table 7 compares the expected estimated values derived through conventional methods (monostatic SARs) with those from FrFT-CSWSF at a signal-to-noise ratio (SNR) of 15 dB, using Pendulum constellation. From the figures, it is obvious that the expected values produced using our method are closer to the default values than those produced by the standard methods. Because the actual signal is discrete with a finite length, the resolutions of the digital FrFT (DFrFT) and match filtering methods are so limited that merely searching for the peak of the output cannot produce results with the necessary precision, especially at small velocities (see the third column in Table 7). In contrast, our method (bottom line in Table 7) improves the resolution of the parameter by using an FrFT (in practice, a DFrFT).

Fig. 6 gives the azimuth focusing results of using the Doppler chirp rate of the static scatter (defocused), estimated via conventional FrFT and FrFT-CSWSF. It is shown that using our method, the azimuth image is better focused than the other, namely the image is finely focused.

Finally, we examined the asymptotic behavior of our method. The root-mean-square-error (RMSE) curves of our estimations following 300 Monte Carlo simulations are plotted in Fig. 7. The fact that the curves approach the Cramer-Rao bound (CRB) as SNR increases confirms that our method is statistically effective. Furthermore, the phenomenon that the RMSE curves of Cartwheel and Pendulum which contain three receive-satellites are lower than those of Helix including only...
two implies that the robustness of the estimation improves with the increase of the number of elements (played by the receive-satellites). Additionally, it is plausible that using improved subspace algorithms in place of the conventional subspace algorithm is a significant factor in moving our RMSE curve closer to the CRB; a more complete demonstration of this will be performed in future work.

Table 7  Cross-range velocity estimation results.

<table>
<thead>
<tr>
<th>Method</th>
<th>Target 1 (m/s)</th>
<th>Target 2 (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default</td>
<td>15</td>
<td>–3</td>
</tr>
<tr>
<td>Match filtering</td>
<td>15.43</td>
<td>–0.59</td>
</tr>
<tr>
<td>FrFT</td>
<td>18.52</td>
<td>–0.65</td>
</tr>
<tr>
<td>FrFT-CSWSF</td>
<td>15.08</td>
<td>–2.94</td>
</tr>
</tbody>
</table>

Fig. 4  Process of FrFT-FIR filtering.

Fig. 5  CSWSF spectra of the two targets.

Fig. 6  Focusing results of azimuth direction.
FrFT-CSWSF: Estimating cross-range velocities of ground moving targets using multistatic synthetic aperture radar

5. Conclusions

In this paper, we present a time-frequency-subspace method for estimating the cross-range velocities of multiple ground moving targets using multistatic SAR. Based on ESTIM, targets within a given range gate can be separated with a set of FrFT-FIR filter banks. The respective filter outputs are then processed using a CSWSF algorithm in order to obtain cross-range velocities. Theoretical analysis and computer simulations with the designed constellations of Cartwheel, Pendulum and Helix have verified that targets can be successfully separated with only a fixed time delay in the phase for each output. Furthermore, the CSWSF algorithm can help obtain better estimates of the cross-range velocity than conventional FrFT and match filtering methods, especially for the target with small cross-range velocity, therefore our method can help obtain a finely focused image. Examination of the asymptotic behavior of our method demonstrated that it is asymptotically efficient, and the robustness of the estimation improves with the increase of the number of the receive-satellites. Although the RMSE of our method is not always extraordinarily close to the CRB bound, we have demonstrated a feasible approach for improving estimation performance that we plan to enhance and make more robust in future work.

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References


Fig. 7 RMSE of the estimates.
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