Modelling social identification and helping in evacuation simulation

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ABSTRACT

Social scientists have criticised computer models of pedestrian streams for their treatment of psychological crowds as mere aggregations of individuals. Indeed most models for evacuation dynamics use analogies from physics where pedestrians are considered as particles. Although this ensures that the results of the simulation match important physical phenomena, such as the deceleration of the crowd with increasing density, social phenomena such as group processes are ignored. In particular, people in a crowd have social identities and share those social identities with the others in the crowd. The process of self categorisation determines norms within the crowd and influences how people will behave in evacuation situations. We formulate the application of social identity in pedestrian simulation algorithmically. The goal is to examine whether it is possible to carry over the psychological model to computer models of pedestrian motion so that simulation results correspond to observations from crowd psychology. That is, we quantify and formalise empirical research on and verbal descriptions of the effect of group identity on behaviour. We use uncertainty quantification to analyse the model's behaviour when we vary crucial model parameters. In this first approach we restrict ourselves to a specific scenario that was thoroughly investigated by crowd psychologists and where some quantitative data is available: the bombing and subsequent evacuation of a London underground tube carriage on July 7th 2005.

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1. Introduction

The importance of evacuation simulations for pedestrians is generally accepted for designing buildings and for ensuring safety at mass events. Multiple models for pedestrian simulation were developed in the past decades. Popular among them are force based models (Helbing and Molnár, 1995; Langston et al., 2006), cellular automata (Gipps and Marksjö, 1985; Blue et al., 1997; Schadschneider, 2001; Kirik et al., 2007), and variations of Reynold's behavioural and steering models (Reynolds, 1987, 1999; Karamouzas et al., 2009). Sveral alternatives have been added to the portfolio in more recent years (Dijkstra et al., 2006; Sud et al., 2008; Seitz and Köster, 2012).

While having a realistic locomotion model is an important basis for simulating pedestrian evacuations, many crucial aspects of social behaviour have been neglected. In particular, the emergence and effects of group behaviour have been examined extensively in empirical research by social psychologists but are often missing in computer models (Templeton et al., 2015).

On the other hand, there are a number of publications on pedestrian motion models which incorporate social and psychological behaviour (Pan et al., 2007; Chu et al., 2011; Tsai et al., 2011; Chu and Law, 2013). Yet, these publications mostly describe computer frameworks designed to incorporate many possible but as yet unspecified behavioural models. That is, their focus is on the implementation of the framework. Notably, they do not attempt to emulate empirical findings from psychology. Furthermore, information on how the behaviours they implemented are modelled in detail is missing in most cases. Another problem is the large number of parameters used to control the interworking between the software modules that instantiate the supposed behavioural models. These three aspects – the distance of the models to the underlying theories, the lack of detailed modelling information, and the number of parameters to calibrate – make it nearly impossible to replicate the models and to check the models against observations. From a safety scientist's point of view, this is a severe drawback.

This is the gap that we attempt to close. With the Social Identity Model Application (SIMA) for pedestrian simulation, we present a

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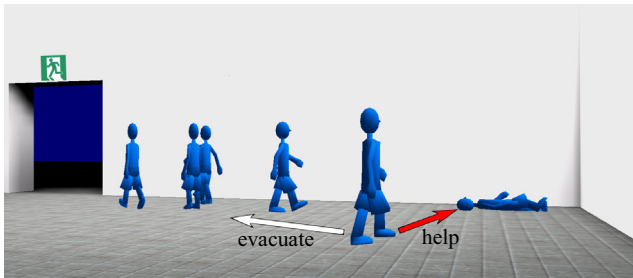E-mail address: isabella.von_sivers@hm.edu (I. von Sivers).

Fig. 1. In an evacuation people have to decide whether to evacuate as quickly as possible or to help others. We explain psychological mechanisms behind these decisions, formalise them in a computer model and show which model parameters are decisive for safety science.

step towards combining research from psychology and computer science. We focus on one pivotal behaviour that was observed in several evacuations: helping others (see Fig. 1). In modelling, we directly follow the ideas of the self-categorisation theory (SCT) (Turner et al., 1987) and the social identity theory (SIT) (Tajfel and Turner, 1979) which are both part of the social identity approach. We describe how to algorithmically formulate helping behaviour in an evacuation and how to choose the parameters so that other scientists can replicate, validate, and use the model. We keep the model independent of the locomotion level, and the parameter space small.

2. Materials and methods

2.1. Social identity theory and self-categorisation theory

2.1.1. Incorporating evidence from crowd psychology

Our model is based upon extensive empirical research on collective behaviour by social psychologists. There are numerous real life examples of collective behaviour where people act together as a group: for example orchestras, football fans, and sports teams. Two prominent theories which provide insight into how this group behaviour emerges are social identity theory (Tajfel and Turner, 1979) and self-categorisation theory (Turner et al., 1987). According to social identity theory, people have multiple social identities which are distinct from the identity of a person as an individual because they refer to ones identity as part of a social group, such as a fan of a certain sports team. Self-categorisation theory refers to the process whereby one categorises oneself as an individual or a group member. It suggests that collective behaviour occurs through the process of depersonalisation, where individuals self-stereotype themselves in line with their group. This occurs through a transformation of one's identity from the *personal* self to the *collective* self. It is this self-categorisation as a group member which makes collective behaviour possible. It can therefore explain the behavioural differences between a *physical* crowd of individuals (who are simply in the same location together) and a *psychological* crowd (where people in a crowd act together).

The effect of social identities on peoples behaviour is crucial to understand for crowd modellers who aim to simulate psychological crowds. Research has shown that a shared social identity amongst crowd members increases the prevalence of supportive behaviours among people in emergency evacuations (Drury et al., 2009c). For example, when they share a social identity people evacuating may be more likely to coordinate their walking behaviours with others and by letting them move first rather than competing for the same exit. Research on emergency mass decontamination has also demonstrated that social identity is key to understanding the coordination of queuing behaviour, showing that members of the public are more likely to participate in

queuing if they identify with the person organising the situation (Carter et al., 2014).

An example of collective behaviour in an emergency evacuation comes from the July 7th London bombings (Drury et al., 2009b). In this paper, we will focus upon on this event which has been analysed by social psychologists.

2.1.2. The event: the London bombings, 7th July 2005

At 8.50 am, during the peak rush hour in central London, three bombs were set off simultaneously in the London underground. The bombs were coordinated so that they detonated on three separate tube lines when the tube trains were between busy stations. The passengers in the tube trains were plunged into darkness and could not know if there were going to be further explosions, with no information when help would arrive. Emergency services did not reach them for some time. Over 700 people were injured in the attacks and 52 people were killed. In this emergency situation, the survivors of the bombings came together to tend to the injured and find a way of evacuating safely. In contrast to portrayals of crowds as panicking and acting selfishly to evacuate, research has shown that the opposite occurred. In the aftermath of the disaster, 140 of the survivors reported seeing helping behaviour and mutual aid, such as offering water to others, providing first aid, and applying makeshift bandages. Only three of the survivors reported witnessing selfish behaviour. This was replicated in internet sources and the public enquiry; first-hand accounts showed that 42 people out of 127 described seeing help and only 11 described seeing people act selfishly.

Crucially, Drury et al. (2009b) found that survivors reported feeling part of a group with the other survivors. Many participants gave accounts of where people cooperated and coordinated to help the group to escape. In fact, most survivors claimed the crowd behaved calmly and orderly, rather than describing panicking behaviour. There was evidence of orderly queuing and people allowing others to go first, which in turn helped people overall by making the evacuation more safe. Other examples of reported behaviour include leaving in a calm manner rather than rushing or pushing past each other which could have caused hindrance to the evacuation of the group overall.

Drury et al. (2009b) demonstrate how survivors emotionally supported one another, allowed others to evacuate first, and stayed behind with people (who were previously strangers) at a personal risk to themselves. This behaviour was relatively common across the survivors, and when combined with the reports of feeling as part of a group it is in line with the idea that the commuters shared a social identity which was invoked through the common fate of the emergency situation. Although there are numerous examples of this collective mutual aid in emergency situations, in this paper we will focus on one key helping behaviour that was documented in the London bombings: assisting injured people to evacuate safely. We argue that in order to adequately simulate collective behaviour in emergency and disaster events, modellers should heed the research conducted by social psychologists. Specifically, modellers should focus on the role of a shared group identity and incidents of ingroup helping by incorporating aspects of self-categorisation theory into their simulations. As such, we use the London bombings scenario to propose a realistic model of collective behaviour which combines the comprehensively validated Optimal Steps Model for locomotion with principles from the well-established social identity theory and self-categorisation theory.

2.1.3. Collecting empirical evidence

The helping behaviour that is modelled in SIMA is taken from accounts given by survivors of the July 7th 2005 London bombings and the behaviour of the crowd in the aftermath as researched by

Drury et al. (2009b). They collated and analysed the survivor's perceptions of people's behaviour and the feelings that they experienced during the event. This was conducted by collecting one hundred and forty-one accounts in contemporaneous newspaper material, in addition to personal archives and accounts from eighty-one survivors which were recorded on the day or in the immediate aftermath of the bombings. Crucially, from this sample of survivors, they were asked about the level of danger that they felt they were in, how the others in the crowd behaved, whether people performed helping behaviour or acted selfishly, and whether their perception towards others changed throughout the course of the event. Notably, support for these findings can be seen in Drury et al. (2009a) which examines the accounts of crowd behaviour given by 21 survivors of 11 emergencies.

2.2. The underlying pedestrian motion model

For the locomotion level of our simulation, we use the Optimal Steps Model (OSM) (Seitz and Köster, 2012; von Sivers and Köster, 2015). As in many models, pedestrians are represented by circles, with radius 20 cm, that represent the solid body. The model deviates from older approaches in its treatment of motion: as in reality, pedestrians make steps to move forward. They do not glide along smooth trajectories that resemble imaginary rails as in force-based models or hop from cell to cell as in cellular automata. For this, each agent searches for the possible next position within a disk of which the radius is the agent's maximum stride length. The maximum stride length is determined individually according to empirical findings that link the stride length to free-flow speed (Grieve and Gear, 1966; Kirtley et al., 1985; Jelić et al., 2012; Seitz and Köster, 2012). The free-flow speed, that is, the speed an agent is supposed to prefer when walking uninhibited on a flat surface, is a standard input parameter in pedestrian simulations.

The search for the next position constitutes a two-dimensional optimisation problem on the step disk. The objective function is a superposition of dedicated utility functions that express closeness to the target, or rather a short travel time to the target (Köster and Zönnchen, 2014), sufficient interpersonal distance to other pedestrians and sufficient distance to obstacles (von Sivers and Köster, 2015; Seitz et al., 2015). The shorter the remaining travel time to the target the higher the utility becomes. The correct travel time, while skirting obstacles that hide the target from direct view, is expressed through the solution of a mathematical equation: the eikonal equation (Kretz, 2009; Kretz et al., 2010; Hartmann, 2010). Originally, the eikonal equation describes the arrival time of a wave front that spreads out from an area, in our case, the target. In our model, the arrival time of this imaginary wave front, at each location, is proportional to the expected remaining travelling time of an agent. All local utility or cost functions are chosen on a compact support, that is, they are truly zero at a suitable distance. This avoids numerical cut-off errors. The update scheme for the pedestrians is event driven (Seitz and Köster, 2014). That is, each agent steps ahead according to its individual stepping frequency which is predefined from the agent's free-flow speed and maximum stride length. Each step is an event in the event queue and occurs at its "natural" time.

To make replication possible, the parameters for the locomotion model are compiled in Table 1. The Optimal Steps model is embedded into the VADERE simulation framework at Munich University of Applied Sciences. Each module of the framework is verified using an automatic test suite. The model itself has been extensively validated (see for example (von Sivers and Köster, 2015; Seitz et al., 2015)) including independent implementations (Kneidl, 2015). Thus it is a reliable basis on which to build modules that instantiate social behaviour.

Table 1
The parameters for the OSM.

Param.	Description	Value
δ_{int}	Intimate distance	0.45 m
δ_{pers}	Personal distance	1.20 m
δ_o	Distance kept from obstacles	0.8 m
μ_p	Strength of 'pedestrian avoidance'	5.0
a_p	Moderation between intimate and personal space	1.0
b_p	Transition between intimate and personal space	1.0
μ_o	Strength of 'obstacle avoidance'	6.0
v_f	Mean free-flow speed	$1.6\frac{m}{s}$
σ_f	Variance of the free-flow speed	$0.26\frac{m}{s}$

2.3. The theory of uncertainty quantification

The field of uncertainty quantification (UQ) addresses the problem that not all parameter values in the simulation are exactly determined: they are uncertain. Uncertainty quantification assumes distributions of the parameters and, thereby, allows to analyse their impact on the solution of the simulation. Probability density functions (PDFs) may be provided directly or statistical moments may be specified.

In recent decades, several methods beyond traditional Monte Carlo simulations have been developed. For a general overview see Smith (2014) and Xiu (2010). Uncertainty quantification distinguishes between forward and inverse uncertainty quantification simulation. The former analyses the impact of the uncertain parameters on the model whereas the latter tries to determine the distribution of the input parameters. In this work, we will use forward uncertainty quantification. Also we will employ non-intrusive methods where, in contrast to intrusive methods, the underlying model remains untouched.

A forward simulation in the context of uncertainty quantification contains three phases (Iaccarino, 2008, p. 17-7): the assimilation phase, the propagation phase, and the certification phase (Fig. 2). In the assimilation phase, the uncertain parameters are defined and prepared with the help of probability theory. For each uncertain parameter, a suitable probability distribution function (PDF) has to be chosen. In the propagation phase the model is evaluated. For this, the parameters are propagated through the model multiple times. The number of propagations necessary to obtain good results depends on the specific uncertainty quantification method. In the certification phase the output of each propagation step is collected. The impact of the uncertain parameters on one or several quantities of interest is observed. Often, the mean and the variance of an output quantity is calculated, but higher statistical moments are also possible (Xiu, 2010, p. 67). Typical quantities of interests (QoI) are physical, domain, or timing values.

Here, we focus on parametric uncertainty: we want to understand the impact of uncertain parameters, such as the number of injured persons, on the simulation results. We choose stochastic collocation (SC) with the pseudospectral approach (Xiu, 2007) as uncertainty quantification method. This fits our requirements because the method is non-intrusive, suitable for the analysis of models with few parameters, and needs less computational effort than other methods (Xiu, 2009).

Stochastic collocation with the pseudospectral approach (SC-GPC) is based on the generalised polynomial chaos (GPC) (Xiu, 2010, p. 57–67) expansion. The idea is to separate the spatio-temporal part of a random process $U(x, t, \zeta)$ from the purely random part (see Smith, 2014, p. 209). In our case, $U(x, t, \zeta)$ is the theoretical 'analytical' solution of the considered pedestrian evacuation scenario.

The generalised polynomial chaos expansion for $U(x, t, \zeta)$ is defined as (Xiu and Karniadakis, 2003, p. 143)
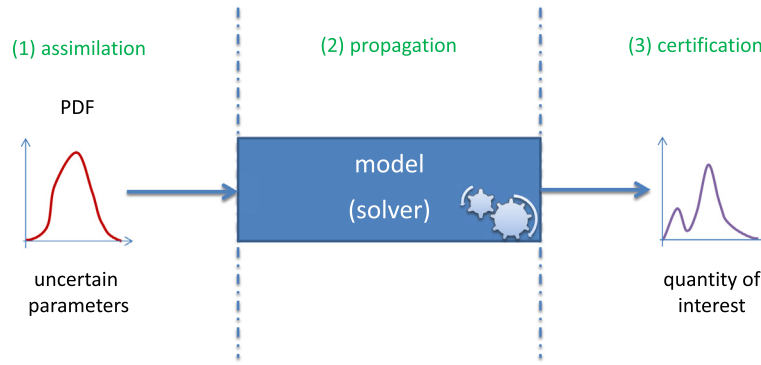
Fig. 2. Illustration of the forward uncertainty quantification phases: assimilation, propagation, and certification.

$$U(x,t,\zeta) = \sum_{j=0}^{\infty} \underbrace{c_j(x,t)}_{\text{spatio-temporal}} \cdot \underbrace{\Phi_j(\zeta)}_{\text{random}}, \tag{1}$$

which depends on space x, time t, and the vector of random parameters $\zeta = (\zeta_1, \ldots, \zeta_M)$ with M independent random variables. $c_j(x,t)$ are the coefficients of the expansion representing the spatio-temporal part and are denoted by c_j for the sake of simplicity in the following. Their computation depends on the specific generalised polynomial chaos method (Smith, 2014, p. 209) one uses. The functions Φ_j are orthogonal polynomial basis functions that correspond to the distributions of the random variables. To evaluate Eq. (1) numerically, the expansion is truncated after N terms.

In the stochastic collocation with the pseudospectral approach, the model is evaluated on so-called collocation points. The coefficients c_j of the generalised polynomial chaos expansion are approximated by applying a projection with an integration rule: once the uncertain parameters are chosen, the collocation points (Xiu, 2007, p. 299) z_i and weights w_i $(i = 1, \ldots, Q)$ for the integration rule can be generated (assimilation). To propagate the uncertainty through the model and to calculate the coefficients c_j, the model $u(z_i)$ (i.e. our pedestrian simulation) and the orthogonal basis functions $\Phi_j(z_i)$ have to be evaluated at the generated collocation points z_i.

Finally, the quantities of interest are extracted (certification). In our example, we have one quantity of interest: the number of pedestrians that have not yet reached safety in an evacuation scenario resembling the London bombings in 2005. To extract the mean μ_{u_N} and the variance σ^2 of the desired quantity of interest, we can directly use the coefficients c_j. For the mean, one has to evaluate (Xiu, 2010, p. 67)

$$\mu_{u_N} \approx \underbrace{\sum_{i=1}^{Q} u(z_i)\Phi_0(z_i)w_i}_{c_0} = c_0, \tag{2}$$

and for the variance σ^2 (Xiu, 2010, p. 67)

$$\sigma^2 \approx \sum_{j=1}^{N} \left(\underbrace{\sum_{i=1}^{Q} u(z_i)\Phi_j(z_i)w_i}_{c_j} \right)^2 = \sum_{j=1}^{N} c_j^2, \tag{3}$$

with σ being the standard deviation.

We implemented a Python program using the chaospy library (Feinberg and Langtangen, 2015) to realise the stochastic collocation with the pseudospectral approach (for results, see Section 4.2.2). Since we are going to use uniform distributions for the computations (see Section 4.2.1), Legendre polynomials are used as orthogonal basis functions Φ (according to Xiu and Karniadakis, 2002, p. 626). We choose the Gauss quadrature as integration rule, and the order of the polynomial chaos expansion is fixed to $N = 6$. The number of required simulation runs depends on the number of collocation points and the number of uncertain parameters. In our example, we use $Q = 21$ collocation points for each uncertain parameter. Hence, for the settings with one uncertain parameter, 21 simulation runs are required, and for the setting with three uncertain parameters 9126 $(= 21^3)$.

3. Results: formalisation of social identity and helping

Self-categorisation theory suggests that, when people categorise themselves as being in the same group, they are more likely to support each other (Levine et al., 2005). For the simulation of emergency evacuations, this basic idea is essential. Other parts of the self-categorisation theory that are important to social psychologists – for example a changing degree of social identity during the evacuation – have to be postponed until there is data to substantiate the mechanisms of the change.

3.1. Social Identity Model Application

We want to enable independent researchers to use the new social model, the *Social Identity Model Application (SIMA)* with any locomotion model and in any simulation framework. The Optimal Steps Model and the VADERE simulation framework of this contribution are merely examples among several well validated choices. We achieve our goal by defining an interface that exclusively consists of the target and the velocity of the pedestrians, the two typical input parameters for a locomotion step. The Social Identity Model Application is called before the execution of the locomotion module in every time step of the simulation. The outcomes of the SIMA call are adjusted targets and velocities of the agents.

The Social Identity Model Application consists of two main components: the social identity component (*Establishing Social Identity*) described in Section 3.2 and the helping behaviour component (*Helping Behaviour*) described in Section 3.4. Fig. 3 shows the main loop of the Social Identity Model Application with these two key components. A new type of agents, badly injured pedestrians, is introduced (see Section 3.3).

The *Establishing Social Identity* component is called the first time a pedestrian recognises the emergency. This follows findings from social psychological research that people categorise themselves as ingroup members when faced with the common fate of an emergency. The component *Helping Behaviour* is relevant during the whole duration of the simulation. It is called for pedestrians who share a social identity. Pedestrians who do not share a social identity head straight for safety, that is, they evacuate without caring
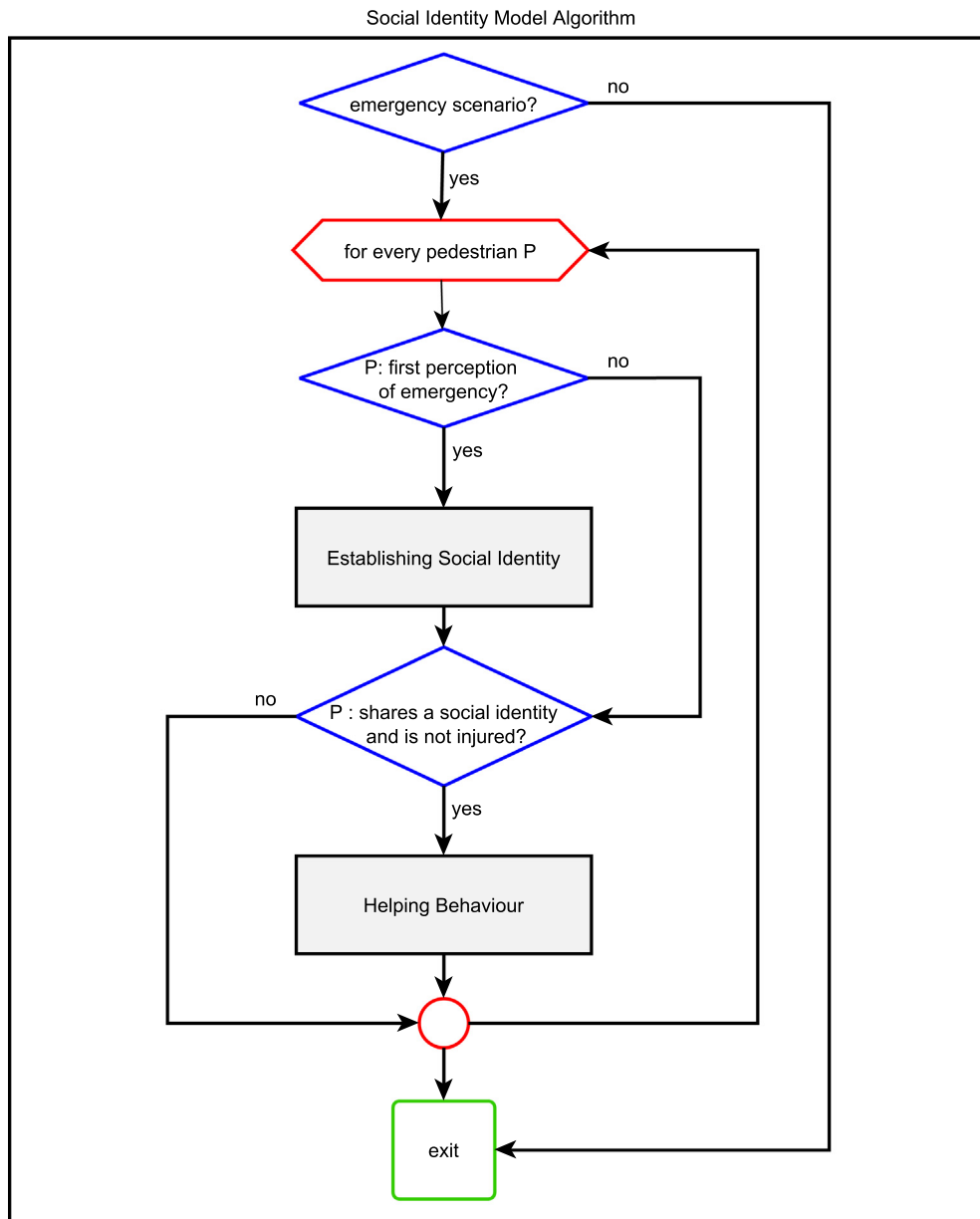
Social Identity Model Algorithm

Fig. 3. General flow chart of the Social Identity Model Application.

for others. With its two states, being an ingroup member or not, and the resulting helping behaviour, the SIMA has similarities to finite-state machines (Kielar et al., 2014) and discrete choice models (Antonini et al., 2006).

3.2. Establishing social identity

The first step in the model is to establish the social identity. However, not every pedestrian in an emergency shares a social identity (Drury et al., 2009a). Thus, we define a parameter $perc_{sharingSI}$. Pedestrians are randomly selected to share a social identity or not according to $perc_{sharingSI}$. The procedure for one pedestrian in the scenario is visualised in Fig. 4.

3.3. Badly injured pedestrians

To model helping behaviour, we first need to introduce another type of agent: a badly injured pedestrian. In emergencies in

general, people can get hurt by fire, bombs, plunging building parts or for other reasons. Although in the London bombings there were some pedestrians who suffered minor injuries we neglect them here. Their behaviour could be easily modelled by reducing their speed. Instead, we focus on badly injured pedestrians who are still able to move with the assistance of others. Dead or deadly wounded passengers could be considered as obstacles in a model. Further, they might hinder the progress of survivors who try to assess the situation and to help. We think that we do not have enough data to formalise this in a meaningful way and thus chose to also neglect completely disabling injuries, accepting it as a limitation of the model.

A pivotal element in evacuation models is the target that each agent moves towards. Usually this is a 'safe area'. We model the immobility of badly injured pedestrians by fixing their target at their current position. Thus they remain stationary. They cannot evacuate without assistance from unharmed pedestrians. As soon as such an aide arrives we turn the aide into the injured agent's
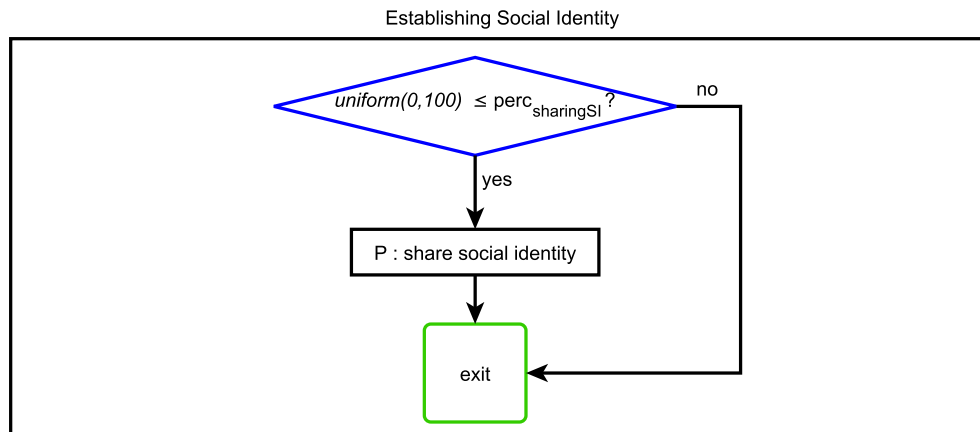
Establishing Social Identity

Fig. 4. Flow chart of how the social identity for pedestrian P is established at the beginning of the simulation. The percentage of pedestrians sharing a social identity is $perc_{sharingSI}$.
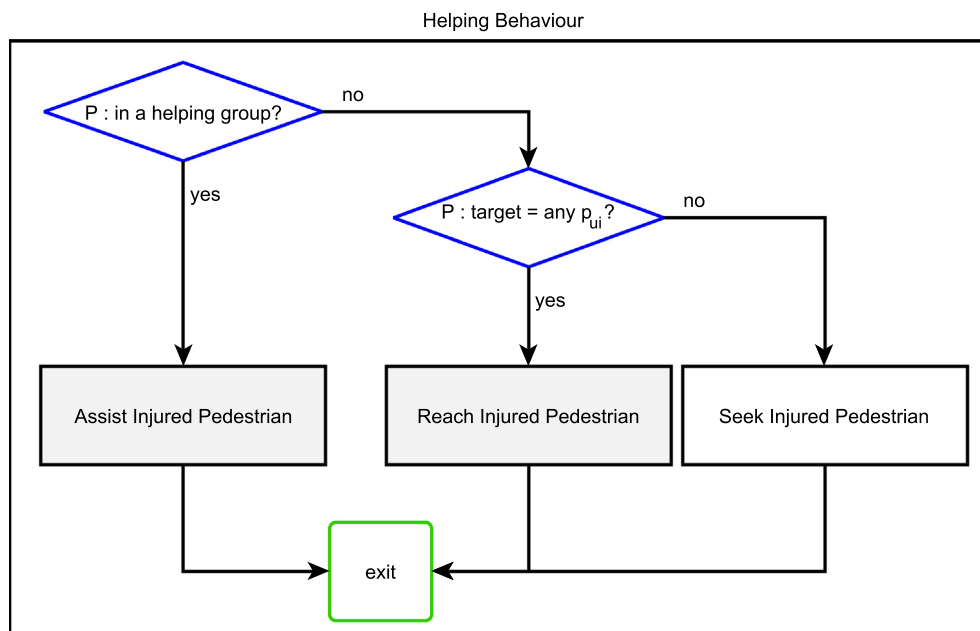
Helping Behaviour

Fig. 5. Flow chart of the helping behaviour for pedestrian P during the emergency; p_{ui} is the abbreviation for unaided and injured pedestrian.

target. By doing this, the injured agent becomes dependent on the aide. The aide's target is set to the 'safe area' and the pair evacuate at a reduced speed v_{inj}. This helping behaviour matches reports by survivors (Baker et al., 2002; Johnson, 2005; Tucker et al., 2007).

3.4. Helping behaviour

Reports on help between strangers and survivors in evacuations usually lack a description of what exactly people do to help. However, one can imagine the typical steps that must be taken. Initially, an injured pedestrian needs to be detected by an aide. Subsequently, this aide must approach the injured pedestrian to finally physically support the injured person while evacuating together. These assumptions are what we implement as helping behaviour (see Fig. 5).

We assume the pedestrians choose the nearest injured and unaided pedestrian as the person to assist. The sub-component *Seek Injured Pedestrian* realises this searching behaviour. Each pedestrian has a range of perception that, in reality, depends on

the scenario or on the pedestrian's abilities. In our simulations, we simplify these dependencies by assuming that each pedestrians is aware of casualties within a radius of 10 m, neglecting visual obstructions or the fact that the train has different compartments and cars. We argue, that even through visual obstructions the shouts of injured people can be heard so the range of perception is far bigger than the range of vision. Each potential helper chooses the nearest unaided and injured pedestrian as new target (see Fig. 6) and approaches this target. If there is no injured pedestrian in the range of perception the pedestrian heads to the 'safe area'.

The sub-component *Reached Injured Pedestrian* controls the situation when a pedestrian reaches the unaided and injured pedestrian. If there is less than an arm length between them they form are group and evacuate together. They both change status and targets. First, the unhelped injured pedestrian p_{ui} becomes a helped injured pedestrian p_{hi}. This status change guarantees that other potential aides stop approaching and search for other casualties (see Fig. 5). As a consequence, assistance is rendered by only one helper. Since it is unclear how many agents would help and how cooperation among helpers alters the evacuation speed, we find
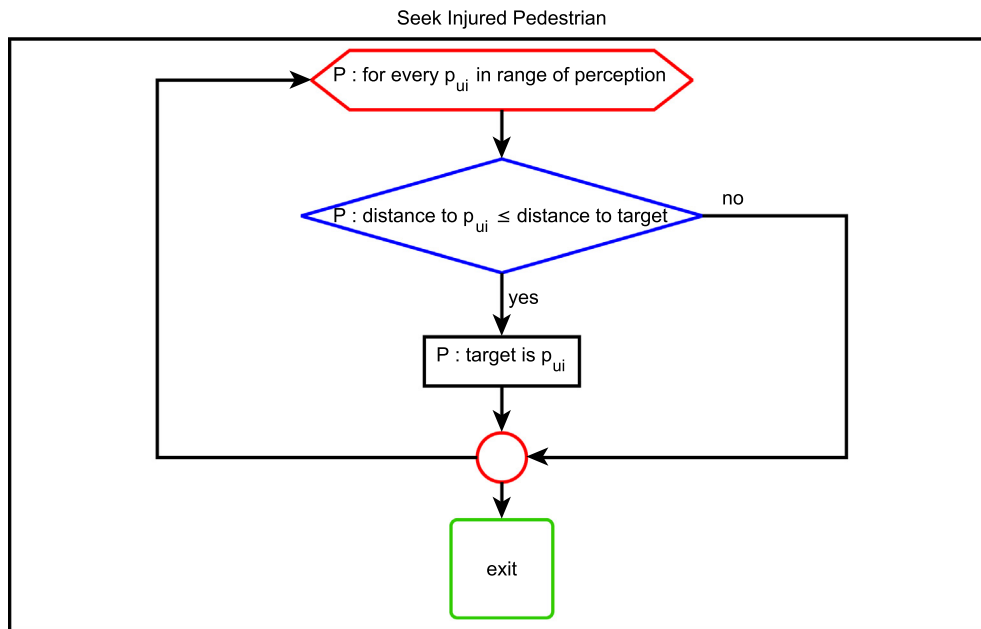
Seek Injured Pedestrian

Fig. 6. Flow chart of the helping behaviour when pedestrian *P* looks for injured pedestrians and chooses whom to help. p_{ui} is the abbreviation for unaided and injured pedestrian. Target means the target of the pedestrian *P*. The range of perception is a radius of 10 m around the pedestrian.
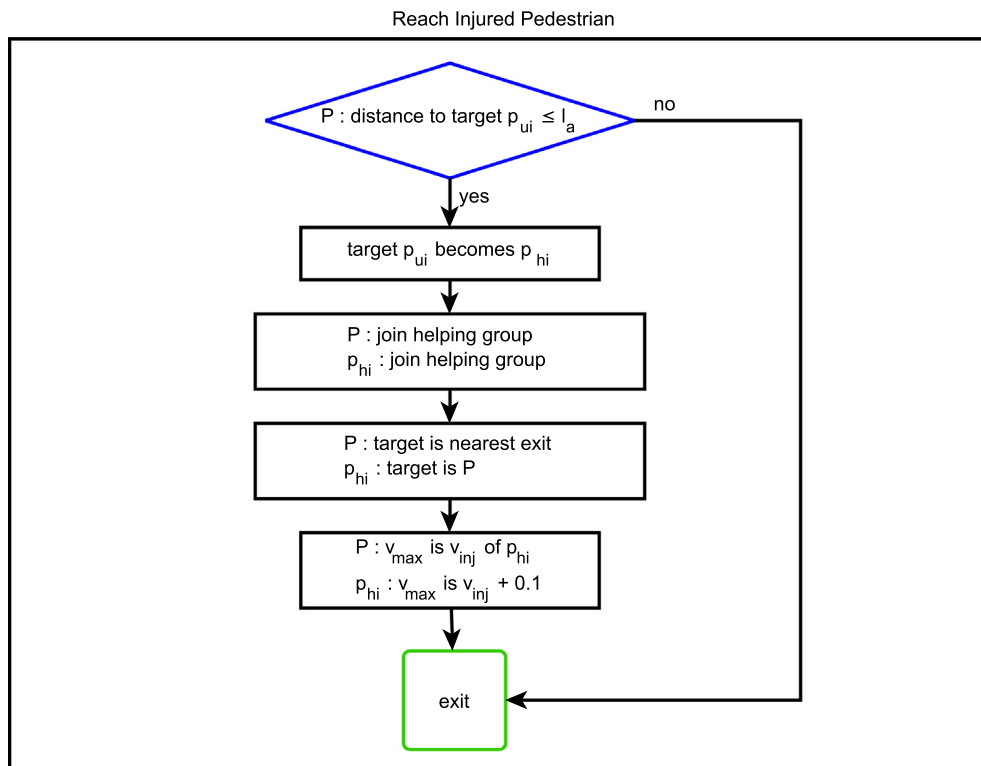
Reach Injured Pedestrian

Fig. 7. Flow chart of the helping behaviour when an injured pedestrian is reached. Parameter l_a is the arm length of a pedestrian, p_{ui} stands for an unaided injured pedestrian, p_{hi} for helped injured pedestrian.

this simplification inevitable. Then, the aide selects the safe location as next target and reduces the free-flow speed to v_{inj}. Since the aide's location is set as target for the injured pedestrian, the latter automatically follows. The injured agent's free-flow speed is set slightly higher than v_{inj} so that it does not fall behind or loose the aide. The arm length l_a is set to 60 cm. The single steps of the procedure are shown in Fig. 7.

The last step of the helping behaviour is the sub-component *Assist Injured Pedestrian*. An agent in this stage evacuates with the injured agent without looking for other casualties. Notably, agents can only help one injured throughout the simulation. From then on, both aide and injured pedestrians keep their respective targets and their reduced speeds. Note that, while the Social Identity Model Application can be applied for almost any locomotion

Table 2
Parameters for the Social Identity Model Application.

Param.	Description	Value
$perc_{sharingSI}$	People sharing a social identity	0.8
$perc_{injPeds}$	Percentage of injured pedestrians	0.1
v_{inj}	Speed of a helper with an injured pedestrian	$0.6\,\frac{m}{s}$

model, the concrete simulation outcomes, such as evacuation times, will most likely vary.

4. Results: quantification of uncertainty in the model

4.1. Simulation of a whole train evacuation

Initially, we focus on the introduction of helping behaviour in a train evacuation scenario of a correctly dimensioned London Underground C69/C77 Circle Line train as in the July 7th London bombings. The evacuation route, which in reality was a path along the tracks of the trains, is modelled as a long and narrow corridor that leaves no room for walking abreast. As during the real bombings, the scenario takes place during rush hour, so that every seat of the 192 seats in the train is occupied, that is, the initial positions of the agents are given by the seat locations. For simplicity, we assume that the standing room is empty.

The simulation parameters of the locomotion model, the Optimal Steps Model, are compiled in Table 1. All parameters of the Social Identity Model Application for this simulation are compiled in Table 2. In the absence of measured behavioural evidence for this scenario, we chose plausible values for the number of pedestrians who share a social identity and for the number of injured pedestrians. The speed of a person assisting a casualty is

set to 0.6 m/s. This value corresponds to observed slowest speeds of pedestrians with a walking handicap (Perry, 1992).

We assume that the bomb detonated in the third car and that, as a consequence, 19 pedestrians are badly injured: 16 casualties are near the bomb and 3 casualties are randomly positioned at other places. Among the remaining pedestrians, those who share a social identity are randomly chosen, according to the percentage set for the simulation run. See Fig. 8 for an illustration of the setting.

After a few seconds of the simulation, the first pairs of aides and injured pedestrians form. In Fig. 9 the helpers are depicted by black striped circles and the injured by light blue circles. At this point, some of the injured pedestrians are still without helpers. A few seconds later, all injured pedestrians are assisted (see Fig. 10). The other pedestrians (indicated by blue circles) evacuate on their own. In this model, those pedestrians who do not help anybody leave faster than those assisting injured pedestrians. This is a result of the reduced speed of the aides and injured pedestrians.

In a later state of the simulation, the escape route becomes congested. In this scenario, overtaking while walking along the evacuation path is not possible. Thus, faster agents get stuck behind aides with their charges (see Fig. 11).

All observations outlined in the simulation match accounts by survivors in emergencies (Johnson, 2005; Drury et al., 2009b). They reported that survivors assisted those who were injured before leaving and formed orderly queues while evacuating. This is the behaviour that emerges in our simulations. We argue that this constitutes a qualitative validation of the Social Identity Model Application. Quantitative validation must be postponed until suitable data is available, possibly from evacuation drills or video footage of future emergencies. For a longer discussion on validation challenges see von Sivers et al. (2014). In addition to qualitative validation, we are able to provide statistical data on
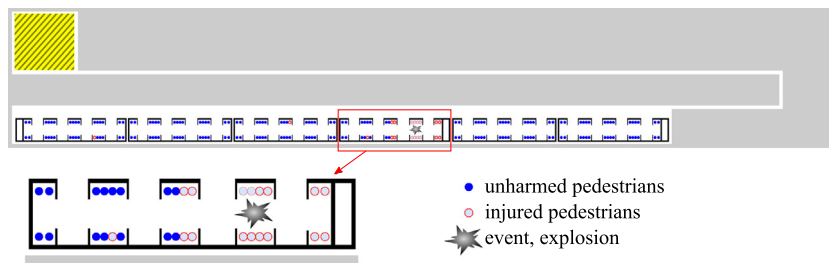

Fig. 8. Evacuation scenario of a train. We assume that every seat is occupied (blue circles for unharmed passengers and red-rimmed light blue circles for injured passengers), but nobody is standing. Black lines indicate the partitioning inside the train and the walls. Light grey areas cannot be stepped upon. The escape route is the narrow white corridor that leads to safety. The safe area, and target, is indicated by the large (yellow, striped) rectangle in the upper left corner. 16 of the 19 injured pedestrians are placed near the event (grey star), three more randomly in the train. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
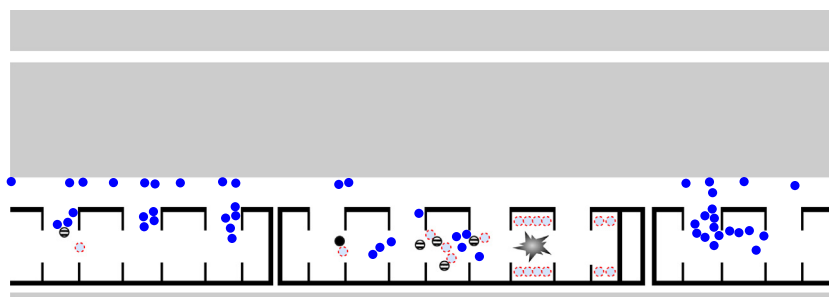

Fig. 9. Evacuation scenario of a train. Close-up after the first seconds of the evacuation. First pairs of injured pedestrians (red-rimmed light blue cirlces) with helpers (black striped circles) form. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
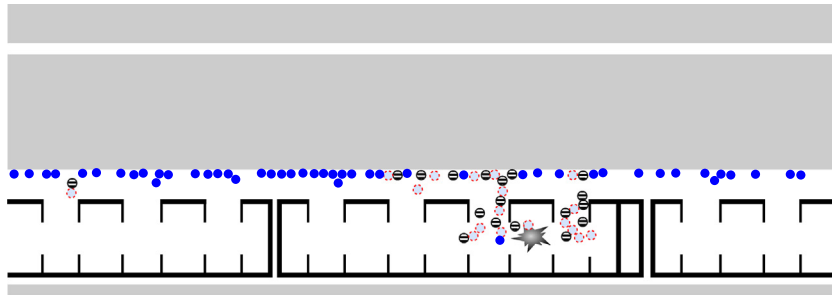
Fig. 10. Evacuation scenario of a train. Close-up after all pairs of aides and charges have formed. The unharmed pedestrians (blue circles) leave the train faster, the helpers (black striped circles) with injured (red-rimmed light blue circles) are slower. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
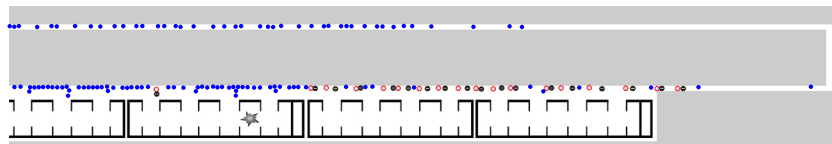

Fig. 11. Evacuation scenario of a train. The slow pairs of helpers and charges (red-rimmed light blue and black striped circles) cause congestions. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the variation of simulation outcomes using the techniques of uncertainty quantification.

4.2. Uncertainty quantification of the Social Identity Model Application

In many applications, the precise value of an important parameter, such as the number of injured pedestrians in emergency planning, is unknown. Moreover, even if a measured value for the parameter is available, it is only correct within a margin of error. If the model is sensitive to the variation of this parameter the predictive power of the model is decreased.

For example, the new helping behaviour in the simulation has a crucial impact on the evacuation time. Clearly, the average evacuation time must depend on the number of injured pedestrians and helpers. In the extreme case, where at least one pedestrian is injured but nobody helps, the evacuation time is infinite. The same occurs if everyone is injured. On the other hand, if there are no casualties then no time is invested in searching for them and nobody is slowed down by helping. Evacuation is much faster. A sample simulation with the setting from Section 4.1 with 10% casualties results in an evacuation time of 467 s. In the same scenario without injuries, the evacuation time is 231 s. Only 10% of casualties in this scenario doubles the evacuation time.

As long as there are uncertain parameters and sensitive parameters, one sample simulation does not give a reasonable estimate of the evacuation time. But how can one reasonably quantify the impact of uncertain parameters such as the number of injured pedestrians? We tackle this challenge using uncertainty quantification.

4.2.1. Simulation setup for uncertainty quantification

For a proof of concept, we focus on one car and make several assumption with respect to the initial states of the agents. Again, every seat is occupied, but this time some people are standing, so that there are 60 persons in the car. They evacuate to a 'safe' platform next to the car. We assume that the bomb detonated at one end of the car and that the 14 people near the event are likely to be injured. As long as the percentage of injured pedestrians is set below 25%, the attribute of being injured is randomly assigned to these pedestrians. Above 25%, all of 14 likely casualties are marked as injured. For the remaining number of casualties other

passengers are randomly chosen. The percentage of passengers sharing a social $perc_{injPeds}$ is completely unknown. It is the first uncertain parameter we investigate. The attribute is randomly assigned to passengers using a uniform distribution according to $perc_{injPeds}$. Fig. 12 illustrates the setting.

Further uncertain parameters are the percentage of casualties and the speed at which an aide and charge evacuate. Since there is no data on parameter distributions from real evacuations or experiments, we need to make plausible assumptions. We assume that the parameters are uniformly distributed with the minimum and maximum values in Table 3.
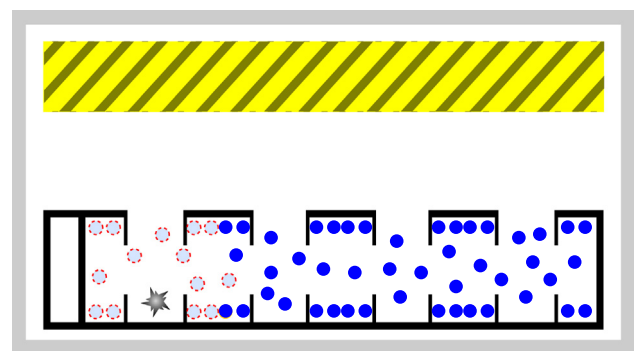

Fig. 12. Scenario for the analysis with uncertainty quantification: evacuation of one car with 60 passengers (blue and light blue circles) to a platform (yellow striped rectangle in the upper part of the picture) after a bomb explosion. The grey star shows the place of the bomb. Thus, the red-rimmed light blue circles on the left side of the train are the passengers with a high risk of injury. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 3
Three uncertain parameters in the Social Identity Model Application. They are uniformly distributed between their minimum and maximum values.

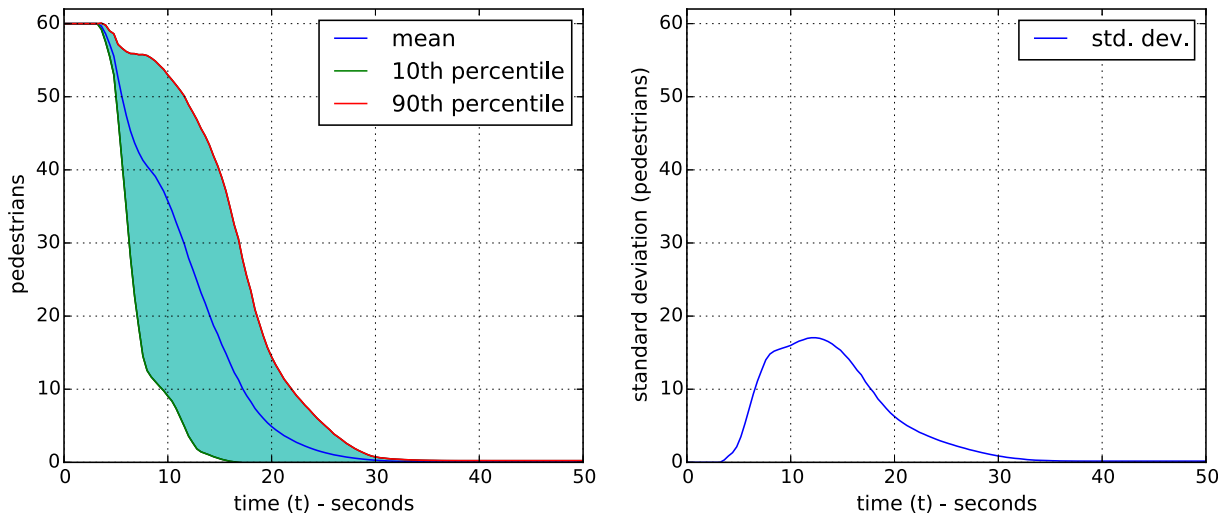| Parameter	Description	Min.	Max.
$perc_{sharingSI}$	People sharing a social identity	0.6	1.0
$perc_{injPeds}$	Percentage of injured pedestrians	0.1	0.3
v_{inj}	Speed of a helper with an injured pedestrian	0.4	0.8

Fig. 13. Uncertainty quantification with three uniformly distributed uncertain parameters. The mean value and the percentiles of the number of pedestrians who remain in the danger zone are plotted on the left. The standard deviation is plotted on the right. In the simulations, the percentage of pedestrians with social identification $perc_{sharingSI}$ varies between 60% and 100%, the percentage of injured pedestrians $perc_{injPeds}$ ranges from 10% to 30%, and the speed of a helper with charge (v_{inj}) ranges from 0.4 m/s to 0.8 m/s.

We choose the maximum evacuation time and the number of people who have not yet reached safety as quantities of interest. At each time step, we analyse the mean and the variance or standard deviation of each quantity of interest, until all persons have evacuated.

The next step is to calculate percentiles which measure which percentage of the determined values are below a specific value. By plotting values of the 10th percentile and the 90th percentile, the space between this values can be interpreted as the area where 80% of the values lie.

4.2.2. Impact of the uncertain parameters

In the context of this contribution, we consider uncertainty in the input parameters of the Social Identity Model Application, that is, the number of pedestrians with social identification $perc_{sharingSI}$, the number of injured pedestrians $perc_{injPeds}$, and the evacuation speed v_{inj}. In the following uncertainty investigation, all three parameters are assumed to be uniformly distributed simultaneously as listed in Table 3.

The quantification results are shown in Fig. 13. In the left plot, the blue line in the filled space is the mean value of the number of pedestrians remaining in the car. Values are plotted from the beginning of the evacuation to the end. Therefore, at the beginning all pedestrians are still in the danger zone, while at the end, everyone has left. The green[1] line on the left of the filled space is the 10th percentile. The red line on the right of the filled space is the 90th percentile. The filled space can be interpreted as follows: with a probability of 80%, the corresponding number of pedestrians remain in the danger zone within the time span. During the very first seconds every agent is evacuating, but nobody has successfully evacuated, because it takes some time to reach the safe area. In the right plot of Fig. 13, the standard deviation for the number of pedestrians who are still in danger is plotted. It illustrates the spread around the mean number of agents at every time step from the beginning of the simulation to the end. The mean for the maximum evacuation time is 21.35 s. The standard deviation is 5.68 s (32.37 variance).

To analyse the impact of a single parameter, only this parameter is disturbed and the others are kept fixed. We choose the average

of the minimum and the maximum values from Table 3 as the fixed values.

Fig. 14 shows the results where the percentage of the injured pedestrians $perc_{injPeds}$ is uncertain. Not surprisingly, we observe that the percentage of injured pedestrians has a strong impact on the number of pedestrians who still remain in the danger zone during the whole simulated evacuation time. During the first seconds of the evacuation, the spread around the mean is small. This is because the number of casualties does not have an impact on the behaviour of agents who are not sharing a social identity and directly evacuate without helping. The mean for the maximum evacuation time is 20.60 s and the standard deviation is 5.43 s.

Next we consider the speed of a helper with an injured pedestrian v_{inj} as uncertain. The result is depicted in Fig. 15. During the first seconds, there is no uncertainty in the number of safely evacuated pedestrians. This can be explained with the immediate evacuation of the unharmed pedestrians who do not share a social identity and who are near the doors. For these pedestrians the speed of a helper with charge plays no role. The mean for the maximum evacuation time is 21.81 s and the standard deviation is 3.29 s.

Finally, Fig. 16 illustrates the results when the percentage of pedestrians with a shared social identity is uncertain. At the beginning of the evacuation, the number of aides has a strong influence on the number of pedestrians remaining in the danger zone. We find this plausible, because many potential helpers make it likely that injured pedestrians are reached quickly and that helping behaviour has an impact on how the situation evolves. Later in the simulation, the influence of the parameter decreases as one might expect: as long as enough unharmed pedestrians share a social identity each injured person will be helped eventually. The values for the maximum evacuation time spread only a little bit around the mean of 21.62 s with a value of 1.11 s for the standard deviation.

Table 4 gives a survey of the results of the four uncertainty quantifications above. From the standard deviation, we see that the percentage of injured pedestrians has the greatest impact on the simulation results. In an earlier sensitivity study (Davidich and Köster, 2013) where the evolution of a passenger stream at a German railway station was simulated and compared to video footage, it was found that changing the targets for the pedestrians had the greatest impact on results compared to other parameters like

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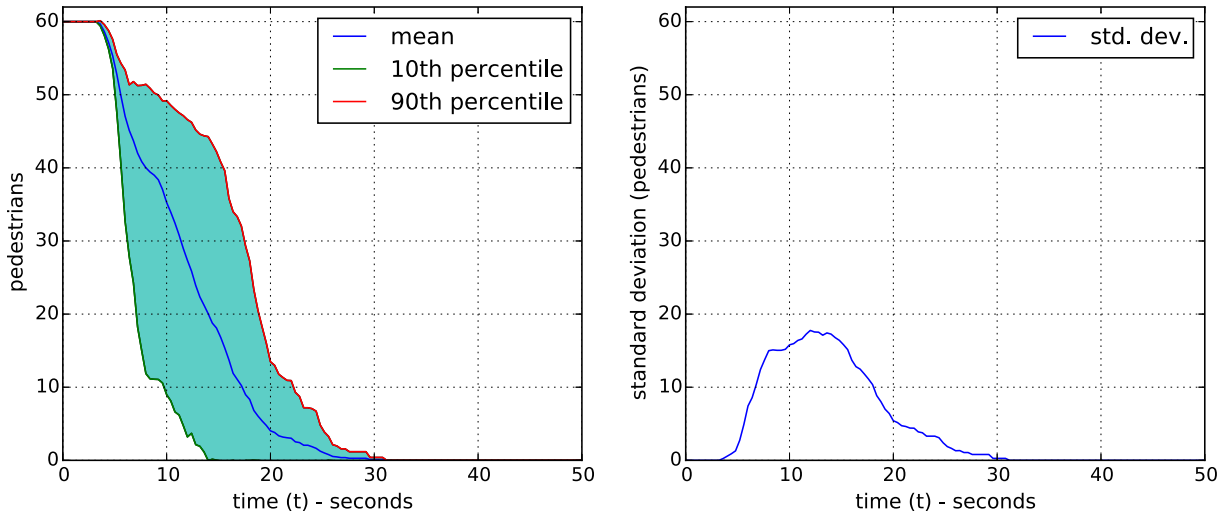[1] For interpretation of color in Fig. 13, the reader is referred to the web version of this article.

Fig. 14. Uncertainty quantification with the percentage of injured pedestrian as uncertain parameter. The mean value and the percentiles of the number of pedestrians who remain in danger are plotted on the left. The standard deviation is plotted on the right. Parameter $perc_{injPeds}$ is uniformly distributed between 10% and 40%.
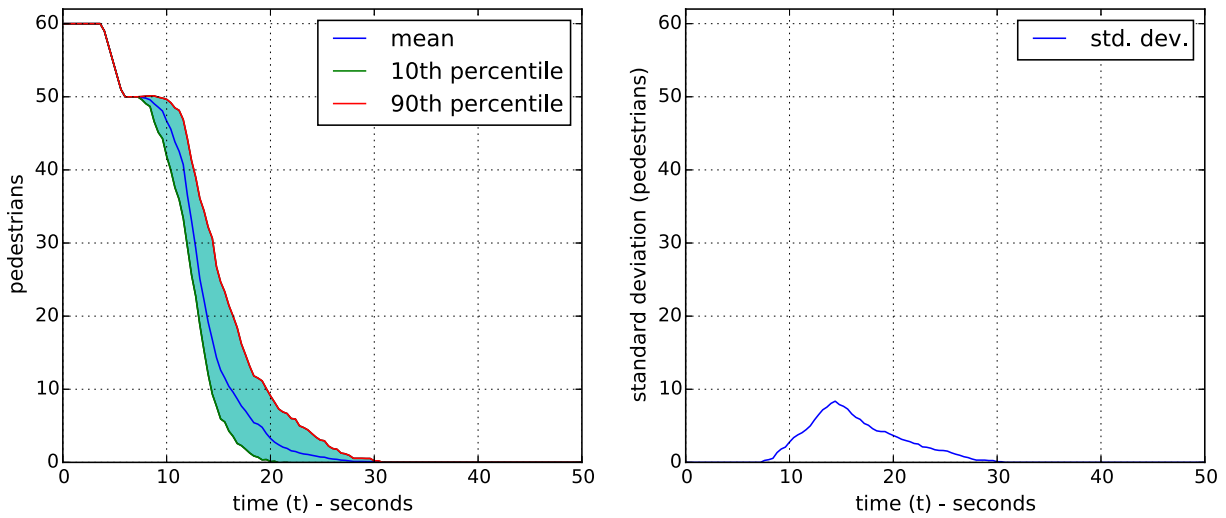

Fig. 15. Uncertainty quantification with the speed of a helper with charge as uncertain parameter. The mean value and the percentiles of the number of pedestrians who remain in danger are plotted on the left. The standard deviation is plotted on the right. Parameter v_{inj} is uniformly distributed between 0.1 m/s and 0.4 m/s.
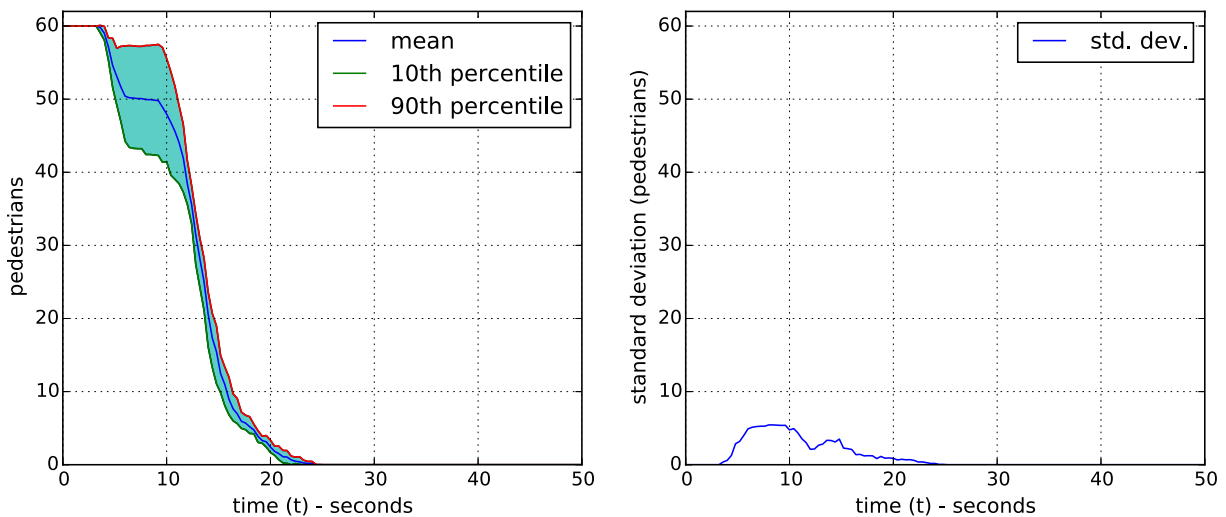

Fig. 16. Uncertainty quantification with the percentage of pedestrians sharing a social identity as uncertain parameter. The mean value and the percentiles of the number of pedestrians who remain in danger are plotted on the left. The standard deviation is plotted on the right. Parameter $perc_{sharingSI}$ is uniformly distributed between 60% and 100%.

Table 4
Comparison of impact of the uncertain parameters in the Social Identity Model Application on the maximum evacuation times.

Uncertain parameter	Max evacuation time	
	Mean	Std. dev.
All three parameters	21.35	5.68
People sharing a social identity	21.62	1.11
Percentage of injured pedestrians	20.60	5.43
Speed of a helper with an injured pedestrian	21.81	3.29

the passengers' preferred walking speeds. Interestingly, this coincides with our new results: in the Social Identity Model Application each injured agent changes, first and above all, targets: for itself and for potential aides. The percentage of pedestrians who share a social identity, on the other hand, has a comparatively small impact. Moreover, if all three parameters are uncertain the impact of the different uncertain parameter does not appear to be cumulative: the standard deviation of 5.68 s in the maximum evacuation time is only marginally higher than the standard deviation of 5.43 s when the percentage of injured pedestrians is the only uncertain parameter.

Through uncertainty quantification we attain a better and deeper understanding of our new model and the parameters – an understanding that cannot be derived from the model itself but is important to safety scientists. And we have good news: while the number of pedestrians sharing a social identity, or the degree of this identification, defies measurement, at least at present, the impact of its variation seems small. Thus, the model retains its predictive power.

Finally, the results quantitatively substantiate our earlier claim that modelling social identity and the helping behaviour that ensues has a very significant impact on evacuation simulations. Neglecting helping behaviour leads to quantitative results at one extreme end of the analysis. In view of the range of the evacuation times in Fig. 14, for example, neglecting injured pedestrians would lead to a serious underestimation of evacuation times.

5. Conclusions

In this paper, we presented an algorithmic formulation of empirical findings from social psychology on human behaviour in a situation of great danger and duress. In particular, we looked at the effect of social identification within a crowd and ensuing helping behaviour during an evacuation. For this, we embedded our algorithm into a pedestrian evacuation simulation. We examined the behaviour in the simulation for a particular scenario that resembled the bomb attack on a metro train in London on July 7th 2005. The computer simulation reproduced observations from the real evacuation. In particular, the agents evacuated in pairs of injured passengers and helpers and the overall behaviour was orderly. We argue, that this constitutes a qualitative validation of the computer model.

Crucially, we went a step beyond qualitative validation: in most cases, one or more model parameters that influence simulation outcomes are uncertain. Either they are entirely unknown, such as the number of injured pedestrians in our virtual evacuation, or they are measured with limited accuracy. We identified three parameters that could be decisive in our model: the percentage of injured pedestrians, the speed at which helpers and charges evacuate, and the percentage of people who share a social identity. We used uncertainty quantification to quantify their influence in an example scenario from which more complex scenarios can be derived. Variations in the percentage of injured pedestrians turned out to have a great influence, whereas variations in the speed had a medium impact, and variations in the percentage of pedestrians

sharing a social identity had a relatively small impact. Since the latter parameter is very hard to measure, this is encouraging news for the safety scientist who needs predictive power of the model to give safety advise on the basis of simulations.

Quantitative validation of our model against measurements, such as trajectories of pedestrians or evacuation times, is still open and must remain open until suitable data is available. Knowing this, we reported all model parameters so that independent researchers can replicate and thus validate – or falsify – our findings.

We consider the instantiation of social identity and helping among strangers in our computer model as a proof of concept that it is indeed possible to carry over findings from social psychology into computer models that possess predictive power. Having more realistic models constitutes progress in itself but there is also an immediate practical application. Safety planners are able to get a much better estimation of evacuation times for scenarios with injured people.

Yet, helping among strangers in emergencies is only one behaviour among many that stem from social identification and that are relevant to safety science. Another important example is the identification with ones own family. Also, with competing social identities the question arises which of the social identities is salient in which situation. Strategies to handle this must be found. Methods from uncertainty quantification which we have introduced to the field of safety science, promise to allow efficient characterisation and quantification of the influence of competing and interworking identities and of further social phenomena.

Acknowledgements

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