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A thermal graviton background from extra dimensions

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Abstract

Inflationary cosmology predicts a low-amplitude graviton background across a wide range of frequencies. This Letter shows that if one or more extra dimensions exist, the graviton background may have a thermal spectrum instead, dependent on the fundamental scale of the extra dimensions. The energy density is shown to be significant enough that it can affect nucleosynthesis in a substantial way. The possibility of direct detection of a thermal graviton background using the 21-cm hydrogen line is discussed. Alternative explanations for the creation of a thermal graviton background are also examined.

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One of the most powerful windows into the early universe is a background of particles whose interactions have frozen-out. The primordial photon background, the primordial baryon background and the primordial neutrino background are all examples of particles that were once in thermal equilibrium. At various times during the history of the universe, the interaction rate of the species in question dropped below the Hubble expansion rate of the universe, causing the species in question to freeze-out. The primordial photon background is observed as the cosmic microwave background (CMB), the baryon background is observed as stars, galaxies, and other normal matter, and the neutrino background, although not observed, is a standard component of big bang cosmology. In addition to these backgrounds, a primordial background of gravitons (or, equivalently, gravitational waves) is expected to exist as well, although it, too, has yet to be detected. The frequency spectrum and amplitude of this background have the potential to convey much information about the early universe. This Letter focuses on using the cosmic gravitational wave background (CGWB) as a probe of extra dimensions.

The success of the inflationary paradigm [1] in resolving many problems associated with the standard

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big bang picture [2] has led to its general acceptance. Inflationary big bang cosmology predicts a stochastic background of gravitational waves across all frequencies [3]. The amplitude of this background is dependent upon the specific model of inflation, but the fractional energy density in a stochastic CGWB is constrained [4] to be

$$\Omega_g \leqslant \mathcal{O}\left(10^{-10}\right). \tag{1}$$

In inflationary cosmology, the predicted CGWB, unlike the CMB and the neutrino background, is non-thermal. Gravitational interactions are not strong enough to produce a thermal CGWB at temperatures below the Planck scale ($m_{\rm pl} \approx 1.22 \times 10^{19}$ GeV). As the existing particles in the universe leave the horizon during inflation, the only major contributions to the energy density will be those particles created during or after reheating, following the end of inflation. Unless the reheat temperature ($T_{\rm RH}$) is greater than $m_{\rm pl}$, gravitational interactions will be too weak to create a thermal CGWB. The measurement of the magnitude of the primordial anisotropies from missions such as COBE/DMR [5] and WMAP [6] provides an upper limit to the energy scale at which inflation occurs [7]. From this and standard cosmological arguments [8], an upper limit on $T_{\rm RH}$ can be derived to be

$$T_{\rm RH} \simeq 6.7 \times 10^{18} (g_*)^{-1/4} \left(\frac{t_{\rm pl}}{t_\phi}\right)^{1/2} \text{GeV}, \tag{2}$$

where g_* is the number of relativistic degrees of freedom at $T_{\rm RH}$, $t_{\rm pl}$ is the Planck time, and t_ϕ is the lifetime of the inflaton. A stronger upper limit on $T_{\rm RH}$ ($\sim 10^8$–10^{10} GeV) can be obtained from nucleosynthesis [9] if supersymmetry is assumed. In all reasonable cases, however, $T_{\rm RH} \ll m_{\rm pl}$, indicating that the CGWB is non-thermal in inflationary cosmology.

If the universe contains extra dimensions, however, predictions about the shape and amplitude of the CGWB may change drastically. Cosmologies involving extra dimensions have been well-motivated since Kaluza [10] and Klein [11] showed that classical electromagnetism and general relativity could be unified in a 5-dimensional framework. More modern scenarios involving extra dimensions are being explored in particle physics, with most models possessing either a large volume [12,13] or a large curvature [14,15]. Any spatial dimensions which exist beyond the standard three must be of a sufficiently small scale that they do not conflict with gravitational experiments. The $(3+1)$-dimensional gravitational force law has been verified down to scales of 0.22 mm [16]. Thus, if extra dimensions do exist, they must be smaller than this length scale.[1] Although there exist many different types of models containing extra dimensions, there are some general features and signals common to all of them.

In the presence of δ extra spatial dimensions, the $(3+\delta+1)$-dimensional action for gravity can be written as

$$\mathcal{S} = \int d^4x \left\{ \int d^\delta y \sqrt{-g'} \frac{\mathcal{R}'}{16\pi G'_N} + \sqrt{-g}\mathcal{L}_m \right\},$$
$$G'_N = G_N \frac{m_{\rm pl}^2}{m_D^{2+\delta}}, \tag{3}$$

where g is the 4-dimensional metric, G_N is Newton's constant, g', G'_N, and $\mathcal{R}'$ denote the higher-dimensional counterparts of the metric, Newton's constant, and the Ricci scalar, respectively, and m_D is the fundamental scale of the higher-dimensional theory. In $3+\delta$ spatial dimensions, the strength of the gravitational interactions scale as $\sim (T/m_D)^{(1+\delta/2)}$. If $\delta = 0$, then $m_D = m_{\rm pl}$, and standard 4-dimensional gravity is recovered.

When energies in the universe are higher than the fundamental scale m_D, the gravitational coupling strength increases significantly, as the gravitational field spreads out into the full spatial volume. Instead of freezing out at $\sim \mathcal{O}(m_{\rm pl})$, as in $3+1$ dimensions, gravitational interactions freeze-out at $\sim \mathcal{O}(m_D)$ [12]. m_D can be much smaller than $m_{\rm pl}$, and may be as small as $\sim$ TeV-scale in some models. If the gravitational interactions become strong at an energy scale below the reheat temperature ($m_D < T_{\rm RH}$), gravitons will have the opportunity to thermalize, creating a thermal CGWB. Fig. 1 illustrates the available parameter space for the creation of a thermal CGWB in the case of large extra dimensions, following the formalism in [18]. Other types of extra dimensions have minor quantitative differences in the shape of their parameter spaces. However, the qualitative result, the creation of a thermal CGWB if $m_D < T_{\rm RH}$, is unchanged by the type of extra dimensions chosen.

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[1] A possible explanation for the vast difference in size between the three known spatial dimensions and any extra dimensions is given in [17].

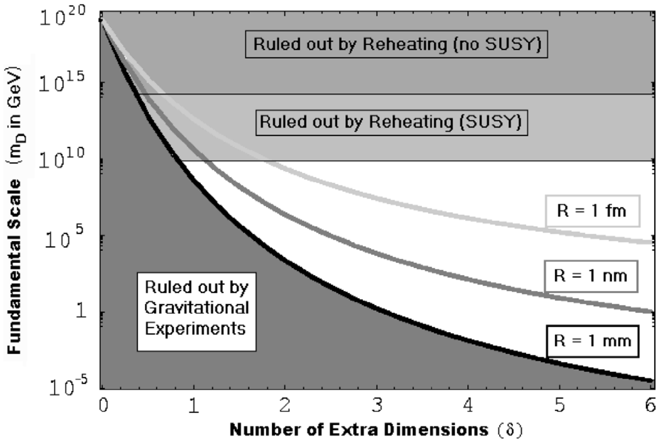

Fig. 1. Parameter space for the creation of a thermal CGWB in the context of large extra dimensions. The shaded areas represent areas ruled out by gravitational experiments and reheating, both with and without the assumption of supersymmetry. Certain assumptions about gravitino physics, as detailed in [9], may significantly lower the bound on reheating with supersymmetry in extra dimensions.

Thus, if extra dimensions do exist, and the fundamental scale of those dimensions is below the reheat temperature, a relic thermal CGWB ought to exist today. Compared to the relic thermal photon background (the CMB), a thermal CGWB would have the same shape, statistics, and high degree of isotropy and homogeneity. The energy density (ρ_g) and fractional energy density (Ω_g) of a thermal CGWB are

$$\rho_g = \frac{\pi^2}{15}\left(\frac{3.91}{g_*}\right)^{4/3}(T_{\mathrm{CMB}})^4, \tag{4}$$

$$\Omega_g \equiv \frac{\rho_g}{\rho_c} \simeq 3.1 \times 10^{-4}(g_*)^{-4/3}, \tag{5}$$

where ρ_c is the critical energy density today, T_{CMB} is the present temperature of the CMB, and g_* is the number of relativistic degrees of freedom at the scale of m_D. g_* is dependent on the particle content of the universe, i.e., whether (and at what scale) the universe is supersymmetric, has a KK tower, etc. Other quantities, such as the temperature (T), peak frequency (ν), number density (n), and entropy density (s) of the thermal CGWB can be derived from the CMB if g_* is known, as

$$n_g = n_{\mathrm{CMB}}\left(\frac{3.91}{g_*}\right), \qquad s_g = s_{\mathrm{CMB}}\left(\frac{3.91}{g_*}\right),$$
$$T_g = T_{\mathrm{CMB}}\left(\frac{3.91}{g_*}\right)^{1/3}, \qquad \nu_g = \nu_{\mathrm{CMB}}\left(\frac{3.91}{g_*}\right)^{1/3}. \tag{6}$$

These quantities are not dependent on the number of extra dimensions, as the large discrepancy in size between the three large spatial dimensions and the δ extra dimensions suppresses those corrections by at least a factor of $\sim 10^{-29}$. As an example, if m_D is just barely above the scale of the standard model, then $g_* = 106.75$. The thermal CGWB then has a temperature of 0.905 K, a peak frequency of 19 GHz, and a fractional energy density $\Omega_g \simeq 6.1 \times 10^{-7}$.

Although the fractional graviton energy density is expected to be small today, it may be detectable either indirectly or directly. Nucleosynthesis provides an indirect testing ground for a thermal CGWB. Standard big bang nucleosynthesis predicts a ^{4}He abundance of $Y_p = 0.2481 \pm 0.0004$ [19]. With a thermal CGWB included, the expansion rate of the universe is slightly increased, causing neutron–proton interconversion to freeze-out slightly earlier. A thermal CGWB can be effectively parameterized as neutrinos, as they serve the same function at that epoch in the universe (as non-collisional radiation). The effective number of neutrino species is increased by $N_{\nu\text{-eff}} \simeq 27.1(g_*)^{-4/3}$, or $\simeq 0.054$ (for $g_* = 106.75$). This would yield a new prediction of $Y_p = 0.2489 \pm 0.0004$. Although observations are not yet able to discriminate between these two values, the constraints are tightening with the advent of recent data [20]. An increase in the precision of various measurements, along with an improvement in the systematic uncertainties, may allow for the indirect detection of a thermal CGWB.

Direct detection of a thermal CGWB is much more challenging, but would provide quite strong evidence for its existence. Conventional gravitational-wave detectors include cryogenic resonant detectors [21], which have evolved from the bars of Weber [22], Doppler spacecraft tracking, and laser interferometers [23]. The maximum frequency that these detectors can probe lies in the kHz regime, whereas a thermal CGWB requires GHz-range detectors. An interesting possibility for detection may lie in the broadening of quantum emission lines due to a thermal CGWB. Individual photons experience a frequency shift due to gravitational waves [24]. For a large sample of radio-frequency photons in a gravitational wave background, the observed line width (W) will broaden by

$$\Delta W \sim h_0 \sim \frac{\sqrt{\Omega_g}}{\nu t_0} \sim 10^{-31}\left(\frac{106.75}{g_*}\right)^{1/3}, \tag{7}$$

where t_0 is the present age of the universe, ν is the peak frequency of the thermal CGWB and h_0 is the metric perturbation today due to the thermal CGWB [25]. As $\mathcal{O}(10^{-31})$ is a very small broadening, a radio line with a narrow natural width is the preferred candidate to observe this effect. One possibility for this type of observation is the 21-cm emission line of atomic hydrogen. So long as the emitting atoms and the detectors are sufficiently cooled, broadening due to thermal noise will be suppressed below ΔW. Because the lifetime $(1/\Gamma)$ of the excited state of hydrogen is large $(\sim 10^7$ yr) and the frequency of the emitted light (ν_γ) is high $(\sim 10^9$ Hz), the natural width (W) is among the smallest known

$$W = \frac{\Gamma}{\nu_\gamma} \simeq \frac{2.869 \times 10^{-15}\ \mathrm{s}^{-1}}{1.42040575179 \times 10^9\ \mathrm{s}^{-1}} \simeq 2.02 \times 10^{-24}. \tag{8}$$

The width of the 21-cm line is regrettably seven orders of magnitude larger than the expected broadening due to a thermal CGWB. Extraordinarily accurate measurements would need to be taken for direct detection of this background. Additionally, temperatures of the atoms and detectors would need to be cryogenically cooled to $\sim 10^{-18}$ K to suppress thermal noise below ΔW. This is far beyond the reach of current technology, and either a major advance or experimental innovation would be required to measure the desired effect using this technique.

Extra dimensions are not the only possible explanation for the existence of a thermal CGWB. Currently, there are three known alternative explanations that would also create a thermal CGWB. They are as follows: there was no inflation, there was a spectrum of low-mass primordial black holes that have decayed by the present epoch, or the gravitational constant is time-varying (the Dirac hypothesis). Each alternative is shown below to face difficulties that may make extra dimensions an attractive explanation for the creation of a thermal CGWB.

The predictions of inflation are numerous [7], and many have been successfully confirmed by WMAP [6]. The major successes of inflation include providing explanations for the observed homogeneity, isotropy, flatness, absence of magnetic monopoles, and origin of anisotropies in the universe. Additionally, confirmed predictions include a scale-invariant matter power spectrum, an $\Omega = 1$ universe, and the spectrum of CMB anisotropies. To explain a thermal CGWB by eliminating inflation would require alternative explanations for each of the predictions above. Although alternative theories have been proposed, as in [26], they have been shown to face significant difficulties [27]. The successes of inflation appear to suggest that it may likely provide an accurate description of the early universe.

Primordial black holes with masses less than 10^{15} g would have decayed by today, producing thermal photons, gravitons, and other forms of radiation. Density fluctuations in the early universe, in order to produce a large mass fraction of low-mass primordial black holes, and not to produce too large of a mass fraction of high-mass ones, favor a spectral index n that is less than or equal to $2/3$ [28]. Accepting the observed scale-invariant $(n \simeq 1)$ spectrum of density fluctuations [29] may disfavor primordial black holes as a reasonable candidate for creating a thermal CGWB.

The Dirac hypothesis states that the difference in magnitude between the gravitational and electromagnetic coupling strengths arises due to time evolution of the couplings [30]. If true, gravitational coupling would have been stronger in the early universe. At temperatures well below the Planck scale, gravity would have been unified with the other forces, creating a thermal CGWB at that epoch. However, this hypothesis produces consequences for cosmological models that are difficult to reconcile [31], and any time variation is severely constrained by geophysical and astronomical observations [32]. The acceptable limits for variation are small enough that they cannot increase coupling sufficiently to generate a thermal CGWB subsequent to the end of inflation. The difficulties faced by each of these alternative explanations points towards extra dimensions as perhaps the leading candidate for the creation of a thermal CGWB.

There exist two major obstacles to the construction of a more complete phenomenological model containing extra dimensions with $m_D < T_{\mathrm{RH}}$. The first of these is the moduli problem [33]. String moduli interactions with standard model fields are highly suppressed, leading to a long lifetime of the string moduli. String moduli decay, however, must be consistent with astrophysical constraints [34]. To accomplish this, string moduli need either a small production amplitude or very specific decay channels, which both require fine-tuning. The second problem is the overproduction of long-wavelength tensor modes from inflation [35,36]. While the short-wavelength modes (the modes inside the horizon when gravitational interactions freeze-out) will thermalize, gravitational waves of longer wavelengths will be unaffected. As the scale of inflation must be above m_D, the amplitude of these waves is expected to be large. This would leave an unacceptable imprint in the CMB. Both problems arise from the fact that at energies above m_D, macroscopic gravity breaks down [37]. Although these problems may not be resolved until a quantum theory of gravity is realized, they do not change the fact that a thermal CGWB would arise from extra dimensions with $m_D < T_{\mathrm{RH}}$.

This work has attempted to show that extra dimensions may be responsible for the production of a thermal gravitational wave background. A thermal CGWB, as opposed to the stochastic CGWB of standard inflationary cosmology, is a prediction of extra dimensions with a scale below the reheat temperature. The detection of a thermal CGWB, although challenging at present, would provide strong evidence for the existence of extra dimensions. The detected absence of a thermal CGWB would conversely disfavor the existence of extra dimensions up to the energy scale of the reheat temperature.

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