More singular? Self-similar dynamics of damage localization and instability in dynamic fracture

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Abstract

Dynamic fracture instability is studied for three characteristic statements: dynamic crack propagation, dynamic fragmentation and failure wave initiation to combine theoretical interpretation and “in-situ” high resolution experimental data for dynamically loaded quasi-brittle materials (PMMA, glasses, ceramics and fused quartz). Specific type of criticality (the structural-scaling transitions) was established in solid with defects in terms of two structural variables: defect density tensor (defect induced strain) and structural scaling parameter. Two critical values for structural scaling parameter were found that separate characteristic nonlinearity of free energy release in damage kinetic equation that allowed one to establish specific self-similar solution (blow-up dissipative structures) responsible for final stage of damage localization and crack initiation. The set of spatial scales of damage localization (that has the image of mirror zones) and corresponding “incubation” time follow from the self-similar solution as the “eigen-values” of non-linear problem. Different scenario of instability of dynamic fracture in mentioned experiments were analyzed as nonlinear dynamic problem in the presence of two self-similar solutions (two attractors): intermediate asymptotical solution of damage evolution equation and describes the blow-up damage localization (as the basement for stress intensity factor) and the set of blow-up self-similar solutions for damage localization kinetics playing the role of collective modes of damage localization. Theoretically predicted flicker noise temporal-spatial statistics was found in dynamic experiments for fused quartz and ceramic rod fragmentation combined with fracto-luminescence recording and analysis of fragment statistics. Resonance excitation of blow-up modes allowed explanation of self-similar pattern of multiple spall kinetics (“dynamic branch” under spall) due to “resonance” excitation of blow-up damage localization kinetics.

Keywords: Damage localization, blow-up self-similar solution, dynamic crack propagation, failure wave, fragmentation statistics

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1. Introduction

Statistically based approach was developed for the constitutive modeling of materials to provide links between defect induced mechanisms of structural relaxation, damage-failure transition and material responses in wide range of load intensities. It is shown that the process of damage-failure transition can be considered as specific type of criticality in out-of-equilibrium system “solid with defects” and wide range constitutive model was proposed as the generalization of the Ginzburg-Landau approach in terms of independent field variables describing typical mesoscopic defects (microshears, microcracks). Specific types of the collective modes of defects were established as self-similar solution of evolution equation for mentioned damage parameter. This solution represents the intermediate asymptotical solution of damage evolution equation and describes the blow-up damage localization kinetics on the set of spatial scales (damage localization areas). The set of blow-up self-similar collective modes of defects can be considered as the independent variables provided universality of nonlinear dynamics of damage-failure transition from steady-state crack propagation to the branching regime with pronounced intermittency in crack propagation velocity, “resonance” excitation of damage localization in shocked materials (“dynamic branch” under spall failure, failure waves), spatial-temporal power law universality in dynamic fragmentation. Original experimental data supported the assumption concerning the role of multiscale blow-up collective modes of defects on self-similar responses of materials in wide range of load intensity

The goal of present study is to link the scenario of damage-failure transition in wide range of load intensity ith self-similar dynamics of damage localization supported by original in-situ experiments.

Nomenclatures mentioned in the article are listed below.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$F$</td>
<td>free energy</td>
</tr>
<tr>
<td>$A, B, C, D, G, m$</td>
<td>material parameters</td>
</tr>
<tr>
<td>$\delta$</td>
<td>structural-scaling parameter</td>
</tr>
<tr>
<td>$\delta_e, \delta_c$</td>
<td>critical values of structural-scaling parameter</td>
</tr>
<tr>
<td>$p_{ik}$</td>
<td>defect density tensor</td>
</tr>
<tr>
<td>$\sigma_{zz}$</td>
<td>stress component</td>
</tr>
<tr>
<td>$\chi$</td>
<td>nonlocality coefficient</td>
</tr>
<tr>
<td>$\Gamma_p, \Gamma_\delta$</td>
<td>kinetic coefficients</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>strain component</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$\hat{C}$</td>
<td>elastic compliance tensor</td>
</tr>
<tr>
<td>$g(t)$</td>
<td>temporal function of blow-up self-similar solution</td>
</tr>
<tr>
<td>$f(\xi)$</td>
<td>spatial function of blow-up self-similar solution</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>blow-up time</td>
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<tr>
<td>$\rho_c$</td>
<td>critical defect density</td>
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<tr>
<td>$L_H, L_c$</td>
<td>self-similar scales of damage localization</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency</td>
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<tr>
<td>$V$</td>
<td>crack velocity</td>
</tr>
<tr>
<td>$V_B, V_C$</td>
<td>characteristic crack velocity</td>
</tr>
<tr>
<td>$V_{fw}$</td>
<td>velocity of failure wave</td>
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2. Structural-scaling transitions. Collective modes of defects

Statistical theory of typical mesoscopic defects (microcracks, microshears) allowed us to establish specific type of critical phenomena in solid with defects – structural-scaling transitions and to propose the phenomenology of damage-failure transition (Naimark (2004)). The key results of the statistical theory and statistically based
The generation of these collective modes under the loading provides statistics. The systems reveal the parameter \( \delta \) are presented in Fig. 3 according to the high speed framing data \( H \). 

Material responses on the defect growth (quasi-brittle, ductile and fine-grain state) in corresponding characteristic values of structural-scaling parameter (bifurcation points) that define the areas of typical nonlinear phenomenology are the establishment of two order parameters responsible for the structure evolution – the defect density tensor \( p_{ik} \) and the structural scaling parameter \( \delta = \left( R/r_0 \right)^3 \), which represents the ratio of the spacing between defects and characteristic size of defects. The value of structural-scaling parameter characterizes the current susceptibility of material to the nucleation and growth of defects. Dependence of solid responses on structural-scaling parameter reflects important feature of damage kinetics, statistical self-similarity, that was established for microshear (microcrack) distribution function for different stages of damage accumulation represented in dimensionless (self-similar) coordinates. Statistically predicted non-equilibrium free energy \( F \) represents generalization of the Ginzburg-Landau expansion in terms of mentioned order parameters – the defect density tensor (defect induced deformation \( p(x) = p_{zz} \) in uni-axial loading deformation in \( z \)-direction) and structural scaling parameter \( \delta \):

\[
F = \frac{1}{2} A(\delta, \delta_z)p^2 - \frac{1}{4} Bp^4 - \frac{1}{6} C(\delta, \delta_z)p^6 - D\sigma p + \chi \left( \frac{\partial p}{\partial x} \right)^2,
\]

where \( \sigma = \sigma_{zz} \) is the stress, \( \chi \) is the non-locality parameter, \( A, B, C, D \) are the material parameters. \( \delta_e \) and \( \delta_c \) are characteristic values of structural-scaling parameter (bifurcation points) that define the areas of typical nonlinear material responses on the defect growth (quasi-brittle, ductile and fine-grain state) in corresponding \( \delta \)–ranges: \( \delta < \delta_c \approx 1, \ \delta_c < \delta < \delta_e, \ \delta > \delta_e \approx 1.3 \). Free energy form (Eq.1) represents multi-wall potential with qualitative different metastability in the ranges \( \delta_e < \delta < \delta_c \) and \( \delta < \delta_c \approx 1 \).

![Fig. 1. Nonlinear responses of material on defect density \( p \) in different ranges of structural-scaling parameter \( \delta \).](image)

The damage kinetics is determined by the kinetic equations for the defect density \( p \) and structural-scaling parameter \( \delta \)

\[
\dot{p} = -\Gamma_p \frac{\Delta F}{\Delta p}, \quad \dot{\delta} = -\Gamma_\delta \frac{\partial F}{\partial \delta},
\]

where \( \Gamma_p, \Gamma_\delta \) are the kinetic coefficients, \( \Delta(\ldots)/\Delta t \) is the variation derivative. Kinetic equations (Eq.2) and the equation for the total deformation \( \varepsilon = \hat{C} \sigma + p \) (\( \hat{C} \) is the component of the elastic compliance tensor) represent the constitutive equations of materials with mesodefects. Material responses on the loading realize as the generation of characteristic collective modes – the autosolitary waves in the range of \( \delta_c < \delta < \delta_e \) and the “blow-up” dissipative structure in the range \( \delta < \delta_c \approx 1 \). The generation of these collective modes under the loading provides the change of the system symmetry according to the group properties of equations in corresponding ranges of structural-scaling parameter and initiates specific mechanisms of the momentum transfer (plastic relaxation) and
damage-failure transition on the scales of damage localization with the blow-up kinetics. The damage-failure scenario includes the “blow-up” kinetics of damage localization as the precursor of crack nucleation according to the self-similar solution (Belyaev (1990)): 

\[ p = g(t)f(\xi), \xi = x/L_H, \quad g(t) = G(1 - t/\tau_c)^{-m}, \]  

(3)

where \( \tau_c \) is the so-called "peak time" ( \( p \to \infty \) at \( t \to \tau_c \) for the self-similar profile \( f(\xi) \) localized on the scale \( L_H \), \( G > 0, m > 0 \) are the parameters of non-linearity, which characterise the free energy release rate for \( \delta < \delta_c \). The self-similar solution Eq.3 describes the blow-up damage kinetics for \( t \to \tau_c \), \( p \to p_c \) (Fig.1) on the set of spatial scales \( L_H = kL_c, \quad k = 1,2,...,K \), where \( L_c \) and \( L_H \) corresponds to the so-called “simple” and “complex” blow-up dissipative structures. Generation of the complex blow-up dissipative structures appears when the distance \( L_s \) between simple structures approaches to the scale \( L_c \). Similar scenario of the “scaling transition” proceeds for the blow-up structures of different complexity to involve in the process of the final stage of damage localization the larger scales of material. The description of damage kinetics as the structural-scaling transition allowed the consideration of solid with defects as a dynamic system with spatial degrees of freedom (corresponding to the set of blow-up dissipative structures of different complexity). Stochastic behavior in this case can be linked with the dynamics of out-of-equilibrium system with the features of flicker noise, or \( 1/f \) - statistics. The systems reveal the so-called self-organized criticality (SOC) with universal behavior that is typical for the late state evolution of dynamic systems, when the correlation will appear on all length of scales. The self-similar nature of mentioned collective modes associated with damage localization zones has the great importance in the case of dynamic loading, when the “excitation” of these modes can lead to the subjection of relaxation and failure to the dynamics of these modes. The examples for this situation are the transition from the steady-state to the branching regimes of crack propagation, qualitative change of the fragmentation statistics with the increase of the energy density imposed into the material.

3. Multicenter failure in spall conditions. Dynamic crack propagation

Considerably interest has recently been attracted to the multiscale damage-failure kinetics under impact loading of quasi-brittle materials (ceramics, glasses, polymers). Experiments have shown that fracture during extension pulse is a multicenter nature with the generation of the mirror zones with characteristic size related to the stress ramp in the corresponding spall cross-section. This scenario has relationship between the development of multicenter fracture and the so-called “dynamic branch effect” under spall failure. Experiments were carried out on rods (10-12 mm in diameter and 100-200 mm long) of PMMA and ultraporcelain (85% Al₂O₃, 15% SiO₂), Fig.2. A compression pulse was excited in the samples by impact on a light-gas gun. Parameters of the compression pulse were measured with a laser differential interferometer. From the results of experimental studies of spall failure the diagrams of fracture time \( t_f \) versus the amplitude \( \sigma_a \) of the tensile stress were plotted. It was established the correspondence of failure hotspots nucleation having the image of mirror zones in experiments with numerous spall failure (Belyaev(1990), Bellendir (1989)). The multiple mirror zones with an equal size were excited on different spall cross sections in the shocked rod when the stress wave amplitude exceeds some critical value corresponding to the transition from the quasi-static to the so-called “dynamic branch”. The point of transition from the quasi-static to dynamic branch corresponds to the qualitative change in the fractography image of fracture surface: generation of failure hotspot (mirror zone area) near the rod surface in quasi-static case and numerous hotspots with characteristic size depending on the stress ramp in spall cross-sections. The low sensitivity of fracture time \( t_f \) on stress amplitude \( \sigma_a \) reflects specific nonlinearity of damage-failure transition corresponding to the self-similar blow-up localization kinetics that provides the “resonance excitation” of failure hotspots (mirror zones) with low sensitive to the stress amplitude. Similar “low sensitivity” to the applied stress was observed in experiment for dynamic crack propagation in preloaded PMMA plate (Naimark (2004a), Naimark (2004b)). The stress field at the area of crack tip in the diagram “crack velocity \( \dot{V} \) versus applied stress \( \sigma \)” are presented in Fig.3 according to the high speed framing data
(REMIX REM 10-8 camera, time lag between pictures 10 μS). Three characteristic regimes of crack dynamics were established in the different ranges of crack velocity: steady-state $V < V_c$, branching $V > V_c$ and fragmenting $V > V_B$, when the multiply branches of main crack have the autonomous behavior (Fig.3, 4).

The scaling properties of failure were studied also under the recording of the stress dynamics using the polarization scheme coupled with the laser system, Fig.6,7. The stress temporal history was measured in the marked point deviated from the main crack path on the fixed (4 mm) distance. This allowed us to investigate the correlation property of the system using the stress phase portrait $\dot{\sigma} \sim \sigma$ for slow and fast cracks.

The existence of three characteristic branches for nonlinear crack dynamics and self-similar features of spall failure initiation was stimulating to consider stochastic aspects of failure (fragmentation statistics) and special case of failure dynamics subject to kinetics of numerous blow-up collective modes of damage localization. It is naturally to assume that the exponential statistics of fragmentation that is generally discussed for the moderate load intensity for failure dynamics subject to kinetics of numerous blow-up collective modes of damage localization. It is naturally to assume that the exponential statistics of fragmentation that is generally discussed for the moderate load intensity for

![Stress Phase portrait for steady-state](image)

![Stress pattern for steady-state](image)

![Crack velocity](image)

![Failure surface](image)
These portraits display the regular stress dynamics (Fig.6) for $\sigma < \sigma_0$ ($V < V_c$) and the stochastic dynamics (Fig.7) for $V > V_c$ related to the second type of the attractor with the set of coordinates corresponding to the blow-up modes of different complexity (mirror zones with different sizes). In the transient regime $V \approx V_c$ the co-existence of two attractors can appear that can lead to the intermittency effect that is characteristic for branching crack dynamics.

4. Fragmentation statistics. Resonance excitation of failure (failure waves)

The existence of three characteristic branches for nonlinear crack dynamics and self-similar features of spall failure initiation was stimulating to consider stochastic aspects of failure (fragmentation statistics) and special case of failure initiation under intensive loading, the so-called, failure wave phenomenon that is observed in shocked glasses. Fragmentation statistics was studied during in situ experiments for impact loaded fused quartz rods and fracture luminescence recording to analyze the temporal sequences of failure hotspots initiation and the following study of fragmentation statistics for recovered samples after the fragment weighing (Fig.8,9) (Grady(2010), Davydova (2010), Davydova (2014)). Temporal fracture luminescence events and fragment size distribution demonstrated the power law statistics (the flicker or $1/f$ - noise) that is characteristic for the out-of-equilibrium critical systems revealing the so-called self-organized criticality (SOC) (Fig.10,11). The comparison with experimental data of nonlinear crack dynamics and self-similar kinetics of failure due to numerous “mirror zone” nucleation at the “dynamic branch” of failure allowed us to conclude that the power law statistics is characteristic for failure dynamics subject to kinetics of numerous blow-up collective modes of damage localization. It is naturally to assume that the exponential statistics of fragmentation that is generally discussed for the moderate load intensity is characteristic for stage when stress intensity factor and damage localization kinetics factor provides the intermediate statistics related to both factors. It is interesting the limit case of failure revealing the temporal-spatial independence of failure evolution on stress. Namely this situation is observed in experiment for failure wave initiation Fig.12.

Experimental study of failure wave generation and propagation was realized for the symmetric Taylor test on fused-quartz rods (Razorenov (1991), Plekhov (2000), Naimark (2003)). Fig. 12 shows the processing of a high-speed photography (upper picture) for the flyer rod travelling at 534 m/s at impact. Three dark zones correspond to the image of impact surface (green triangle), failure wave (red square) and (blue diamond) the shock wave. The initial slope for the failure wave gives the front velocity $V_{fw} \approx 1.57 \text{ km/s}$ that is close to traditionally measured in the plate impact test (Naimark (2003)). However, the experiment revealed the increase of failure front velocity up to the value $V_{fw} \approx 4\text{ km/s}$. Approaching of failure wave front velocity to the shock front velocity supports theoretically based result concerning the failure wave nature as “delayed failure” with the limit of “delay time” corresponding to the “peak time” in the self-similar solution, Eq.3. Theoretical analysis of this situation was proposed in (Plekhov (2000)) and allowed the interpretation of damage kinetics as the “resonance excitation” of blow-up damage.
localization modes.

3. Conclusion

Scaling aspects and nonlinearity of damage-failure transition were explained as the consequence of subjection of damage kinetics to intermediate asymptotical (self-similar) solution. This solution has the nature of multiscale blow-up dissipative structures, represents the set of collective modes of defects responsible for the damage localization stage. Spatial-temporal kinetics of nucleation and interaction of collective modes of defects allowed us to link qualitative different scenarios of multiscale damage-failure transitions in wide range of load intensities with dynamics of these modes. Original experiments were used to verify theoretical results and modeling: “in-situ” observation of crack dynamics in preloaded PMMA plates–transition from the steady to branching regimes and fragmentation; failure wave initiation in shocked fused quartz rods; multicenter spall failure in PMMA and ceramic
roads corresponding to the “spall dynamic branch” regime; scaling analysis of fragmentation statistics based on the “in-situ” recorded fracture luminescence signals in dynamically loaded glass and ceramic rods and analysis of fragment mass statistics in recovered samples. It was shown that crack branching dynamics, failure wave, multicenter spall failure, fragmentation statistics are related to the specific class of the universality for multiscale damage localization phenomena – the excitation of multiscale blow-up collective modes of defects, which can be considered as the independent collective variables. Morphological image of these modes on the fracture surface is the mirror zones. The failure wave and “dynamic branch” phenomena correspond to the case of “resonance excitation” of blow-up regime of damage localization with pronounced features of “delayed failure” Qualitative changes in the fragmentation statistics (both temporal and fragment size distribution) were found: transition from exponential to power law statistics with increasing of load intensity. This result supported theoretical prediction concerning self-criticality scenario of fragmentation caused by multiscale blow-up damage localization kinetics

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