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Shear viscosity and Chern–Simons diffusion rate from hyperbolic horizons

George Koutsoumbas [a], Eleftherios Papantonopoulos [a,*], George Siopsis [b]

[a] Department of Physics, National Technical University of Athens, GR 157 73 Athens, Greece
[b] Department of Physics and Astronomy, The University of Tennessee, Knoxville, TN 37996-1200, USA

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ABSTRACT

We calculate the shear viscosity and anomalous baryon number violation rate in quantum field theories at finite temperature having gravity duals with hyperbolic horizons. We find the explicit dependence of these quantities on the temperature. We show that the ratio of shear viscosity to entropy density is below $1/(4\pi)$ at all temperatures and can be made arbitrarily small in the low temperature limit for hyperbolic surfaces of sufficiently high genus so that the hydrodynamic limit remains valid.

In certain finite-temperature quantum field theories having gravity duals with black brane solutions in higher spacetime dimensions, the hydrodynamic behaviour of the thermal field theory is identified with the hydrodynamic behaviour of the dual gravity theory [1]. It was shown [2] that for these field theories, the ratio of the shear viscosity to the volume density of entropy has a universal value $\eta/s = 1/(4\pi)$ and it was further conjectured that this is the lowest bound on the ratio η/s for a large class of thermal quantum field theories.

This conjecture was tested against a wide range of thermal field theories having gravity duals: in gauge theories with chemical potentials studying their R-charged black hole duals [3], in field theories with stringy corrections [4] and also in field theories with gravity duals of Einstein-Born–Infeld gravity [5]. In all these theories it was found that the lower bound is satisfied. However, in conformal field theories dual to Einstein gravity with curvature square corrections it was found that the bound is violated [6] but the physical implication of the violation of the bound is still not clear.

In the gravity sector of this gravity/gauge duality, maximally symmetric spaces naturally arise as the near-horizon region of black brane geometries [7]. Spherically symmetric spaces have been extensively investigated. Also hyperbolic geometries involving n-dimensional hyperboloids $\mathbb{H}^n$ or $\mathbb{H}^n/\Gamma$ cosets, where Γ is a discrete subgroup of the isometry group of $\mathbb{H}^n$, arise naturally in supergravity as a result of string compactifications [8]. However, the presence of the discrete group Γ introduces another scale which breaks all supersymmetries. $\mathcal{N}=0$ conformal field theories can be constructed having gravity duals with constant negative curvature [8,9].

The hydrodynamic properties of the boundary conformal field theory can be inferred from the lowest frequency quasinormal modes of the gravity sector [10]. The lowest-lying gravitational quasinormal modes for a Schwarzschild–AdS solution were numerically calculated in four and five dimensions and were shown to be in agreement with hydrodynamic perturbations of the gauge theory plasma on the AdS boundary [11]. For AdS_5 this was understood as a finite "conformal soliton flow" after the spherical AdS_5 boundary obtained in global coordinates was conformally mapped to the physically relevant flat Minkowski spacetime. This study was extended to black holes with a hyperbolic horizon. It was shown in [12] that the quasinormal modes obtained agreed with the frequencies resulting from considering perturbations of the gauge theory fluid on the boundary.

Recently, interesting features have shown up in the study of topological black holes (TBH). The spectrum of the quasinormal modes of TBH [13] has been studied extensively [14]. For large black holes this spectrum is similar to the Schwarzschild–AdS spectrum. For small black holes however the quasinormal modes spectrum is quite different. It was found [15] that there is a critical temperature, below which there is a phase transition of the TBH to AdS space. This has been attributed entirely to the properties of the hyperbolic geometry.

In this work we will show that the hyperbolic geometry allows us to calculate hydrodynamic transport coefficients like shear viscosity and the Chern–Simons diffusion rate of the boundary thermal field theory at any temperature under certain conditions. This should be contrasted with the case of a spherical black hole where low temperature is invariably associated with small horizon area and therefore the hydrodynamic approximation breaks down. In the hyperbolic case, the area of the horizon can be large even at low temperatures provided the hyperbolic surface is of high genus.

Topological black holes are solutions of the Einstein equations for vacuum AdS space. Consider the action

* Corresponding author.
E-mail address: lpapa@central.ntua.gr (E. Papantonopoulos).

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$$I = \frac{1}{16\pi G}\int d^d x\sqrt{-g}\left[R + \frac{(d-1)(d-2)}{l^2}\right], \qquad (1)$$

where G is the Newton's constant, R is the Ricci scalar and l is the AdS radius. The presence of a negative cosmological constant ($\Lambda = -\frac{(d-1)(d-2)}{2l^2}$) allows the existence of black holes with topology $\mathbb{R}^2 \times \Sigma$, where Σ is a $(d-2)$-dimensional manifold of constant negative curvature. These black holes are known as topological black holes (TBHs) [13]. The simplest solution of this kind in four dimensions reads

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}dr^2 + r^2\,d\sigma^2,$$
$$f(r) = r^2 - 1 - 2G\mu/r, \qquad (2)$$

where we have set the AdS radius $l = 1$, μ is a constant which is proportional to the mass and $d\sigma^2$ is the line element of the two-dimensional manifold Σ, which is locally isomorphic to the hyperbolic manifold $\mathbb{H}^2$ and of the form

$$\Sigma = \mathbb{H}^2/\Gamma, \quad \Gamma \subset O(2,1), \qquad (3)$$

where Γ is a freely acting discrete subgroup (i.e. without fixed points) of isometries. The line element $d\sigma^2$ of Σ is

$$d\sigma^2 = d\theta^2 + \sinh^2\theta\,d\varphi^2, \qquad (4)$$

with $\theta \geqslant 0$ and $0 \leqslant \phi < 2\pi$ being the coordinates of the hyperbolic space $\mathbb{H}^2$ or pseudosphere, which is a non-compact two-dimensional space of constant negative curvature. This space becomes a compact space of constant negative curvature with genus $g \geqslant 2$ by identifying, according to the connection rules of the discrete subgroup Γ, the opposite edges of a 4g-sided polygon whose sides are geodesics and is centered at the origin $\theta = \varphi = 0$ of the pseudosphere [13,17]. An octagon is the simplest such polygon, yielding a compact surface of genus $g = 2$ under these identifications. Thus, the two-dimensional manifold Σ is a compact Riemann 2-surface of genus $g \geqslant 2$. The configuration (2) is an asymptotically locally AdS spacetime.

This construction can be generalized to higher dimensions and our aim in this work is to elucidate the effect of hyperbolic horizons on the gauge theory on the AdS boundary. In five spacetime dimensions the metric takes the form

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Sigma_3^2, \quad f(r) = r^2 - 1 - \frac{2\mu}{r^2}, \qquad (5)$$

where $\Sigma_3 = \mathbb{H}^3/\Gamma$. The horizon radius r_+ is found from

$$2\mu = r_+^4\left(1 - \frac{1}{r_+^2}\right). \qquad (6)$$

The Hawking temperature is

$$T = \frac{2r_+^2 - 1}{2\pi r_+}, \qquad (7)$$

while the mass and entropy of the black hole are given respectively by

$$M = \frac{3V}{16\pi G}r_+^2\left(r_+^2 - 1\right), \qquad S = \frac{V}{4G}r_+^3 \qquad (8)$$

where V is the volume of the hyperbolic space Σ_3. Note that in the horizon radius range $1/2 \leqslant r_+^2 \leqslant 1$ the mass of the black hole is negative [16]. The lower bound corresponds to its maximum negative value where the temperature is zero. The upper bound corresponds to zero mass, where as shown in [15] there is a phase transition of the TBH to AdS space, while above that value the mass takes positive values.

The energy of the dual CFT is [9]

$$E_{\rm CFT} = \frac{3V}{16\pi G}\left(r_+^2 - \frac{1}{2}\right)^2 \qquad (9)$$

which is shifted with respect to the black hole energy by a positive amount (Casimir energy due to counterterms one needs to add to the action to cancel infinities). Notice that the minimum energy ($E_{\rm CFT} = 0$) is at $T = 0$, therefore the energy of the CFT is never negative, unlike its dual black hole.

For the study of perturbations, we need the behaviour of harmonic functions on Σ_3. In general, they obey

$$\left(\nabla^2 + k^2\right)\mathbb{T} = 0. \qquad (10)$$

Without identifications (i.e., in $\mathbb{H}^3$), the spectrum is continuous. We obtain [14]

$$k^2 = \xi^2 + 1 + \delta \qquad (11)$$

where ξ is arbitrary and $\delta = 0, 1, 2$ for scalar, vector and tensor perturbations, respectively. When a compactification scheme is chosen, the spectrum becomes discrete. Depending on the choice of Γ, the discretized eigenvalues ξ^2 may be made as small as desired, i.e., zero is an accumulation point of the spectrum of ξ [17]. We also obtain negative values of ξ^2. As ξ^2 approaches its minimum value, the complexity of the set of isometries Γ increases and the volume V of the hyperbolic space Σ_3 can be made arbitrarily large (hence also the mass and entropy of the black hole).

Using the harmonics on Σ_3, we may write the wave equation for gravitational perturbations in the general Schrödinger-like form [18]

$$-\frac{d^2\Phi}{dr_*^2} + V\left[r(r_*)\right]\Phi = \omega^2\Phi, \qquad (12)$$

in terms of the tortoise coordinate r_* defined by $\frac{dr_*}{dr} = \frac{1}{f(r)}$ where $f(r)$ is defined in (5). The potential takes different forms for different types of perturbation.

To calculate the Chern–Simons diffusion rate one needs to solve the wave equation for a massless scalar field. The radial wave equation is

$$\frac{1}{r^3}\left(r^3 f(r)\Phi'\right)' + \frac{\omega^2}{f(r)}\Phi - \frac{k_S^2}{r^2}\Phi = 0. \qquad (13)$$

By defining $\Phi = r^{-\frac{3}{2}}\Psi$ it can be cast into the Schrödinger-like form (12) with the potential given by

$$V_S(r) = f(r)\left\{\frac{15}{4} + \frac{k_S^2 - \frac{3}{4}}{r^2} + \frac{9\mu}{2r^4}\right\}. \qquad (14)$$

We may solve the wave equation in terms of a Heun function and use the latter to determine the spectrum exactly albeit numerically [15]. However, such explicit expressions will not be needed for our purposes.

If the hyperbolic space Σ_3 is infinite, then $k_S^2 \geqslant 1$ (Eq. (11)). However, if Σ_3 is finite, then it is easy to see that the minimum eigenvalue is $k_S^2 = 0$. The corresponding hyperspherical harmonic is a constant. Above $k_S^2 = 0$, the spectrum is discrete.

For the AdS/CFT correspondence, we need the flux

$$\mathcal{F} = \frac{N^2}{16\pi^2}\sqrt{-g}g^{rr}\frac{\Phi^*\partial_r\Phi}{|\Phi|^2}\bigg|_{r\to\infty}. \qquad (15)$$

The imaginary part is independent of r (conserved flux). It is convenient to evaluate it at the horizon where the wavefunction behaves as

$$\Phi(r) \approx \left(1 - \frac{r_+}{r}\right)^{-\frac{i\omega}{4\pi T}}. \qquad (16)$$

We obtain

$$\sqrt{-g}g^{rr}\Im\left(\Phi^*\partial_r\Phi\right) = -\omega r_+^3 \qquad (17)$$

therefore

$$\Im\mathcal{F} = -\frac{N^2 r_+^3 \omega}{16\pi^2 |\Phi(\infty)|^2}. \tag{18}$$

It is related to the imaginary part of the retarded Green function,

$$\Im\tilde{G}^R(\omega, k_S^2) = -2\Im\mathcal{F} \tag{19}$$

of some scalar operator $\mathcal{O}$ ($G(x) = \langle\mathcal{O}(x)\mathcal{O}(0)\rangle$, where $x \in \mathbb{R}\times\Sigma_3$). We readily obtain

$$\Im\tilde{G}^R(\omega, k_S^2) = \frac{N^2 r_+^4 \hat{\omega}}{8\pi^2 |\Phi(\infty)|^2}. \tag{20}$$

For $\mathcal{O} = \frac{1}{4} F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$, we may define the Chern–Simons diffusion rate

$$\Gamma = \left(\frac{g_{YM}^2}{8\pi^2}\right)^2 \int dt \int_{\Sigma_3} d^3\sigma \, \langle\mathcal{O}(x)\mathcal{O}(0)\rangle. \tag{21}$$

Γ determines the rate of anomalous baryon number violation at high temperatures in the Standard Model. Expanding $G(x)$ in hyperspherical harmonics, the integral over Σ_3 projects onto the lowest harmonic ($k_S^2 = 0$). The integral over time then yields the Fourier transform at $\omega = 0$. Using

$$\tilde{G}(0, k_S^2) = -\lim_{\omega\to 0} \frac{2T}{\omega} \Im\tilde{G}^R(\omega, k_S^2) \tag{22}$$

we deduce

$$\Gamma = \left(\frac{g_{YM}^2}{8\pi^2}\right)^2 \tilde{G}(0,0) = \frac{(g_{YM}^2 N)^2}{256\pi^6} \left.\frac{T r_+^3}{|\Phi(\infty)|^2}\right|_{\omega=0, k_S^2=0}. \tag{23}$$

Evidently,

$$\Phi(r)|_{\omega=0, k_S^2=0} = 1 \tag{24}$$

at any temperature, therefore

$$\Gamma = \frac{(g_{YM}^2 N)^2}{256\pi^7} r_+^4 \left(1 - \frac{1}{2r_+^2}\right). \tag{25}$$

At high temperatures, $\Gamma \sim T^4$ whereas as $T \to 0$, $\Gamma \sim T \to 0$, i.e., anomalous baryon number violation is suppressed at low temperatures.

To calculate the shear viscosity, we need to discuss vector gravitational perturbations. The lowest eigenvalue of the angular equation (10) for a vector harmonic $\mathbb{V}_i$ ($\nabla_i \mathbb{V}^i = 0$) on a finite hyperbolic space Σ_3 is found by observing that

$$\nabla^j(\partial_i \mathbb{V}_j - \partial_j \mathbb{V}_i) = (k_V^2 + 2)\mathbb{V}_i$$

where we used $R_{ij} = -2\gamma_{ij}$ (γ_{ij} being the metric on Σ_3). Therefore, we have a constant vector harmonic if $k_V^2 + 2 = 0$. The minimum eigenvalue is $k_V^2 = -2$. Above it, we have a discrete spectrum of eigenvalues

$$k_V^2 = -2 + \Delta, \quad \Delta \geqslant 0. \tag{26}$$

The radial wave equation is of the form (12) with potential

$$V_V(r) = f(r)\left\{\frac{3}{4} + \frac{k_V^2 - \frac{7}{4}}{r^2} - \frac{27\mu}{2r^4}\right\}. \tag{27}$$

We may solve the radial equation and obtain a solution in terms of a Heun function. Since we are interested in the hydrodynamic behaviour, we shall solve the radial equation only for small ω and Δ using perturbation theory.

More precisely, the hydrodynamic approximation is valid provided

$$\omega, \sqrt{\Delta} \ll r_+ \tag{28}$$

(recall that we are working in units in which the AdS radius $l = 1$). At high temperatures, this constraint is equivalent to $\omega, \sqrt{\Delta} \ll T$. Also, the area of the horizon ($A_+ \sim r_+^2$) is large and the constraint (28) is satisfied for eigenvalues $\Delta \sim \mathcal{O}(1)$ because then $\Delta \ll A_+$. This is similar to the case of a sphere. In both cases, the hydrodynamic limit is valid at high temperature (large black hole) [11, 12].

At low temperatures, in the case of a spherical horizon, its area becomes small. Even with $A_+ \sim \mathcal{O}(1)$, it is no longer possible to satisfy the constraint (28) because the low-lying eigenvalues $\Delta \sim \mathcal{O}(1)$ regardless of the size of the horizon. Thus, for a small spherical black hole the hydrodynamic approximation is invalid.

For a hyperbolic horizon at low temperature, we have $r_+ \sim \mathcal{O}(1)$ ($r_+ \geqslant 1/\sqrt{2}$ at all temperatures), so the hydrodynamic constraint (28) is not always satisfied, as in the case of a spherical horizon. However, unlike in the case of a sphere, a hyperbolic space Σ_3 of high genus can have a large volume $V \gg 1$. The low lying eigenvalues are

$$\sqrt{\Delta} \sim \frac{1}{V^{1/3}} \tag{29}$$

and therefore can be small ($\sqrt{\Delta} \lesssim \mathcal{O}(1)$) if V is large. Thus, for topological black holes of high genus hyperbolic horizons the hydrodynamic approximation is valid even in the low temperature (small horizon radius) limit owing to the complexity of the horizon surface.

To solve the radial wave equation, it is convenient to introduce the coordinate

$$u = \left(\frac{r_+}{r}\right)^2. \tag{30}$$

In terms of the wavefunction $F(u)$ defined by

$$\Psi(u) = (1-u)^{-\frac{i\omega}{4\pi T}} F(u) \tag{31}$$

we have

$$\mathcal{A}F'' + \mathcal{B}F' + \mathcal{C}F = 0, \tag{32}$$

where

$$\mathcal{A} = u\hat{f}, \qquad \mathcal{B} = u\hat{f}' + \frac{3}{2}\hat{f} + \frac{i\omega}{4\pi T}\frac{u\hat{f}}{1-u},$$
$$\mathcal{C} = -\frac{\hat{V}}{4u^2\hat{f}} + \frac{i\omega}{4\pi T}\frac{u\hat{f}' + \frac{3}{2}\hat{f}}{1-u} + \frac{i\omega}{4\pi T}\frac{u\hat{f}}{(1-u)^2} + \mathcal{O}(\omega^2/T^2), \tag{33}$$

where prime denotes differentiation with respect to u and we have defined

$$\hat{f}(u) \equiv \frac{f(r)}{r_+^2} = \frac{1}{u} - \frac{1}{r_+^2} - \frac{2\mu}{r_+^4}u,$$
$$\hat{V}_V(u) \equiv \frac{V_V(r)}{r_+^2} = \hat{f}(u)\left\{\frac{3}{4} + \frac{\Delta - \frac{15}{4}}{r_+^2}u - \frac{27\mu}{2r_+^4}u^2\right\}. \tag{34}$$

We obtain the zeroth order equation by setting $\omega = 0$, $\Delta = 0$. The acceptable solution is

$$F_0 = u^{3/4} \tag{35}$$

independent of the temperature.

Expanding the wavefunction,

$$F = F_0 + F_1 + \cdots, \tag{36}$$

at first order the wave equation reads

$$\mathcal{H}_0 F_1 = -\mathcal{H}_1 F_0, \tag{37}$$

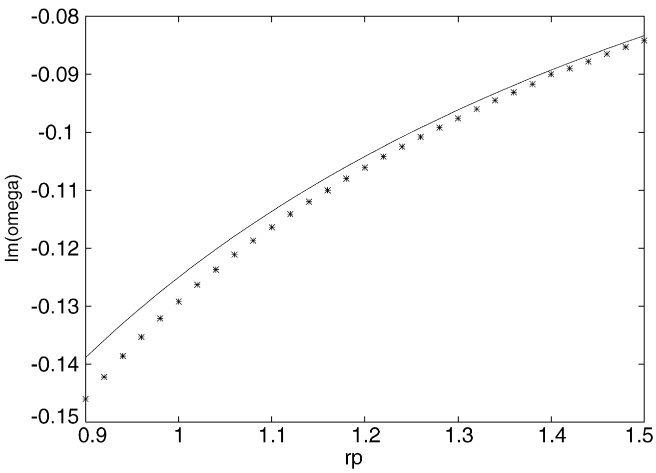

Fig. 1. The imaginary part of the lowest, purely dissipative, mode versus r_+. The continuous line and the points represent the perturbative and numerical results, respectively.

where

$$\mathcal{H}_1 F_0 = \frac{i\omega}{4\pi T}\left\{\frac{2}{u} + 3\left(1 - \frac{1}{r_+^2}\right)\right\}F_0 - \frac{\Delta}{4r_+^2 u}F_0. \tag{38}$$

The solution may be written as

$$F_1 = F_0 \int \frac{\mathcal{W}}{F_0^2} \int \frac{F_0 \mathcal{H}_1 F_0}{\mathcal{A}\mathcal{W}}, \tag{39}$$

where $\mathcal{W} = 1/(u^{3/2}\hat{f})$ is the Wronskian. The limits of the inner integral may be adjusted at will because this amounts to adding an arbitrary amount of the unacceptable zeroth-order wavefunction. To ensure regularity at the horizon, we should choose one of the limits of integration at $u = 1$. Then by demanding that the singularity vanish at the boundary ($u = 0$), we arrive at the first-order constraint

$$\int_0^1 du \frac{F_0 \mathcal{H}_1 F_0}{\mathcal{A}\mathcal{W}} = 0. \tag{40}$$

After some straightforward algebra, this leads to the dispersion relation

$$\omega = -i\frac{\Delta}{4r_+} \tag{41}$$

in agreement with Ref. [12] at high temperatures and matching numerical results at all temperatures (Fig. 1). From (41) we read off the diffusion coefficient

$$D = \frac{1}{4r_+} \tag{42}$$

which is related to the viscosity coefficient via

$$D = \frac{\eta}{\epsilon + p}. \tag{43}$$

This is known to be valid in flat space. It is also valid in our case, as can be seen by writing the hydrodynamic equations $\nabla_\mu T^{\mu\nu} = 0$ for a static fluid of constant pressure perturbed by a small velocity field $u^i = e^{-i\omega t}\mathbb{V}^i$. The conservation law of the hydrodynamic equations yields [12]

$$-4i\omega p + \eta\left(k_{\mathrm{V}}^2 + 2\right) = 0. \tag{44}$$

Eq. (43) then follows if we use (41), (42) together with $\epsilon = 3p$ which is valid for a conformal fluid.

From the expression for the energy (9), we obtain the energy density $\epsilon = E_{\mathrm{CFT}}/V$ and the shear viscosity coefficient

$$\eta = \frac{4}{3}\epsilon D = \frac{1}{16\pi G r_+}\left(r_+^2 - \frac{1}{2}\right)^2. \tag{45}$$

Dividing by the entropy density ($s = S/V$, where the entropy is given by (8)), we obtain

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2r_+^2}\right)^2. \tag{46}$$

At high temperature (large r_+), $\frac{\eta}{s} \approx \frac{1}{4\pi}$. As $T \to 0$, $\frac{\eta}{s} \sim T^2 \to 0$. At all temperatures, the ratio is below $\frac{1}{4\pi}$.

In flat spacetime, one also obtains the viscosity coefficient from the Kubo formula [1], which agrees with the result obtained via the diffusion coefficient. The former is derived by considering tensor perturbations. This is not possible in our case, because tensor perturbations of the static fluid in hyperbolic space do not exist due to their being traceless and divergenceless [12].

In conclusion. Using the AdS/CFT correspondence we have calculated the anomalous baryon number violation rate and the ratio of shear viscosity to entropy density in thermal field theories having gravity duals with hyperbolic horizons. We found the explicit temperature dependence of the anomalous baryon number violation and we showed that it is suppressed at low temperatures. For high genus hyperbolic spaces the hydrodynamic approximation is valid at low temperatures, and the ratio of shear viscosity to entropy density is found to be below $1/(4\pi)$ at all temperatures. It can be made arbitrarily small in the low temperature limit.

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