Computers and Mathematics with Applications 58 (2009) 369–379

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

The effect of thermal radiation on the flow of a second grade fluid

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ARTICLE INFO

Article history:
Received 31 August 2008
Received in revised form 24 December 2008
Accepted 12 January 2009

Keywords:
Thermal radiation
Series solution
Stretching
Skin friction coefficient
Nusselt number

ABSTRACT

This paper reports the magnetohydrodynamic (MHD) flow and heat transfer characteristics of a second grade fluid in a channel. Analytic technique namely the homotopy analysis method (HAM) is used to solve the momentum and energy equations. The important findings in this paper are the effects of second grade parameter, Hartmann number, Reynolds number, thermal radiation parameter, Prandtl and local Eckert numbers on the velocity, temperature, skin friction coefficient and Nusselt number.

1. Introduction

The flow behavior of rheological fluids is a topic of current interest because of its engineering and industrial importance; for instance processing of polymers, biomechanics, enhanced oil recovery and food products etc. Different from the viscous fluids, it is impossible to characterize all the rheological fluids by a single constitutive equation. Therefore several constitutive equations of such fluids have been reported in the literature. Amongst these there is the simplest one which is known as the second grade fluid. In view of its simplicity, various investigators [1–10] in the field have recently used it in different flow descriptions. It is noticed that the second grade fluids can predict the normal stress effects. However, such fluids do not exhibit the shear thinning/thickening effects [11–13]. These fluids also do not possess the characteristics of relaxation and retardation phenomena.

The influence of complex rheological parameters on the heat transfer is a topic of great interest to the researchers nowadays. Such analysis has potential applications in extrusion process, hemodialysis and oxygenation. In the light of such motivations, the present work studies the effects of thermal radiation [14] on the MHD flow in a channel with stretching walls. Constitutive equations of second grade fluid are considered. Series solutions for velocity and temperature are constructed by homotopy analysis method [15,16]. This method has been already used by different authors for many problems [17–30]. Here the convergence of the derived solutions is ensured. The variation of emerging parameters are discussed on the flow quantities of interest.

2. Development of the flow problem

Let us consider the two-dimensional and steady flow of a second grade fluid in a channel bounded by the planes $y = \pm a$. The x- and y-axes are chosen parallel to the channel walls and perpendicular to the flow respectively. A constant magnetic

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0898-1221/$ – see front matter
doi:10.1016/j.camwa.2009.01.040

field $\mathbf{B}_0$ is applied in a direction transverse to the flow. Symmetry in the flow is taken into account about the line $y = 0$. The flow is generated due to stretching of the channel walls. The heat transfer in the channel is because of the constant temperature to the channel walls. In addition heat radiation effects are included. Constitutive expressions in a second grade fluid are

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{1}$$

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^{\mathrm{T}}, \tag{2}$$

$$\mathbf{A}_2 = \frac{\mathrm{d}\mathbf{A}_1}{\mathrm{d}t} + \mathbf{A}_1(\nabla\mathbf{V}) + (\nabla\mathbf{V})^{\mathrm{T}}\mathbf{A}_1, \tag{3}$$

in which p is pressure, $\mathbf{I}$ is unit tensor, $\mathbf{V}$ is the velocity and $\mathrm{d}/\mathrm{d}t$, μ, α_1, α_2 are material derivative, dynamic viscosity, viscoelasticity, cross viscosity respectively. Further more μ, α_1 and α_2 satisfy [13]

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \qquad \alpha_1 + \alpha_2 = 0. \tag{4}$$

The relevant boundary layer problems are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u + \frac{\alpha_1}{\rho}\begin{bmatrix} u\dfrac{\partial^3 u}{\partial x\partial y^2} + v\dfrac{\partial^3 u}{\partial y^3} \\ +\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2} - \dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x\partial y} \end{bmatrix}, \tag{5}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{c_p\rho}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{c_p\rho}\frac{\partial q_r}{\partial y} + \frac{\alpha_1}{\rho c_p}\begin{bmatrix} u\dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x\partial y} + v\dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial y^2} \\ +\dfrac{\partial v}{\partial y}\left(\dfrac{\partial u}{\partial y}\right)^2 + \dfrac{\partial u}{\partial x}\left(\dfrac{\partial u}{\partial y}\right)^2 \end{bmatrix}, \tag{6}$$

$$\begin{aligned} &u = bx, \qquad v = 0 \quad \text{at } y = a; \quad b > 0, \\ &\frac{\partial u}{\partial y} = 0 \qquad v = 0 \quad \text{at } y = 0, \\ &T = T_w, \quad \text{at } y = a, \\ &\frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0, \end{aligned} \tag{7}$$

where $u, v, T, T_w, \nu, \sigma, \rho, k, c_p$ and q_r are components of velocity in x and y directions, temperature, the wall temperature, kinematic viscosity, electrical conductivity, mass density, thermal conductivity, specific heat and heat flux respectively. All the fluid properties are taken constant. By the Rosseland approximation the radiative heat flux can be reduced in the form

$$q_r = -\frac{4\Gamma}{3k^*}\frac{\partial T^4}{\partial y}, \tag{8}$$

where Γ and k^* are the Stefan–Boltzmann constant and the mean absorption coefficient respectively. Invoking Taylor series, one has

$$T^4 \approx 4T_0^3 T - 3T_0^4, \tag{9}$$

where in the above equation T_0 is the temperature at the central line $y = 0$ and terms of higher order are neglected.

Employing the following transformations

$$u = bxf'(\eta), \qquad v = -abf(\eta), \qquad \eta = \frac{y}{a}, \qquad \theta = \frac{T}{T_w}, \tag{10}$$

we have

$$f''' - \mathrm{Re}\left(\left(f'\right)^2 - ff''\right) - M^2 f' + \alpha\begin{bmatrix} 2f'f''' - ff^{(iv)} \\ -\left(f''\right)^2 \end{bmatrix} = 0, \tag{11}$$

$$f(0) = 0, \qquad f(1) = 0, \qquad f'(1) = 1, \qquad f''(0) = 0, \tag{12}$$

$$\left(\frac{3N_R + 4}{3N_R}\right)\theta'' + \mathrm{Pr}\,\mathrm{Re} f\theta' + \mathrm{Pr}\,Ec\left(f''\right)^2 + \alpha\,\mathrm{Pr}\,Ec\begin{pmatrix} f'f''^2 \\ -ff''f''' \end{pmatrix} = 0, \tag{13}$$

$$\theta'(0) = 0, \qquad \theta(1) = 1, \tag{14}$$

in which

$$M=\sqrt{\frac{\sigma}{\mu}}B_0a,\qquad \mathrm{Re}=\frac{a^2b}{\nu},\qquad \mathrm{Pr}=\frac{\mu c_p}{k},$$
$$Ec=\frac{b^2x^2}{T_w c_p},\qquad N_R=\frac{kk^*}{4\Gamma T_0^3},\qquad \alpha=\frac{\alpha_1 b}{\mu},$$

are respectively the Hartmann number, Reynolds number, Prandtl number, local Eckert number, thermal radiation and viscoelastic parameters. The local skin friction coefficient C_f and Nusselt number Nu are

$$C_f=\left(\frac{1+\alpha}{\mathrm{Re}_x}\right)f''(1), \tag{15}$$

$$Nu=-\theta'(1), \tag{16}$$

where Re_x is local Reynolds number.

3. Solution by homotopy analysis method (HAM)

3.1. Zeroth-order deformation problem

In order to obtain series solutions, $f(\eta)$ and $\theta(\eta)$ can be expressed by the set of base functions

$$\{\eta^n, n\geq 0\} \tag{17}$$

in the form

$$f(\eta)=\sum_{n=0}^{\infty}a_{n,n}\eta^{2n+1}, \tag{18}$$

$$\theta(\eta)=\sum_{n=0}^{\infty}b_{n,n}\eta^{2n}, \tag{19}$$

where $a_{n,n}$ and $b_{n,n}$ are coefficients. The initial guesses and auxiliary linear operators are chosen as follows

$$f_0(\eta)=\frac{1}{2}\left(\eta^3-\eta\right), \tag{20}$$

$$\theta_0(\eta)=\eta^2, \tag{21}$$

$$L_1(f(\eta))=\frac{\mathrm{d}^3f}{\mathrm{d}\eta^3}, \tag{22}$$

$$L_2(\theta(\eta))=\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2}, \tag{23}$$

with

$$L_1(C_1+C_2\eta^2+C_3\eta^2+C_4\eta^3)=0, \tag{24}$$

$$L_2\left(C_5+C_6\eta^2\right)=0, \tag{25}$$

and C_i $(i=1-6)$ are the arbitrary constants. Now the problems corresponding to zeroth-order deformation are

$$(1-q)L_1\left[\Phi(\eta;q)-f_0(\eta)\right]=q\,\hbar_1\,N_1\left[\Phi(\eta;q)\right], \tag{26}$$

$$(1-q)L_2\left[\Psi(\eta;q)-\theta_0(\eta)\right]=q\,\hbar_2\,N_2\left[\Psi(\eta;q)\right], \tag{27}$$

$$\Phi(0;q)=0,\qquad \left.\frac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\right|_{\eta=0}=0,$$
$$\left.\frac{\partial\Phi(\eta;q)}{\partial\eta}\right|_{\eta=1}=1,\qquad \left.\frac{\partial\Phi(\eta;q)}{\partial\eta}\right|_{\eta=1}=0, \tag{28}$$

$$\Psi(1;q)=1,\qquad \left.\frac{\partial\Psi(\eta;q)}{\partial\eta}\right|_{\eta=0}=0, \tag{29}$$

$$N_1\left[\Phi(\eta;q)\right] = \frac{\partial^3\Phi(\eta;q)}{\partial\eta^3} - M^2\frac{\partial\Phi(\eta;q)}{\partial\eta} + \alpha\begin{bmatrix} 2\dfrac{\partial\Phi(\eta;q)}{\partial\eta}\dfrac{\partial^3\Phi(\eta;q)}{\partial\eta^3} \\ -\Phi(\eta;q)\dfrac{\partial^4\Phi(\eta;q)}{\partial\eta^4} \\ -\left(\dfrac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\right)^2\end{bmatrix} - \mathrm{Re}\left[\left(\frac{\partial\Phi(\eta;q)}{\partial\eta}\right)^2 - \Phi(\eta;q)\frac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\right], \quad (30)$$

$$N_2\left[\Psi(\eta;q)\right] = \left(\frac{3N_R+4}{3N_R}\right)\frac{\partial^2\Psi(\eta;q)}{\partial\eta^2} + \mathrm{Pr}\,\mathrm{Re}\,\Phi(\eta;q)\frac{\partial\Psi(\eta;q)}{\partial\eta} + \mathrm{Pr}\,Ec\left(\frac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\right)^2 + \alpha\,\mathrm{Pr}\,Ec\begin{bmatrix}\dfrac{\partial\Phi(\eta;q)}{\partial\eta}\left(\dfrac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\right)^2 \\ -\Phi(\eta;q)\dfrac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\dfrac{\partial^3\Phi(\eta;q)}{\partial\eta^3}\end{bmatrix}, \quad (31)$$

where $q \in [0, 1]$ and $\hbar_i \neq 0$ $(i = 1, 2)$ are the respective embedding and auxiliary parameters such that $\Phi(\eta; 0) = f_0(\eta)$, $\Phi(\eta; 1) = f(\eta)$ and $\Psi(\eta; 0) = \theta_0(\eta)$, $\Psi(\eta; 1) = \theta(\eta)$. Obviously when q varies from 0 to 1, $\Phi(\eta; q)$ changes from the initial guess $f_0(\eta)$ to an exact solution $f(\eta)$ and $\Psi(\eta; q)$ from $\theta_0(\eta)$ to $\theta(\eta)$. By Taylor series one has

$$\Phi(\eta;q) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)\,q^m, \quad (32)$$

$$\Psi(\eta;q) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)\,q^m, \quad (33)$$

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m\Phi(\eta;q)}{\partial\eta^m}\bigg|_{q=0}, \quad (34)$$

$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m\Psi(\eta;q)}{\partial\eta^m}\bigg|_{q=0}. \quad (35)$$

3.2. Higher-order deformation problem

The mth-order deformation problems are

$$L_1\left[f_m(\eta) - \chi_m f_{m-1}(\eta)\right] = \hbar_1 R_{1m}\left(f_{m-1}(\eta)\right), \quad f_m(0) = 0, \quad f_m(1) = 0, \quad f_m'(1) = 0, \quad f_m''(0) = 0, \quad (36)$$

$$L_2\left[\theta_m(\eta) - \chi_m\theta_{m-1}(\eta)\right] = \hbar_2 R_{2m}\left(\theta_{m-1}(\eta)\right), \quad \theta_m(1) = 0, \quad \theta_m'(0) = 0, \quad (37)$$

whence

$$R_{1m}\left(f_{m-1}(\eta)\right) = f_{m-1}'''(\eta) - M^2 f_{m-1}'(\eta) + \alpha\sum_{n=0}^{m-1}\begin{bmatrix}2f_n'(\eta)f_{m-1-n}'''(\eta) \\ -f_n(\eta)f_{m-1-n}^{(iv)}(\eta) \\ -f_n''(\eta)f_{m-1-n}''(\eta)\end{bmatrix} - \mathrm{Re}\sum_{n=0}^{m-1}\begin{bmatrix}f_n'(\eta)f_{m-1-n}'(\eta) \\ -f_n(\eta)f_{m-1-n}''(\eta)\end{bmatrix}, \quad (38)$$

$$R_{2m}\left(\theta_{m-1}(\eta)\right) = \left(\frac{3N_R+4}{3N_R}\right)\theta_{m-1}''(\eta) + \mathrm{Pr}\,\mathrm{Re}\sum_{n=0}^{m-1} f_n(\eta)\,\theta_{m-1-n}'(\eta) + \mathrm{Pr}\,Ec\sum_{n=0}^{m-1} Ec f_n''(\eta) f_{m-1-n}''(\eta) + \alpha\,\mathrm{Pr}\,Ec\sum_{l=0}^{n}\sum_{n=0}^{m-1} f_{m-1-n}'(\eta) f_{n-l}''(\eta) f_l''(\eta) + \alpha\,\mathrm{Pr}\,Ec\sum_{l=0}^{n}\sum_{n=0}^{m-1} -f_{m-1-n}(\eta) f_{n-l}''(\eta) f_l'''(\eta), \quad (39)$$

$$\chi_m = \begin{cases}0, & m \le 1 \\ 1, & m > 1.\end{cases} \quad (40)$$

Table 1
The values of skin friction coefficient for different values of M, Re and α.

M	Re	α	$\mathrm{Re}_x C_f$
6	10	0.0	7.7958
		0.4	15.4094
		0.6	22.1710
		1.0	25.3590
6	0.0	0.6	17.7101
	5		18.2930
	15		19.4301
	20		19.9843
0.0	10	0.6	9.9075
2			11.5515
4			14.9731
6			18.8663

Table 2
The values of Nusselt number for different values of M, Re, α, Pr, N_R and Ec.

M	Re	α	Pr	N_R	Ec	Nu
6	10	0.6	0.0	10	0.6	0.0000
			0.3			1.0189
			0.6			2.0635
			1.0			3.4988
6	10	0.6	0.3	10	0.0	0.0000
					0.3	0.5094
					0.9	1.5284
					1.5	2.5474
6	5	10	0.6	0.3	0.6	0.4552
	10					0.5094
	15					0.5305
	20					0.5414
0	10	0.6	0.3	10	0.3	0.3553
2						0.3742
4						0.4314
6						0.5094
6	10	0.0	0.3	10	0.3	0.3653
		0.4				0.4625
		0.8				0.5555
		1.0				0.6010
6	0.0	0.6	0.3	10	0.3	0.5701
	5					0.5857
	15					0.6161
	20					0.6309

If $f_m^*(\eta)$ and $\theta_m^*(\eta)$ are the particular solutions of Eqs. (36) and (37) then the solutions are

$$f_m(\eta) = f_m^*(\eta) + C_1^m + C_2^m\eta + C_3^m\eta^2 + C_4^m\eta^3, \tag{41}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_5^m + C_6^m\eta \tag{42}$$

where coefficients C_i^m $(i = 1 - 6)$ can be determined by the boundary conditions in Eqs. (36) and (37) *i.e.* Mathematica has been used in solving the problems (36) and (37) for $m = 1, 2, 3 \ldots$.

4. Results and discussion

As proposed by Liao [15], the convergence of homotopy series solutions (18) and (19) depend upon the values of convergence — control parameters $\hbar_1$ and $\hbar_2$. Therefore $\hbar$-curves (Figs. 1–3) are plotted to determine the region of convergence. The range of admissible values of $\hbar_1$ and $\hbar_2$ here are $-1.70 \le \hbar_1 \le -0.2$ and $-0.55 \le \hbar_2 \le -0.3$ respectively. The series solutions (18) and (19) converge in the interval $0 \le \eta \le 1$ when $\hbar_1 = -0.3$ and $\hbar_2 = -0.4$ respectively.

Figs. 4–9 are made to analyze the effects of Hartmann number M, Reynolds number Re and the parameter α on the velocity. Figs. 10–15 illustrate the influence of M, Re, Pr, N_R and α on the temperature.

Fig. 3 shows that $f(\eta)$ decreases with an increase in Hartmann number M. From Fig. 5 one can see that for different values of M, the magnitude of velocity $f'(\eta)$ decreases. This is due to the effect of the magnetic force against the direction of the flow. Figs. 6 and 7 illustrate the variations of velocity components $f(\eta)$ and $f'(\eta)$ for various values of the viscoelastic

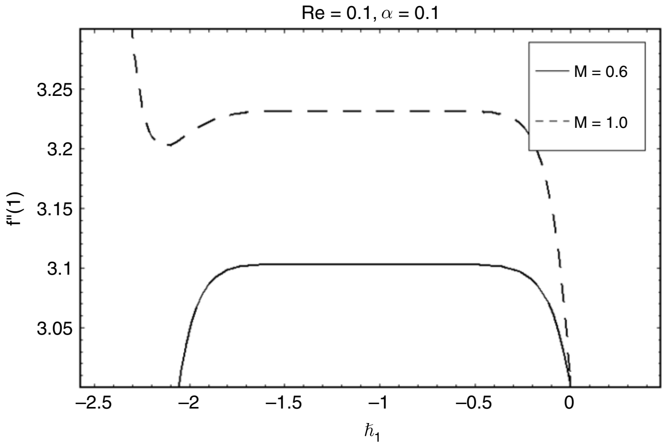

Fig. 1. $\hbar$-curves for 11th-order approximation when $M = 0.6$ and $M = 1$.

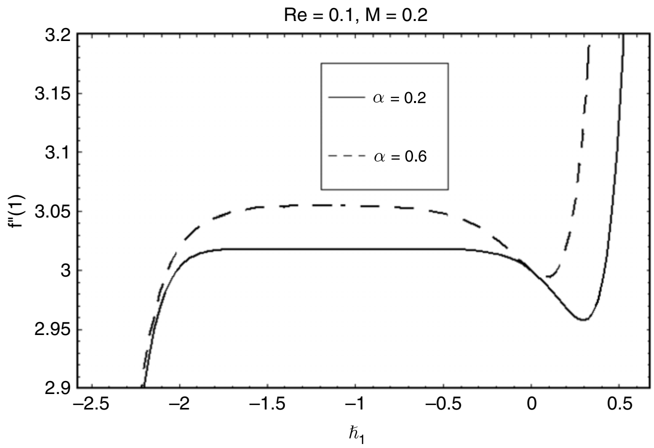

Fig. 2. $\hbar$-curves for 11th-order approximation when $\alpha = 0.2$ and $\alpha = 0.6$.

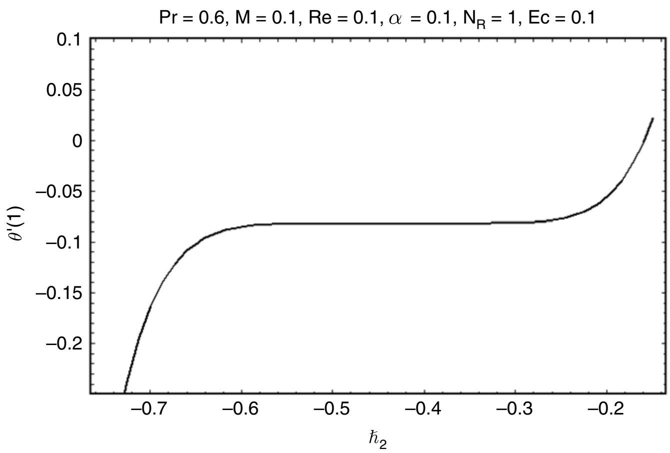

Fig. 3. $\hbar$-curve for 11th-order approximation.

parameter α. It is noticed from these figures that the velocity components increase when α increases. Fig. 8 reveals that velocity component $f(\eta)$ decreases when Re increases. Fig. 9 depicts that magnitude of $f'(\eta)$ increases for $\eta > 0.6$ but decreases for $\eta < 0.6$ with an increase of Reynold number Re. Figs. 9–12 indicate that the temperature is an increasing function of α, Pr, Ec and Re. Figs. 13 and 14 depict that the temperature decreases as M and N_R increase. Tables 1 and 2

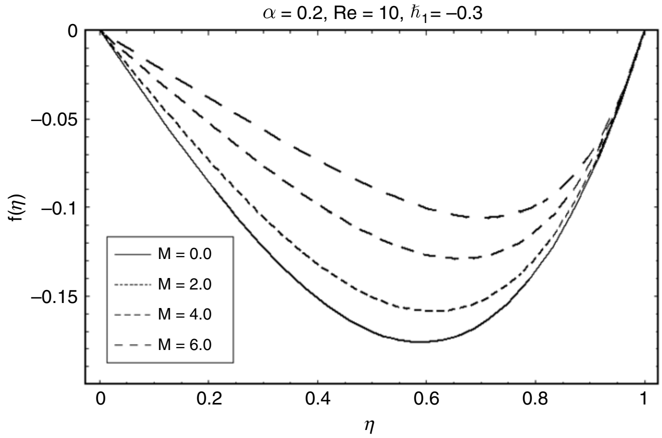

Fig. 4. Variation of $f(\eta)$ with η for different values of M.

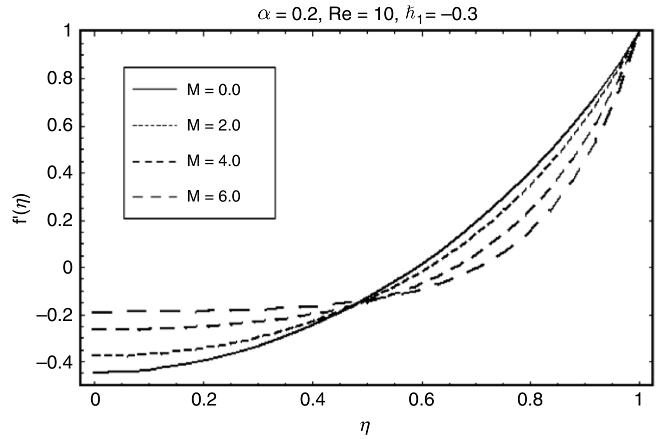

Fig. 5. Variation of $f'(\eta)$ with η for different values of M.

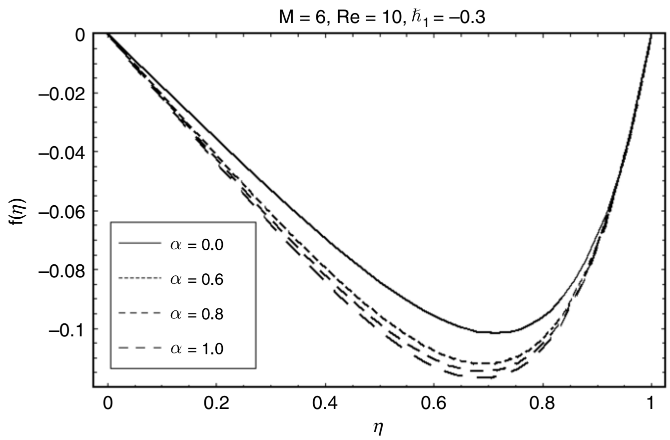

Fig. 6. Variation of $f(\eta)$ with η for different values of α.

are drawn to analyze the effects of involved physical constants on the skin friction coefficient and Nusselt number. From Table 1, it is clear that the skin friction coefficient C_f increases by increasing M, Re and α. Table 2 shows that Nu increases with an increase of M, Re, α, Pr, N_R and Ec.

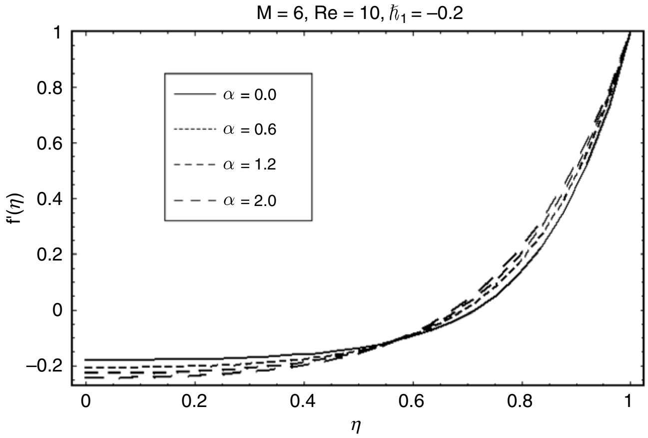

Fig. 7. Variation of $f'(\eta)$ with η for different values of α.

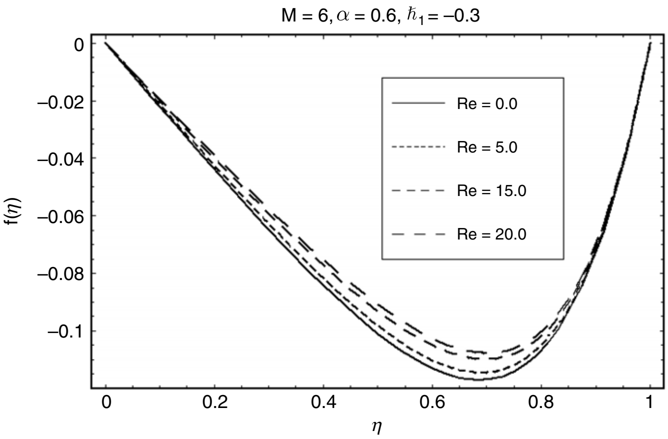

Fig. 8. Variation of $f(\eta)$ for different values of Re.

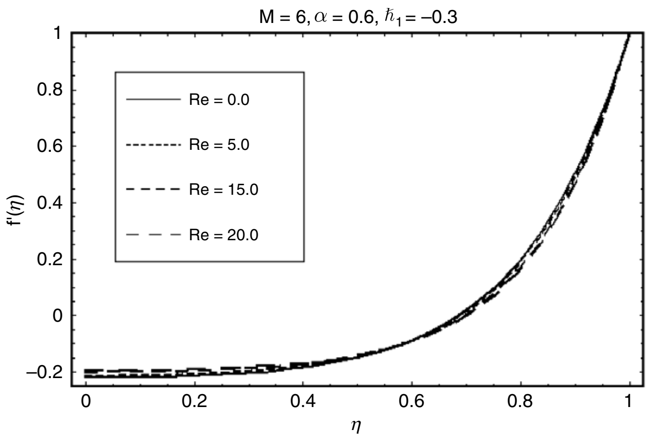

Fig. 9. Variation of $f'(\eta)$ for different values of Re.

5. Concluding remarks

The present study describes the analytic solution of the MHD second grade fluid flow in a channel. The nonlinear problem of velocity is solved by HAM. Using this velocity, energy equation is also solved analytically. The effects of various key parameters including the second grade parameter (α), Hartmann number (M), Reynolds number (Re), thermal radiation

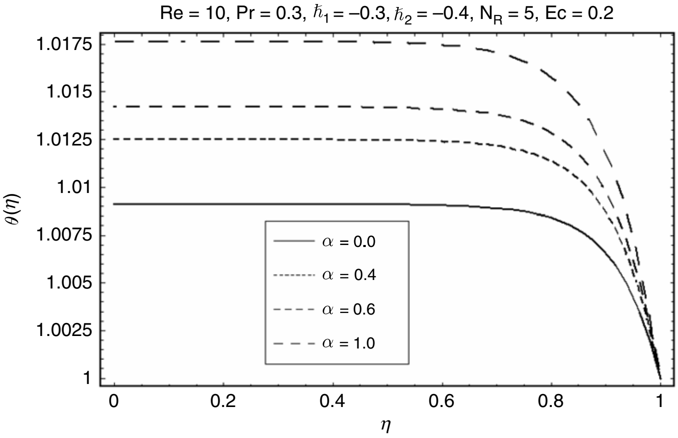

Fig. 10. Variation of $\theta(\eta)$ with η for different values of α.

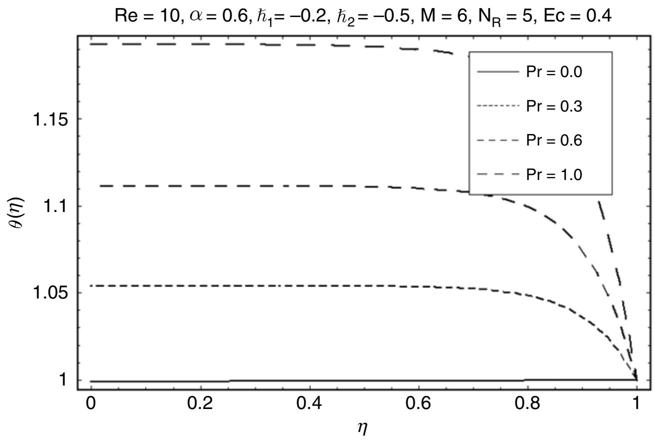

Fig. 11. Variation of $\theta(\eta)$ with η for different values of Pr .

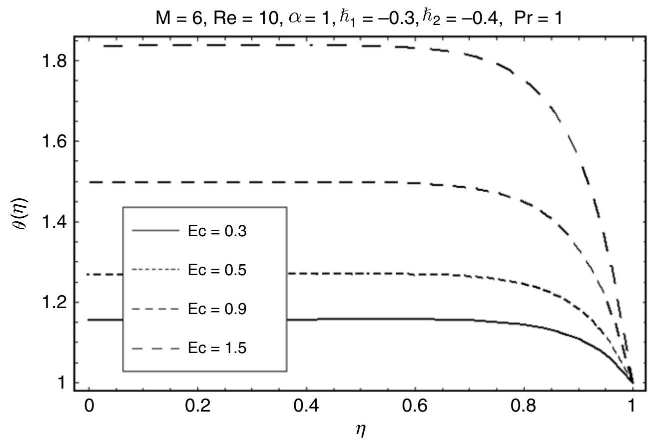

Fig. 12. Variation of $\theta(\eta)$ with η for different values of Ec.

parameter (N_R), Prandtl number (Pr) and local Eckert number (Ec) are examined. The convergent series solutions are obtained. The main results of the present analysis are as follows.

1. The velocity components $f(\eta)$ and $f'(\eta)$ are decreasing functions of M. This is because of the fact that magnetic force acts against the direction of flow.
2. The velocity components are increasing functions of α.

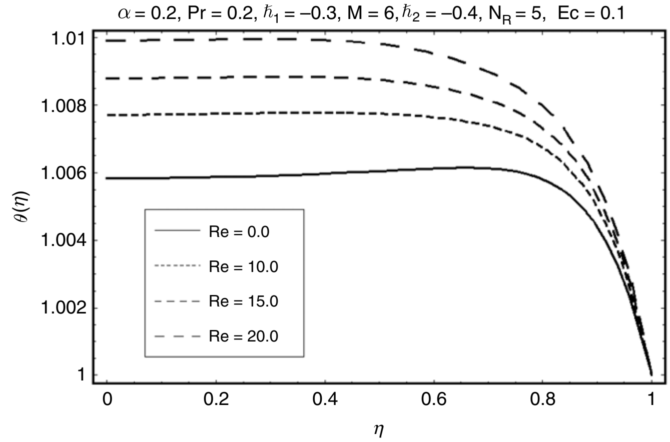

Fig. 13. Variation of $\theta(\eta)$ with η for different values of Re.

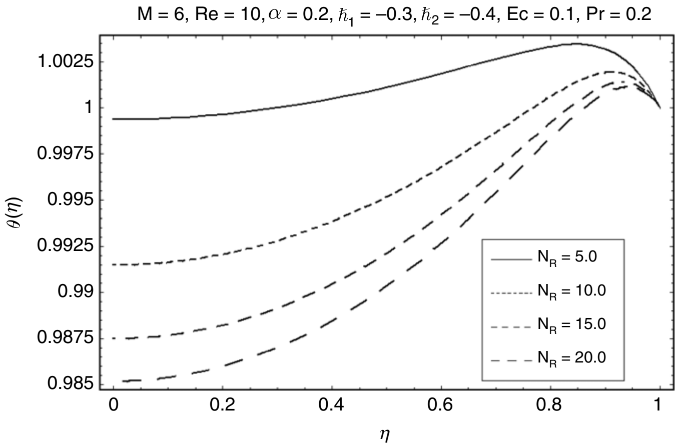

Fig. 14. Variation of $\theta(\eta)$ with η for different values of N_R.

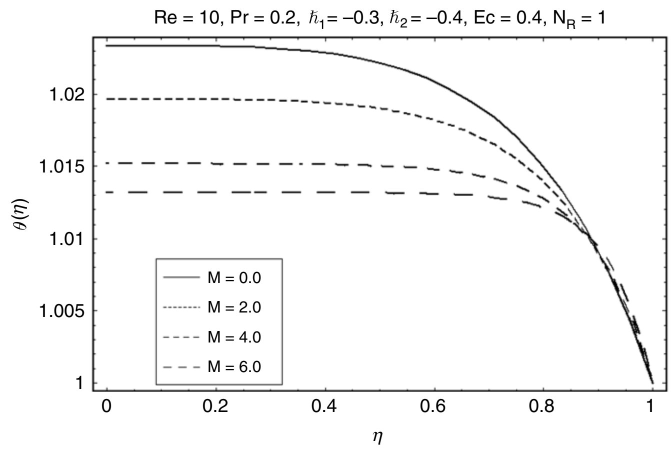

Fig. 15. Variation of $\theta(\eta)$ with η for different values of M.

3. The velocity components $f(\eta)$ and $f'(\eta)$ decrease when Re increases. Such effects occur due to decrease in viscosity.
4. Temperature increases with an increase of α, Ec and Re but decreases with an increase of M and N_R. The increase in the temperature by increasing Pr is due to increase in viscous effects.
5. The skin friction coefficient C_f increases by increasing M, Re and α.
6. The Nusselt number Nu is an increasing function of M, Re, α, Pr, N_R and Ec.

Acknowledgements

The authors are thankful to the Higher Education Commission of Pakistan for their financial assistance. We are further grateful to the reviewers for their useful comments.

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