



# Thermal fluctuations in viscous cosmology

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## ABSTRACT

In this Letter we investigate the power spectrum of thermal fluctuations in very early stage of viscous cosmology. When the state parameter as well as the viscous coefficient of a barotropic fluid is properly chosen, a scale invariant spectrum with large non-Gaussianity can be obtained. In contrast to the results previously obtained in string gas cosmology and holographic cosmology, we find the non-Gaussianity in this context can be  $k$ -independent such that it is not suppressed at large scale, which is expected to be testified in future observation.

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## 1. Introduction

Recent observed Cosmic Microwave Background (CMB) anisotropy [1–3] and the large scale structure of the universe [4–8] can be viewed as the results of the primordial density perturbations, which is characterized by a nearly scale invariant and Gaussian power spectrum [9]. Though at present completely understanding the origin of the primordial density perturbations is still an open question, one traditional point of view is thinking of quantum fluctuations in vacuum as the seed of such classical perturbations. Nevertheless, there is an alternative conjecture arguing that it may be due to thermal fluctuations of matter sources during the inflationary stage [10–13]. Unfortunately, thermal fluctuations in standard inflation models always generate a power spectrum with spectral index  $n_s = 4$  [14,15]. Recently progress has been made to overcome this difficulty. People show that once some new physics is introduced, for instance in the context of the noncommutative

models since the non-Gaussianity has to be greatly suppressed at  $|f_{\text{NL}}| < 1$  in most of these models [21,22]. Contrasting to fluctuations originated from vacuum, the fluctuations in thermal scenario are always not strictly Gaussian. This may provide an alternative way to seek the observable non-Gaussianity. Therefore, the thermal non-Gaussianity has been widely discussed in recent literatures [23–34].

In this Letter we intend to investigate the power spectrum of thermal fluctuations in the very early stage of viscous cosmology, where the matter source is a viscous barotropic fluid (the state parameter  $w = \frac{p}{\rho} \in [-1, 1]$  is a constant). Since the viscous effect has an impact on the perturbation modes after they cross the thermal horizon, it is expected that the power spectrum should be different from the previous results even without other new mechanism or extra structure introduced. As a matter of fact, we find that provided the state parameter as well as the viscous coefficient of a barotropic fluid is properly chosen, a scale invariant spectrum with large non-Gaussianity is indeed.

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we can find [11,14–20].

Recently the thermal origin of the primordial density perturbation has received more attention since the non-Gaussianity of CMB has been further disclosed in the latest observation data. It indicates that at 95% confidence level, the primordial non-Gaussianity parameters for the local and equilateral models are in the region  $-9 < f_{\text{NL}}^{\text{local}} < 111$  and  $-151 < f_{\text{NL}}^{\text{equil}} < 253$ , respectively [9]. If this result is confirmed by future experiments such as the Planck satellite, then it will be a great challenge to many slow-roll inflation

In the next section we present a brief review on the thermal fluctuations in the FRW universe filled with a perfect fluid and demonstrate the difficulty of gaining a scale-invariant spectrum in this model. Then in Section 3 we show that this difficulty can be overcome by introducing a viscous fluid and properly choosing its viscous coefficient. Its non-Gaussianity is investigated in Section 4. Finally the comparison with other cosmological models with thermal fluctuations is given in Section 5.

## 2. A review on thermal fluctuations in the FRW universe with a perfect fluid

In this section, we briefly review the thermal fluctuations in very early universe and derive its power spectrum based on ther-

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modynamical consideration. First of all, we consider the FRW universe filled with a perfect fluid with state parameter  $w$ . For a perfect fluid, the thermal system can reach equilibrium state through interactions at sub-horizon scales. In the thermal scenario of cosmological fluctuations, this process is called thermalization. Since during this process both of the extensive internal energy  $U$  and entropy  $S$  are well defined for a global equilibrium state, we have the following form of the first law of thermodynamics

$$dU = T dS - p dV. \quad (1)$$

For barotropic fluid, one finds that the relation between the energy density and the temperature is uniquely fixed by this thermodynamical law [14,15]

$$\rho = AT^m, \quad (2)$$

where  $m = 1 + \frac{1}{w}$  and  $A$  is an integral constant. For radiation with  $w = \frac{1}{3}$ , this equation is nothing but the famous Stefan–Boltzmann law. On the other hand, the partition function of the thermal fluid is defined as

$$Z = \sum_r e^{-\beta E_r}, \quad (3)$$

where  $\beta = T^{-1}$ . The internal energy  $U$  inside a volume  $V$  is given by

$$U = \langle E \rangle = \frac{\sum_r E_r e^{-\beta E_r}}{\sum_r e^{-\beta E_r}} = -\frac{d \log Z}{d\beta}. \quad (4)$$

Therefore, the two-point correlation function for the density fluctuation  $\delta\rho \equiv \rho - \langle \rho \rangle$  can be obtained as

$$\begin{aligned} \langle \delta\rho^2 \rangle &= \frac{\langle \delta E^2 \rangle}{V^2} = \frac{\langle E^2 \rangle - \langle E \rangle^2}{V^2} = \frac{1}{V^2} \frac{d^2 \log Z}{d\beta^2} \\ &= -\frac{1}{V^2} \frac{dU}{d\beta} = \frac{T^2 C_V}{R^6}, \end{aligned} \quad (5)$$

where  $C_V = (\frac{\partial U}{\partial T})_V = V \frac{d\rho}{dT} \equiv V\rho'$  is the specific heat and  $R \sim V^{\frac{1}{3}}$  is the size of the thermal horizon. Thermal fluctuations generated from the matter inside  $R$  can be described by the thermodynamics above. But when thermal modes are pushed outside the horizon, they are frozen and become non-thermal governed by the theory of perturbations. Next we calculate the power spectrum of perturbations. Following most of literatures we identify the thermal horizon  $R$  with the Hubble horizon  $H^{-1}$  [14,15,18,19,25–27], which means our calculation of spectrum is always taken at the Hubble scale.

If perturbations are deeply in the horizon, the 0–0 component of the perturbative Einstein equation will reduce to the Poisson equation which relates the curvature fluctuations  $\Phi_k$  and the density perturbations  $\delta\rho_k$  as [35]

$$k^2 \Phi_k = 4\pi G a^2 \delta\rho_k, \quad (6)$$

where  $\delta\rho_k = k^{-\frac{3}{2}} \delta\rho$ . Thus the power spectrum can be obtained by combining Eqs. (5) and (6)

$$\mathcal{P}_\Phi(k) \equiv \frac{k^3}{2\pi} \langle \Phi_k^2 \rangle \sim \frac{a}{k} T^2 \rho', \quad (7)$$

where we have used the condition  $R = H^{-1} = \frac{a}{k}$ . When the  $\Phi$  modes leave the horizon, their amplitudes get fixed at whatever thermal amplitudes they have at crossing  $k = aH$ . For simplicity, we consider the spatially flat universe. Then using the Friedmann equation  $H^2 \propto \rho$ , we have

$$\mathcal{P}_\Phi(k) \sim \left[ \frac{T^2 \rho'}{\sqrt{\rho}} \right]_{k=aH}. \quad (8)$$

For a constant  $w$ , substituting the thermodynamical relation (2) into above equation leads to

$$\frac{d \ln \mathcal{P}_\Phi}{d \ln T} = 1 + \frac{m}{2}. \quad (9)$$

Furthermore, from the conservation equation of the fluid one has  $\rho \propto a^{-3(1+w)}$  such that

$$a \propto T^{-\frac{m}{3(1+w)}}. \quad (10)$$

Thus we have

$$\frac{d \ln k}{d \ln T} = \frac{m(1+3w)}{6(1+w)}, \quad (11)$$

where we have used  $k = aH \propto a\sqrt{\rho}$ . Using (9) and (11), it is straightforward to calculate the spectral index as

$$n_s - 1 = \frac{d \ln \mathcal{P}_\Phi}{d \ln k} = \frac{d \ln \mathcal{P}_\Phi}{d \ln T} \frac{d \ln T}{d \ln k} = 3 \frac{2+m}{m} \frac{w+1}{3w+1} = 3. \quad (12)$$

In this equation the relation  $m = 1 + \frac{1}{w}$  has been applied. Therefore the spectrum is always blue and independent of the value of  $w$ . Of course this “no-go result” [15] is not consistent with the current experiments, in which  $n_s$  is restricted at  $n_s = 0.960_{-0.013}^{+0.014}$  [9]. Then, to obtain a scale invariant spectrum in thermal scenario, one need introduce new physics to either relax some constraints due to thermodynamics, for instance as presented in [14,19], or modify the standard cosmological equations, as presented in [15]. Different from above considerations, in next section we would like to argue that the no-go result above can also be avoided if the matter source is a viscous fluid rather than a perfect one.

### 3. Realization of scale-invariant fluctuations in viscous cosmology

Viscous cosmology has been widely applied to investigate the structure and the evolution of the universe [36–50] (for recent review we refer to [51]). In this context the energy–momentum tensor of the fluid is given by

$$T_{\mu\nu} = (\rho + p - 3\xi H)u_\mu u_\nu + (p - 3\xi H)g_{\mu\nu}, \quad (13)$$

where the bulk viscosity coefficient  $\xi = \xi(\rho)$  is usually a function of the energy density  $\rho$  of the fluid. Moreover, as pointed out in [52],  $\xi$  should be positive if the second law of thermodynamics is respected. In FRW universe the conservation equation  $\nabla^\mu T_{\mu\nu} = 0$  becomes

$$\dot{\rho} + 3H(\rho + p - 3\xi H) = 0. \quad (14)$$

In this Letter, we choose a special kind of viscous fluids with a bulk viscosity coefficient  $\xi(\rho) \propto \rho^{\frac{1}{2}}$ , which has already been investigated in some literatures [39,43,53]. In this case,  $\frac{3\xi H}{\rho} \equiv \alpha$  is a positive constant which is relevant to the viscosity of the fluid. Then given a constant  $w$ , we have a relation between the energy density  $\rho$  and the scale factor from the equation above

$$\rho \propto a^{-3(1+w-\alpha)}. \quad (15)$$

Obviously, the evolution of the universe depends not only on the state parameter  $w$  but also on  $\alpha$ .

Now we discuss the thermalization of the matter source. Usually, the thermal scenario of cosmological fluctuations is established on the thermodynamics of the equilibrium state. Unlike the case with a perfect fluid, the viscosity brings dissipative effect such that the global description of the first law of thermodynamics in Eq. (1) no longer stands in general. Nevertheless, each particle in a viscous fluid satisfies the Gibbs relation in a local equilibrium state

$$T ds = d\frac{\rho}{n} + p d\frac{1}{n}, \quad (16)$$

where  $s$  is the entropy and  $n$  is the particle number in a local equilibrium element. For the barotropic case, it can be rewritten as

$$ds = \frac{1}{Tn} d\rho - \frac{\rho(1+w)}{Tn^2} dn. \quad (17)$$

Employing the integrability condition

$$\frac{1}{T} \left( \frac{\partial 1}{\partial n n} \right)_\rho = - \frac{(1+w)}{n^2} \left( \frac{\partial \rho}{\partial \rho T} \right)_n, \quad (18)$$

one can still derive the Stefan–Boltzmann law  $\rho \propto T^m$  with  $m = 1 + \frac{1}{w}$  [45,54]. This result indicates the viscosity does not change the relation between the energy density and the temperature of the fluid.

In a heuristic manner we may argue that the anisotropy and the inhomogeneity of our universe totally originate from the fluctuations in such a local equilibrium element. Consider a quanta of the fluctuation with a momentum  $P_r$ . Since the momentum uncertainty of a quantum particle  $\Delta P_r$  is of order of  $P_r$  [55–57], from the uncertainty principle we have

$$P_r = \hbar k_{rph} \sim \Delta P_r \sim \frac{\hbar}{\Delta x_r}, \quad (19)$$

where  $k_{rph} = \frac{k_r}{a}$  is the physical wavenumber. Then we have  $\Delta x_r \sim \frac{a}{k_r}$ . Using the crossing condition  $k_r = a_r H_r$ , we derive its position uncertainty  $\Delta x_r \sim H_r^{-1}$ . Since all of the fluctuations happen in the local equilibrium element, the position uncertainty should not be larger than the scale of the element  $\Delta x_r \leq R$ . We find

$$H^{-1} \sim \Delta x \leq R, \quad (20)$$

when each  $k$  crosses the horizon. In other words, this local equilibrium element within a radius of the Hubble scale can be viewed as the cradle of the cosmological perturbations. Moreover, we point out that both the background equation  $H^2 \propto \rho$  and the Poisson equation (6) will not change due to the viscosity effects and this can be understood from the fact that the viscous term does not appear in the 0–0 component of the energy–momentum tensor, as shown in Eq. (13). Therefore the correlation functions can be similarly calculated as in previous section. Inserting the relation in Eq. (2) into (15), we find the variation of the scale factor with the temperature is modified as

$$a \propto T^{\frac{-m}{3(1+w-\alpha)}}. \quad (21)$$

Repeating the calculation in previous section, we derive

$$\frac{d \ln k}{d \ln T} = \frac{m(1+3w-3\alpha)}{6(1+w-\alpha)}. \quad (22)$$

As a result, the spectral index for a viscous fluid can be obtained as

$$\begin{aligned} n_s - 1 &= \frac{d \ln \mathcal{P}_\Phi}{d \ln k} = 3 \frac{2+m}{m} \frac{w-\alpha+1}{3w-3\alpha+1} \\ &= 3 \frac{3w+1}{w+1} \frac{w-\alpha+1}{3w-3\alpha+1}. \end{aligned} \quad (23)$$

We find that the effect of the viscosity does imprint on the power spectrum of the cosmological perturbations. Especially, if we choose  $w \approx \alpha - 1$  or  $w \approx -\frac{1}{3}$  with any non-vanishing  $\alpha$ , a nearly scale-invariant power spectrum of the perturbations can be implemented. The case with  $w \approx \alpha - 1$  is nothing but the standard inflation, because  $\dot{H} = -4\pi G \rho(1+w-\alpha) \approx 0$  implies a nearly de Sitter expansion. When  $w \approx -\frac{1}{3}$ , since  $\alpha > 0$  as required by the second law of thermodynamics, we have  $\frac{\dot{a}}{a} = -4\pi G \rho(1+3w-3\alpha) > 0$ . In this case, the universe undergoes an accelerating

expansion as well. Therefore, in contrast to models with a perfect fluid, the thermal scenario in viscous cosmology may provide an alternative interpretation on the scale invariance of the power spectrum. However, to be well consistent with the current observation constraints, we need further study the non-Gaussianity in this thermal scenario, and this is what we intend to do in next section.

#### 4. Thermal non-Gaussianity in viscous cosmology

Another important quantity which is expected to be measured precisely in coming experiments is the non-Gaussianity of CMB. The amount of the non-Gaussianity is usually estimated by the quantity  $f_{\text{NL}}$  which is defined by the three-point correlation function for the curvature perturbations  $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$ . Generally, two cases are considered. one is the local non-Gaussianity labeled by  $f_{\text{NL}}^{\text{local}}$  with  $k_1 \ll k_2 \approx k_3$ , and the other is the equilateral non-Gaussianity denoted by  $f_{\text{NL}}^{\text{equil}}$  with  $k_1 \approx k_2 \approx k_3$ . More detailed discussions on these quantities can be found in [58,59]. Recent WMAP-5 data shows the possibility that there exist large non-Gaussianities of cosmological perturbations [9]. If this is true, thermal scenario may give a more consistent result. In this section, we will calculate the equilateral non-Gaussianity of thermal fluctuations in the context of viscous cosmology.

The non-Gaussianity is quantitatively described by the non-Gaussianity estimator

$$f_{\text{NL}}^{\text{equil}} = \frac{5}{18} k^{-\frac{3}{2}} \frac{\langle \zeta_k^3 \rangle}{\langle \zeta_k^2 \rangle \langle \zeta_k^2 \rangle}, \quad (24)$$

where  $\zeta_k$  is the curvature perturbation on slices of uniform energy density, while  $\langle \zeta_k^2 \rangle$  and  $\langle \zeta_k^3 \rangle$  are two-point and three-point correlation functions, respectively. Since no entropic perturbations occur in single component case [60], the cosmological perturbations are adiabatic. Then  $\zeta$  can be given as

$$\zeta = \Phi + H \frac{\delta \rho}{\rho}. \quad (25)$$

It remains constant on super-horizon scales in the adiabatic case even if  $w$  is changing. Considering the fact that  $\frac{\delta \rho}{\rho}$  is always equal to  $-2\Phi$  on super-horizon scales [61] and using (14), we can rewrite Eq. (25) as

$$\zeta = \frac{5+3w-3\alpha}{3+3w-3\alpha} \Phi. \quad (26)$$

Since both  $w$  and  $\alpha$  are constants,  $\Phi$  is also a constant. Therefore, we can obtain  $f_{\text{NL}}^{\text{equil}}$  by evaluating  $\langle \Phi_k^2 \rangle$  and  $\langle \Phi_k^3 \rangle$ . From Eq. (2), we have the two-point correlation

$$\langle \delta \rho^2 \rangle = \frac{\langle \delta E^2 \rangle}{V^2} = -\frac{1}{V^2} \frac{dU}{d\beta} = \frac{mAT^{m+1}}{V}, \quad (27)$$

and the three-point correlation

$$\langle \delta \rho^3 \rangle = \frac{\langle \delta E^3 \rangle}{V^3} = \frac{1}{V^3} \frac{d^2 U}{d\beta^2} = \frac{m(m+1)AT^{m+2}}{V^2}. \quad (28)$$

Applying the horizon crossing condition  $k = a/R = aH$  and the Poisson equation (6), the two-point correlation function for  $\Phi_k$  is given as

$$\langle \Phi_k^2 \rangle = (4\pi G)^2 R^4 \langle \delta \rho_k^2 \rangle = (4\pi G)^2 R k^{-3} m A T^{m+1}, \quad (29)$$

while the three-point correlation function is

$$\langle \Phi_k^3 \rangle = (4\pi G)^3 R^6 \langle \delta \rho_k^3 \rangle = (4\pi G)^3 k^{-\frac{9}{2}} m(m+1) A T^{m+2}. \quad (30)$$

Then the non-Gaussianity estimator  $f_{\text{NL}}^{\text{equil}}$  can be calculated with the use of Eq. (24). It turns out

$$\begin{aligned} f_{\text{NL}}^{\text{equil}} &= \frac{5}{72\pi GAT^m R^2} \frac{3+3w-3\alpha}{5+3w-3\alpha} \frac{(m+1)}{m} \\ &= \frac{5}{72\pi G\rho R^2} \frac{3+3w-3\alpha}{5+3w-3\alpha} \frac{2w+1}{w+1}. \end{aligned} \quad (31)$$

Obviously the amount of non-Gaussianity also depends on both  $w$  and  $\alpha$ . In the previous section, we have concluded that the state parameter should be restricted at  $w \approx \alpha - 1$  or  $w \approx -\frac{1}{3}$  in order to obtain a scale invariant spectrum.

In the case of  $w \approx \alpha - 1$ , we get a small non-Gaussianity

$$f_{\text{NL}}^{\text{equil}} \approx 0, \quad (32)$$

which is not surprising since it leads to de Sitter expansion as in the slow-roll inflation scenario.

In the case of  $w = -\frac{1}{3}$ , we have

$$f_{\text{NL}}^{\text{equil}} = \frac{5}{144\pi G\rho R^2} \frac{2-3\alpha}{4-3\alpha} = \frac{5}{54} \frac{2-3\alpha}{4-3\alpha}. \quad (33)$$

As a result, if we choose the value of  $\alpha$  close to  $\frac{4}{3}$ , a large non-Gaussianity can be obtained. Furthermore, we find the non-Gaussianity is scale-invariant as well. That is to say, it is not suppressed at large scale, which is in contrast to the previous results in string gas cosmology and holographic cosmology where the non-Gaussianity is  $k$  dependent [24,25]. Finally, it is worthwhile to point out that in this case we have  $\dot{H} > 0$ , which implies a phantom like universe.

## 5. Conclusion and discussion

In this Letter, we have investigated the power spectrum of thermal fluctuations in very early stage of viscous cosmology with a special viscous coefficient. Firstly, with the conjecture that the origin of the structure of our universe is completely thermal, we have demonstrated that a nearly scale invariant spectrum can be achieved when the state parameter and the viscous coefficient are properly selected. Different from previous literatures, we have only introduced the viscosity of the fluid, without modifying the Einstein equation or the thermodynamical relations of matter sources. Moreover, the results of thermal scenario under the condition of  $w \approx \alpha - 1$  is closely analogous to those obtained in standard inflation scenario.

Secondly, we analyze the non-Gaussianity of the thermal fluctuations in this scenario. It is shown that there will be a suppressed non-Gaussianity in the case  $w \approx \alpha - 1$ . It is just analogous to the usual inflationary phase where  $w \approx -1$  is required. However, if we choose  $w \approx -\frac{1}{3}$ , a large and  $k$ -independent non-Gaussianity can be achieved by tuning  $\alpha$  near  $\frac{4}{3}$  which implies a phantom like universe. It is of interesting to notice that a phantom phase is also required in holographic cosmology and the near Milne cosmology when a large non-Gaussianity can be obtained.

In conclusion, due to the viscosity effects of non-perfect fluids, the *no-go result* of the thermal fluctuations can be avoided and a large non-Gaussianity can also be achieved. Therefore it is desirable to investigate furthermore the influence of viscosity effects on the structure formation and anisotropy of the CMB.

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