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Procedia Computer Science 34 (2014) 23 – 30

Procedia Computer Science

The 9th International Conference on Future Networks and Communications (FNC-2014)

Outage and BER Performance Analysis of Cascade Channel in Relay Networks

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Abstract

In this paper, the outage and bit error rate (BER) performance of a relay networks is studied based on the cascade channel. Firstly, the distribution of the cascade channel, which is the integration of L double-Rayleigh complex random variables (RV), is derived in close form. To overcome the computational difficulty of the arising special function, the probability density function (pdf) of the channel is approximated by the concise Gamma distribution. Through matching the moments of both pdfs, the corresponding shape and scale parameters of the Gamma pdf are obtained. Based on the pdf approximation, the closed-form and accurate expressions of the metrics: outage probability and BER of the cascade channel, are deduced. Numerical results illustrate the accuracy of the proposed performance evaluation.

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Keywords: relay networks; cascade channel; outage probability; bit error rate.

1. Introduction

As a spectral efficient scheme for wireless communication system, relay networks has been recently attracting considerable investigation interesting [1]. It is a means of attaining larger cell coverage, mitigating channel impairments and increasing the channel capacity without extra transmission power. As a promising technique for future communication, virtual array formed by relays with different locations can gain spatial diversity, even the source and destination are only equipped with a single antenna. This spatial diversity is known as the cooperative diversity. During the signal transmission, information is conveyed to the destination via relays in two time phases which are the multiple access channel (MAC) phase and the broadcast channel (BC) phase.

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Selection and peer-review under responsibility of Conference Program Chairs
doi:10.1016/j.procs.2014.07.010

In consideration of the practical concerns, the relay networks performance analysis is a critical work before the application. Therefore, many study results were obtained including the outage probability, information theoretic metrics and the bit error rate (BER)[2-6]. In reference 2, BER analysis of amplify-and-forward (AF) relay networks is addressed. The channels between the terminals and the relay nodes were modelled as normally Rayleigh distribution. To overcome the severe fading, the author proposed an extended multiple input multiple output (MIMO) system. The author of reference 3 constructed a scenario where the two signal transmitters are both equipped multiple antennas. The transmitted signal was amplified by the best relay with single-antenna which was selected before the networks operation. Two metrics, the overall outage probability and the symbol error probability, were investigated over the Nakagami-m fading channels. This work was extended to references 4 and 5, where another two metrics, the overall symbol error rate and the ergodic sum rate, were obtained in the form of an exact and an upper bound expressions, respectively. However, the study[7,8] showed that the exact knowledge of the individual channels, which connect the terminal and the relay node, is not required for data detection. Instead, the cascade channel, from the information source to the destination via the relay nodes, is more convenient and straightforward for applications. In reference 6, the author approximated the cascade channel by the harmonic mean of two independent and identically distributed (i.i.d) random variables (RV). Then, the average sum rate and the upper bound for pairwise error probability were derived. To the best of the authors' knowledge, little performance discussion is on the scenario where the number of the cooperative relay is larger than 2, which is mainly considered in this paper.

The remainder of this paper is organized as follows: the relay networks model is described in Section II; as an indispensable foundation, the closed-form probability density function (pdf) of the cascade channel is proposed Section III; by approximating the original pdf with Gamma distribution, the closed-form and accurate expressions of the outage and BER performance of the cascade channel are deduced in Section IV; numerical results in Section V are presented to demonstrate the proposed analysis; some conclusions are made in the final section.

2. The relay networks model

Consider a relay networks where two terminals T_1 and T_2 are equipped with single antenna respectively. They communicate with each other via L single antenna relays $\boldsymbol{R}_L$. We further assume that the individual channels of both hops are reciprocal and subject to independent block Rayleigh fading. Let $\boldsymbol{h}=\left(h_1,h_2,\ldots,h_L\right)^{\mathrm{T}}$ and $\boldsymbol{g}=\left(g_1,g_2,\ldots,g_L\right)^{\mathrm{T}}$ denote the channel vectors from T_1 and T_2 to $\boldsymbol{R}_L$. According to the Rayleigh assumption, element of the two vectors follows zero-mean circularly symmetric complex Gaussian random variable with variance σ_h^2 or σ_g^2. We assume that the transmit power at the source T_1 and $\boldsymbol{R}_L$ are P and P_r, respectively. The whole transmission can be divided into two phases. In the first phase, T_1 transmits signal s to $\boldsymbol{R}_L$. Then $\boldsymbol{R}_L$ scale the received signals by a fixed factor $\alpha \triangleq \sqrt{P_r/\left(\sigma_h^2 P+\sigma_n^2\right)}$ where σ_n^2 stands for the variance of the additive white Gaussian noise (AWGN) in each relay node, and broadcast to T_2. The received signal at T_2 can be expressed as

$$y=\alpha \boldsymbol{h}^{\mathrm{T}}\boldsymbol{g}s+\alpha \boldsymbol{g}^{\mathrm{T}}\boldsymbol{n}_1+n_2 \tag{1}$$

without loss of generality, we assume that AWGN at $\boldsymbol{R}_L$ and T_2 share the same variance.

Observing (1) we realize that the exact knowledge of individual channels $\boldsymbol{h}$ and $\boldsymbol{g}$ are not required for data detection. Instead, the cascade channel which is the first term in the right-hand side (RHS) is only needed. The other two terms denote the entire noise, and can be represented by a zero-mean AWGN with variance $\left(\alpha^2 L\sigma_g^2+1\right)\sigma_n^2$.

3. The pdf of the amplitude of cascade channel

It can be also observed that the cascade channel $a \triangleq \boldsymbol{h}^{\mathrm{T}}\boldsymbol{g}$ denotes the cross-product of two independent complex

Gaussian vectors distributed as $\mathcal{CN}\left(0,\sigma_h^2 \boldsymbol{I}_L\right)$ and $\mathcal{CN}\left(0,\sigma_g^2 \boldsymbol{I}_L\right)$ where $\boldsymbol{I}_L$ denotes the $L \times L$ identity matrix. Let a_{R} and a_{I} stands for the real and imaginary parts of a, then their distributions conditioned on $\boldsymbol{h}$ are independent and normal, which are given as $a_{\mathrm{R}} \left|\mathbf{h} \sim \mathcal{N}\left(0,\sigma_g^2 \|\mathbf{h}\|^2/2\right)\right.$ and $a_{\mathrm{I}} \left|\mathbf{h} \sim \mathcal{N}\left(0,\sigma_g^2 \|\mathbf{h}\|^2/2\right)\right.$ where " $\sim$ " means "subjects to", $\|\cdot\|$ means the Euclidean norm. The joint characteristic function of the conditioned a_{R} and a_{I} is expressed as [9]

$$\Psi_{a_{\mathrm{R}},a_{\mathrm{I}}|\mathbf{h}}\left(j\omega_1, j\omega_2 \left|\mathbf{h}\right.\right) = E\left\{\exp\left[j\left(\omega_1 a_{\mathrm{R}} + \omega_2 a_{\mathrm{I}}\right)\right]\left|\mathbf{h}\right.\right\} = \exp\left[-\frac{1}{4}\left(\omega_1^2+\omega_2^2\right)\sigma_g^2 \|\mathbf{h}\|^2\right] \tag{2}$$

Meanwhile, the pdf of $\mathbf{h}$ is

$$p_{\mathbf{h}}\left(\mathbf{h}\right) = \frac{1}{\pi^L \sigma_h^{2L}} \cdot \exp\left(-\|\mathbf{h}\|^2/\sigma_h^2\right) \tag{3}$$

Therefore, the joint characteristic function of a_{R} and a_{I} is

$$\begin{aligned}\Psi_{a_{\mathrm{R}},a_{\mathrm{I}}}\left(j\omega_1, j\omega_2\right) &= \frac{1}{\pi^L \sigma_h^{2L}} \int_{\mathbf{h}} \exp\left[-\left(\frac{1}{\sigma_h^2} + \frac{\omega_1^2+\omega_2^2}{4}\sigma_g^2\right)\cdot \|\mathbf{h}\|^2\right] d\mathbf{h} \\ &= \left(1 + \frac{\omega_1^2+\omega_2^2}{4}\sigma_h^2\sigma_g^2\right)^{-L}\end{aligned} \tag{4}$$

We assume the pdf of a is $p_a\left(x,y\right)$ where x and y are its real and imagine parts. After transforming the Cartesian coordinates to the Polar coordinates $p_a\left(r_a,\theta_a\right)$, we obtain the pdf of fading amplitude r_a as

$$\begin{aligned}p_a\left(r_a\right) &= \int_0^{2\pi} p_a\left(r_a,\theta_a\right) d\theta_a \\ &= \frac{1}{4\pi^2}\int_{\theta_a=0}^{2\pi} r_a \cdot \int_{\omega_1=-\infty}^{+\infty}\int_{\omega_2=-\infty}^{+\infty} \Psi_{a_{\mathrm{R}}a_{\mathrm{I}}}\left(j\omega_1, j\omega_2\right) \\ &\cdot \exp\left[-jr_a\cdot\left(\omega_1\cos\theta + \omega_2\sin\theta\right)\right] d\omega_1 d\omega_2 d\theta_a\end{aligned} \tag{5}$$

Insert the expanded form of $\Psi_{a_{\mathrm{R}}a_{\mathrm{I}}}\left(j\omega_1, j\omega_2\right)$ into (5), and it yields

$$\begin{aligned}p_a\left(r_a\right) &= \frac{r_a}{4\pi^2}\int_0^{\infty} t\cdot\left(1+\frac{t^2}{4}\sigma_h^2\sigma_g^2\right)^{-L} \cdot \int_0^{2\pi}\int_0^{2\pi} \exp\left[-jr_a t\cos\left(\theta_a - \phi\right)\right] d\theta_a d\phi dt \\ &= \frac{4r_a^L}{\Gamma\left(L\right)\cdot\left(\sigma_h\sigma_g\right)^{L+1}}\cdot K_{L-1}\left(\frac{2r_a}{\sigma_h\sigma_g}\right) \quad r_a \ge 0\end{aligned} \tag{6}$$

where $\Gamma\left(\cdot\right)$ means Gamma function, $K_v\left(\cdot\right)$ means the second modified Bessel function with the order v. It is straightforward to obtain the pdf of the power of the channel by substituting variable r_a as $\sqrt{R_a}$, which is

$$p_{a^2}\left(R_a\right)=\frac{2R_a^{\frac{L-1}{2}}}{\Gamma(L)\cdot\left(\sigma_h\sigma_g\right)^{L+1}}\cdot K_{L-1}\left(\frac{2\sqrt{R_a}}{\sigma_h\sigma_g}\right)\quad R_a\ge 0 \tag{7}$$

The amplitude pdf of the cascade channel a in (6) is known as the generalized- K model[10]. When $L\to\infty$, it approaches the Nakagami-m distribution[11]. The corresponding cumulative distribution function (cdf) of a was derived[12] as

$$P_a(r)=\int_0^r p_a(t)\,dt=\frac{1}{\Gamma(L)}G_{1,3}^{2,1}\left[\frac{r^2}{\sigma_h^2\sigma_g^2}\Big|_{L,1,0}^{1}\right] \tag{8}$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer-G function[13]. Unfortunately, it is not an easy work for further analytical calculation due to the special functions contained in (6) and (8). Therefore, it is necessary to explore a handy approximation with the original pdf expression.

4. Approximation with Gamma pdf

It is convenient to approximated the generalized- K distribution with some more tractable pdf by the moment matching method, which saves plenty of computational difficulties[14]. It is also suggested that the Gamma distribution is a preferable candidate. The reason is that the generalized- K distribution, which corresponds to the product of two Gamma RVs[10], is dominate by the one with larger shape parameter.

The moments of the generalized- K pdf can be expressed as

$$\mu_K(n)=\frac{\Gamma(L+n/2)\Gamma(1+n/2)}{\Gamma(L)}\left(\sigma_h\sigma_g\right)^n\quad n=0,1,\dots \tag{9}$$

Assume V is a Gamma RV with a shape parameter k and a scale parameter θ, the positive moments of V can be denoted as

$$\mu_V(n)=\frac{\Gamma(k+n)\theta^n}{\Gamma(k)}\quad n=0,1,\dots \tag{10}$$

Besides, the first negative moments of the two distributions can be obtained as

$$\frac{1}{(k-1)\theta}=\frac{\Gamma(L-1/2)\Gamma(1/2)}{\Gamma(L)}\left(\sigma_h\sigma_g\right)^{-1} \tag{11}$$

Since the parameters k and θ can be calculated from moment matching equations, three representative pairs of (k,θ) are obtained by matching moments with orders $(-1,1)$, $(1,2)$ and $(2,3)$, which are listed in Table 1.

Table 1. The calculation results of Gamma parameters by matching moments.

Orders	(-1,1)	(1,2)	(2,3)

Shape parameter k	$\frac{\mu_1\mu_{-1}}{\mu_1\mu_{-1}-1}$	$\frac{\mu_1^2}{\mu_2-\mu_1^2}$	$\frac{\mu_3^2-4\mu_2^3+\sqrt{\left(4\mu_2^3-\mu_3^2\right)^2-16\mu_2^3\left(\mu_2^3-\mu_3^2\right)}}{2\left(\mu_2^3-\mu_3^2\right)}$
Scale parameter θ	$\frac{\mu_1\mu_{-1}-1}{\mu_{-1}}$	$\frac{\mu_2}{\mu_1}-\mu_1$	$\frac{2\mu_3\left(\mu_2^3-\mu_3^2\right)}{\mu_2\left[\sqrt{\left(4\mu_2^3-\mu_3^2\right)^2-16\mu_2^3\left(\mu_2^3-\mu_3^2\right)}-3\mu_3^2\right]}$

Remarks:

- Since the moments can be regarded as weighted integrals of pdf, the n th positive moment uses the weight function r^n, which all monotonically increase with r. Thus, the larger matching moment selected, the more approximated errors near the tail region are penalized. On the other hand, matching the negative moment means lower penalty to errors in the pdf tail, and exhibits better tracking around the head portion of the generalized-K distribution.
- It needs to be noted that the parameter L, which denotes the number of relay nodes involved to cooperate the communication, is an integer. Therefore, the $2(L+n)$ th ($n=0,1,\ldots$) negative moment can not obtained owning to the inexistence of Gamma function.
- It is not appropriate to calculate the parameters of the approximated Gamma pdf by matching higher orders such as $n \geq 4$. On one hand, much effort would devoted to derive the involved expressions of k and θ; on the other hand, matching higher orders may lead to the obvious divergence from the average power values of (6).

5. Approximation with Gamma pdf

With the aid of the approximation introduced above, some critical performance of the cascade channel can be evaluated by the tractable Gamma distribution, instead of the laborious generalized-K pdf. More specifically, these metrics are the outage probability and BER.

5.1. Outage probability

The outage probability denotes the probability that the received signal power falls below a given power threshold γ_{th}, which can be expressed as

$$P_{outage}\left(\gamma_{th}\right)=\Pr\left\{a^2\leq\gamma_{th}\right\}=\frac{1}{\Gamma(L)}G_{1,3}^{2,1}\left[\frac{\gamma_{th}}{\sigma_h^2\sigma_g^2}\middle|_{L,1,0}^{1}\right] \tag{12}$$

whereas, the outage probability can be computed by the concise cdf of the approximating Gamma distribution as

$$P_{outage}\left(\gamma_{th}\right)\doteq\Pr\left\{V^2\leq\gamma_{th}\right\}=\frac{1}{\Gamma(k)}\gamma\left(k,\sqrt{\gamma_{th}}/\theta\right) \tag{13}$$

where $\gamma(\alpha,x)$ is the incomplete Gamma function[13].

5.2. BER

An important performance measure in communication networks is the bit error probability which can be expressed as

$$P_e = \int_0^\infty P_e(x) p_\gamma(x) dx \tag{14}$$

where $P_e(x)$ is the BER in an AWGN channel.

For BPSK, M-QAM and for high values of the average input SNR, $P_e(x) = A \cdot erfc\left(\sqrt{Bx}\right)$ where $erfc(\cdot)$ is the complementary error function[13], and constants A and B depending on the specific modulation scheme. The accurate and approximated BER of BPSK ($A = 1/2$, $B = 1$) can be expressed as follows

$$P_{eK}^{BPSK} = \int_0^\infty \frac{1}{2} erfc\left(\sqrt{\mathrm{SNR} \cdot r^2}\right) \cdot p_a(r) dr = \frac{1}{2\sqrt{\pi}\Gamma(L)} G_{3,2}^{2,2}\left[\mathrm{SNR} \cdot \left(\sigma_h \sigma_g\right)^2 \middle| {}^{1-L,0,1}_{0,1/2}\right] \tag{15}$$

$$\begin{aligned} P_{e\Gamma}^{BPSK} &= \int_0^\infty \frac{1}{2} erfc\left(\sqrt{\mathrm{SNR} \cdot r^2}\right) \cdot p_V(r) dr \\ &= \frac{\theta^{-k} \cdot \mathrm{SNR}^{-\frac{k}{2}}}{2\sqrt{\pi}\Gamma(k)} \cdot \left[\frac{1}{k}\Gamma\left(\frac{k+1}{2}\right) {}_2F_2\left(\frac{k+1}{2}, \frac{k}{2}; \frac{1}{2}, \frac{k+2}{2}; \frac{1}{4\theta^2 \mathrm{SNR}}\right)\right. \\ &\quad \left. - \frac{1}{k+1} \cdot \frac{1}{\sqrt{\theta^2 \mathrm{SNR}}} \Gamma\left(\frac{k+2}{2}\right) {}_2F_2\left(\frac{k+1}{2}, \frac{k+2}{2}; \frac{3}{2}, \frac{k+3}{2}; \frac{1}{4\theta^2 \mathrm{SNR}}\right)\right] \end{aligned} \tag{16}$$

where ${}_2F_2(\cdot,\cdot;\cdot,\cdot;\cdot)$ denotes the generalized hypergeometric series[13].

For non-coherent BFSK and DPSK, $P_e(x) = A \cdot \exp(-Bx)$, Since $A = 1/2$, $B = 1$ is for the DPSK modulation scheme. The accurate and approximated BER of DPSK yield

$$\begin{aligned} P_{eK}^{DPSK} &= \int_0^\infty \frac{1}{2} \exp\left(-\mathrm{SNR} \cdot r^2\right) \cdot p_a(r) dr \\ &= \frac{1}{\Gamma(L)\left(\sigma_h \sigma_g\right)^{L+1}} \cdot \mathrm{SNR}^{-\frac{L+1}{2}} G_{1,2}^{2,1}\left[\frac{1}{\mathrm{SNR} \cdot \left(\sigma_h \sigma_g\right)^2} \middle| {}^{0}_{\frac{L-1}{2}, \frac{1-L}{2}}\right] \end{aligned} \tag{17}$$

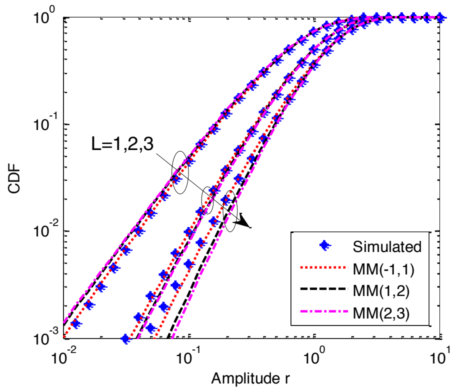
$$\begin{aligned} P_{e\Gamma}^{DPSK} &= \int_0^\infty \frac{1}{2} \exp\left(-\mathrm{SNR} \cdot r^2\right) \cdot p_V(r) dr \\ &= 2^{-\frac{k+2}{2}} \cdot \left(\theta^2 \mathrm{SNR}\right)^{-\frac{k}{2}} \cdot \exp\left(\frac{1}{8\theta^2 \mathrm{SNR}}\right) \cdot D_{-k}\left(\frac{1}{\sqrt{2\theta^2 \mathrm{SNR}}}\right) \end{aligned} \tag{18}$$

where $D_v(\cdot)$ is the parabolic function[13].

6. Numerical results

In this section, some numerical results are exhibited to illustrate the performance of the proposed metric approximation. In our analysis, the variances of individual and AWGN are all normalized to 1. The number of the relay nodes L is restricted to 3 or less due to the consideration of the acceptable complexity of the relay networks. SNR is defined as the ratio of the signal power to the whole noise, which is $\sqrt{P/\left(L\sigma_g^2 + \alpha^{-2}\right)\sigma_n^2}$. The different pairs of the moment-matching (MM) Gamma coefficients are calculated based on Table 1. The simulated data was obtained via Monte-Carlo method by generating 10^6 random samples.

Fig.1 displays the simulated and approximated cdf of the amplitude r of the cascade channel. It can be seen that the result of the first positive and negative moment matching method outperforms the other two, which is calculated by the expressions in Table 1. With a larger L, the slope of cdf turns more steep, but the (-1,1) pair provides a good fit for approximating the head portion of the cdf. While in Fig.2, the outage probability is depicted. It can be seen that the outage performance improves as L increases. The reason is that the more relay nodes cooperates, the less opportunity that the cascade channel falls below a given power threshold.



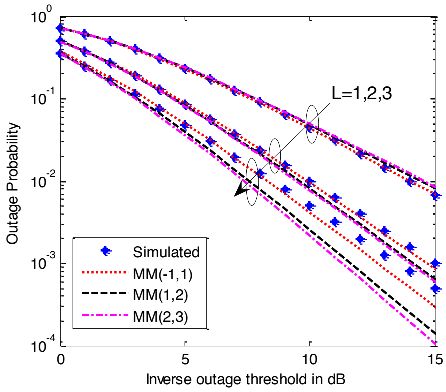
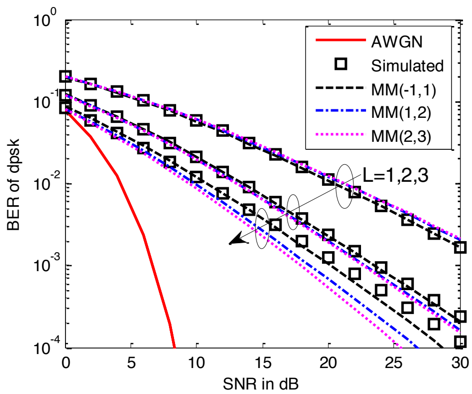


Fig.1 comparison of CDF with various L Fig.2 comparison of Outage probability with various L



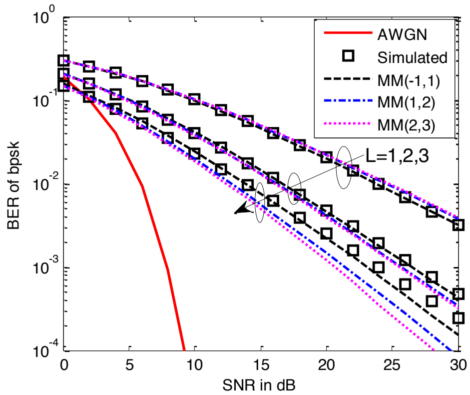


Fig.3 BER for dpsk with various L Fig.4 BER for bpsk with various L

In Fig.3 and Fig.4, the BER performance of the cascade channel with DPSK and BPSK modulation are plotted, respectively. As a comparison, the BER of AWGN channel scheme is also presented. It is obvious that the drop of the cascade channel in error probability with SNR is much less than AWGN channel because of its double-Rayleigh mechanism. Similarly, the BER performance improves with more relay nodes. The curve obtained with (-1,1) pair fits the simulated points better than the other two counterparts. This trend exhibits more obviously during the higher SNR region.

Based on the numerical results above, it can be noticed that the approximated Gamma distribution obtained through the (-1,1) pair leads to a good match to the original generalized-K pdf. The reason is that this pair penalizes the errors in the head portion of the pdf mainly. In the meantime, the metrics of outage probability, outage capacity and BER weigh the head portion of pdf more than the tail part. This coherence of the (-1,1) pair and the metrics results in a better accuracy than the (1,2) pair and the (2,3) pair.

7. Conclusions

In this paper, the pdf of the cascade channel of the relay networks, which comprised of several i.i.d double-Rayleigh complex RVs, is derived. The amplitude of the channel is shown to subject to the generalized- K distribution. To avoid computational difficulty, this pdf is approximated by the Gamma distribution, whose corresponding parameters are obtained through matching moments. With this replacement, closed-form expressions for the outage probability and BER are deduced. The numerical results verified the accuracy of the proposed performance evaluation.

References

1. Picinbono B, Kerilis JM. Some properties of prediction and interpolation errors. IEEE Transactions on Acoustics, Speech and Signal Processing; 1988. 36(4):525-531.
2. PremKumar M, SenthilKumaran VN, Thiruvengadam SJ, editors. Performance analysis of Diversity Techniques and amplify and forward wireless relay networks in multipath fading channels. International Conference on Recent Trends In Information Technology (ICRTIT); 2012. p.19-21.
3. Hui G, Jianghua G, Haiyang D. Symbol Error Probability of Two-Way Amplify-and-Forward Relaying. IEEE Communications Letters; 2011. 15(1):22-24.
4. Yadav S, Upadhyay PK, Prakriya S. Performance Evaluation and Optimization for Two-Way Relaying with Multi-Antenna Sources. IEEE Transactions on Vehicular Technology; 2014. (99):1-7.
5. Upadhyay PK, Shankar PM. Performance of Two-Way Opportunistic Relaying With Analog Network Coding Over Nakagami-m Fading. IEEE Transactions On Vehicular Technology; 2011. 60(4):1965-1971.
6. Han Y, Ting SH, Ho CK, Chin WH. Performance Bounds for Two-way Amplify-and-Forward Relaying. IEEE Transactions On Wireless Communications; 2009. 8(1):432-439.
7. Feifei G, Rui Z, Ying-Chang L. Optimal channel estimation and training design for two-way relay networks. IEEE Transactions on Communications; 2009. 57(10):3024-3033.
8. Feifei G, Tao C, Nallanathan A. On channel estimation and optimal training design for amplify and forward relay networks. IEEE Transactions on Wireless Communications; 2008. 7(5):1907-1916.
9. Mallik RK, Sagias NC. Distribution of Inner Product of Complex Gaussian Random Vectors and its Applications. IEEE Transactions on Communications; 2011. 59(12):3353-3362.
10. Lewinski D. Nonstationary probabilistic target and clutter scattering models. IEEE Transactions on Antennas and Propagation; 1983. 31(3):490-498.
11. Sen I, Matolak DW, Xiong W, editors. Wireless Channels that Exhibit "Worse than Rayleigh" Fading: Analytical and Measurement Results. IEEE Military Communications Conference; 2006. p. 23-25.
12. Peppas KP. Accurate closed-form approximations to generalised-K sum distributions and applications in the performance analysis of equal-gain combining receivers. IET Communications; 2011. 5(7):982-989.
13. Gradshteyn IS, Ryzhik IM. Tables of Integrals, Series and Products(7 edition). San Diego: Academic; 2007.
14. Al-Ahmadi S, Yanikomeroglu H. On the approximation of the generalized-K distribution by a gamma distribution for modeling composite fading channels. IEEE Transactions on Wireless Communications; 2010. 9(2):706-713.