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## Graviton modes in multiply warped geometry

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## ABSTRACT

The negative results in the search for Kaluza–Klein graviton modes at the LHC, when confronted with the discovery of the Higgs, have been construed to have severely limited the efficacy of the Randall–Sundrum model as an explanation of the hierarchy problem. We show, though, that the presence of multiple warping offers a natural resolution of this conundrum through modifications in both the graviton spectrum and their couplings to the Standard Model fields.

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## 1. Introduction

Despite the spectacular success of the Standard Model (SM) of elementary particles, the search for new physics beyond the SM continues. One of the primary motivations for this is to resolve the well-known gauge hierarchy/naturalness problem in connection with the fine tuning of the higgs mass against large radiative corrections. Among several proposals to address this problem, models with extra spatial dimensions draw special attention. In this context, the warped geometry model proposed by Randall and Sundrum (RS) [1] turned out to be particularly successful for (i) it resolves the gauge hierarchy problem without bringing in any other intermediate scale in the theory in contrast to the large extra dimensional models; (ii) the modulus of the extra dimensional model can be stabilized to a desired value by the Goldberger–Wise mechanism [2], and (iii) a similar warped solution can be obtained from a more fundamental theory like string theory where extra dimensions appear naturally [3]. As a result, several search strategies at the LHC were designed specifically [4–7] to detect the indirect/direct signatures of these warped extra dimensions e.g. through the dileptonic decays of Kaluza–Klein (KK) excitations of the graviton which appear in these models at the TeV scale.

The original RS model was defined as a slice of AdS<sub>5</sub> space with an S<sup>1</sup>/Z<sub>2</sub> orbifolding and a pair of three-branes located at the orbifold fixed points, viz.  $y = 0, \pi$  (with the SM fields being localized on the last mentioned). The parameters characterizing the

theory are the 5-dimensional fundamental (gravitational) scale  $M_5$  and the bulk cosmological constant  $\Lambda_5$ . The solution to Einstein's equations, on demanding a (1 + 3)-dimensional Lorentz symmetry, then leads to a warp-factor in the metric of the form  $\exp(-k_5 r_c y)$  where  $r_c$  is the compactification radius and  $k_5 = \sqrt{-\Lambda_5/24 M_5^3}$ . Clearly, the applicability of the semiclassical treatment (as opposed to a full quantum gravity calculation) requires that the bulk curvature  $k_5$  be substantially smaller than  $M_5$ . An analogous string theoretic argument [8] relating the D3 brane tension to the string scale (related, in turn, to  $M_5$  through Yang–Mills gauge couplings) demands the same, leading to  $k_5/M_5 \lesssim 0.1$ . On the other hand, too small a value for this ratio would, typically, necessitate a considerable hierarchy between  $r_c^{-1}$  and  $M_5$ , thereby taking away from the merits of the scenario. Thus, it is normally accepted that one should consider only  $0.01 \leq k_5/M_5 \leq 0.1$ . Indeed, this constraint plays a crucial role in most of the phenomenological studies of this scenario, and certainly for the aforementioned results reported by the ATLAS and the CMS groups. Throughout our analysis we shall impose an analogous condition on the bulk curvature as an important restriction to ensure the applicability of our semiclassical calculations.

In the context of the original RS model, the large exponential warping is held responsible for the apparent lightness of the Higgs vacuum expectation value  $v$  (and its mass), as perceived on our brane, related as it is to some naturally high scale  $\tilde{v} \sim \mathcal{O}(M_5)$ , applicable at the other brane, through the relation

$$v = \tilde{v} e^{-\pi k_5 r_c}. \quad (1)$$

Here  $\tilde{v}$  is determined by the natural scale of higher dimensional model  $\sim$  five dimensional Planck scale  $M_5$  and  $k_5 r_c \approx 12$  would explain the hierarchy with  $r_c$  being stabilized to this value by

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some mechanism [2]. The compactification leads to a non-trivial KK tower of gravitons with the levels being given by

$$m_n = x_n k_5 e^{-\pi k_5 r_c} \quad (2)$$

where  $x_n$ 's are the roots of the Bessel function of order one. With only the lowest (massless) graviton wavefunction being localized away from our brane, its coupling to the SM fields is small, viz.  $\mathcal{O}(M_5^{-1})$ . As the couplings of the others to the SM fields suffer no such suppression, they are, presumably, accessible to collider searches. The ATLAS Collaboration [5], though, has reported negative results ruling out a level-1 KK graviton in the mass range below 1.03 (2.23) TeV, with the exact lower bound depending on the value chosen for  $k_5/M_5$ .

This result immediately brings forth a potential problem for the model, for Eqs. (1) and (2) together demand that

$$\frac{m_1}{m_H} \sim \frac{m_1}{v} = x_1 \frac{k_5}{\tilde{v}} = x_1 \frac{k_5}{M_5} \frac{M_5}{\tilde{v}} \quad (3)$$

Since  $k_5/M_5 \lesssim 0.1$ , it is immediately apparent that, unless  $\tilde{v}$  is at least two orders of magnitude smaller than  $M_5$ , a 126 GeV Higgs [9,10] would cry out for a KK graviton below a TeV. Indeed, this argument has been inverted in the literature [11] to argue for a much lower cutoff (in other words  $\tilde{v}$ ) in the theory. In other words, some new physics would need to appear at least two orders of magnitude below the fundamental scale  $M_5$ , which, in the RS scenario is very close to the four-dimensional Planck scale itself.

Let us remind ourselves of the nature of cutoffs in the effective four-dimensional theory, considered as a theory of the SM fields augmented by the RS gravitons. While the SM is operative below the scale of the first KK graviton, the *new* four-dimensional theory is operative all the way up to the compactification scale  $\sim r_c^{-1}$  when each of the KK graviton is expected to take part in the amplitude estimation as the beam energy is increased. Beyond the energy  $\sim r_c^{-1}$ , we indeed encounter new physics by probing into the extra dimension where the theory can no longer be defined as an effective theory in four dimensions defined by standard model and KK gravitons.

It is important to realize, at this stage, that part of the aforementioned problem lies in the very restrictive nature of the RS model as it is impossible to lower  $r_c^{-1}$  by two orders without disturbing the value of the warped factor significantly. This, in turn, would introduce a little hierarchy necessitating a fine tuning of 2–3 orders so that the Higgs mass may be kept  $\sim 125$  GeV. This feature would worsen further if a graviton KK mode continues to elude us in the forthcoming runs of the LHC, as well as in future collider experiments.

On the other hand, within the context of a generalization of the RS model with additional warped extra dimensions, a lower cutoff appears *naturally*, in the form of a larger compactification radius. In other words, the problem is circumvented without the need for any additional (small) fine tuning. Indeed, once we admit more than four dimensions, there is no particular reason to restrict the number to five, especially with constructs such as string theoretic models arguing in favour of many more. Such variants of the RS model have been proposed earlier [12–15,28] with these, typically, considering several independent  $S^1/Z_2$  orbifolded dimensions along with  $M^{(1,3)}$ . For example, codimension-2 brane models [16] have been invoked to address aspects like Hubble expansion and inflation [17–19], Casimir densities [20,21], little RS hierarchy [22], gravity and matter field localizations [23,24], fermion mass generations [25,26], moduli stabilization [27], etc.

We begin our study, with a brief discussion of the basic features of warped geometry model in 6-dimension with two successive  $S^1/Z_2$  orbifoldings.

## 2. Multiply warped brane world model in 6D

Consider a doubly warped compactified six-dimensional space-time with successive  $Z_2$  orbifolding in each of the extra dimensions, viz.  $M^{1,5} \rightarrow [M^{1,3} \times S^1/Z_2] \times S^1/Z_2$ . Demanding four-dimensional ( $x^\mu$ ) Lorentz symmetry within the set up, requires the line element to be given by [28]

$$ds_6^2 = b^2(z)[a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2, \quad (4)$$

where the compact directions are represented by the angular coordinates  $y, z \in [0, \pi]$  with  $R_y$  and  $r_z$  being the corresponding moduli. Just as in the RS case, non-trivial warp factors  $a(y)$  and  $b(z)$ , when accompanied by the orbifolding necessitates the presence of localized energy densities at the orbifold fixed points, and in the present case, these appear in the form of tensions associated with the four end-of-the-world 4-branes.

The total bulk-brane action for the six dimensional space time is, thus,

$$\begin{aligned} \mathcal{S} &= \mathcal{S}_6 + \mathcal{S}_5 \\ \mathcal{S}_6 &= \int d^4x dy dz \sqrt{-g_6} (M_6^4 R_6 - \Lambda) \\ \mathcal{S}_5 &= \int d^4x dy dz \sqrt{-g_5} [V_1(z)\delta(y) + V_2(z)\delta(y-\pi)] \\ &\quad + \int d^4x dy dz \sqrt{-\tilde{g}_5} [V_3(y)\delta(z) + V_4(y)\delta(z-\pi)], \end{aligned} \quad (5)$$

where  $\Lambda$  is the (six dimensional) bulk cosmological constant and  $M_6$  is the natural scale (quantum gravity scale) in six dimensions. The five-dimensional metrics in  $\mathcal{S}_5$  are those induced on the appropriate 4-branes, which accord a rectangular box shape to the space. Furthermore, the SM (and other) fields may be localized on additional 3-branes located at the four corners of the box, viz.

$$\mathcal{S}_4 = \sum_{y_i, z_i=0, \pi} \int d^4x dy dz \sqrt{-g_4} \mathcal{L}_i \delta(y-y_i) \delta(z-z_i).$$

These terms, however, are not germane to the discussions of this paper, and we shall not discuss  $\mathcal{S}_4$  any further.

For a negative bulk cosmological constant  $\Lambda$ , the solutions for the 6-dimensional Einstein field equations are given by [28]

$$\begin{aligned} a(y) &= e^{-c|y|} & c &= \frac{R_y k}{r_z \cosh k\pi} \\ b(z) &= \frac{\cosh(kz)}{\cosh(k\pi)} & k &= r_z \sqrt{\frac{-\Lambda}{10M_6^4}} \equiv r_z k'. \end{aligned} \quad (6)$$

The Israel junction conditions specify the brane tensions. The smoothness of the warp factor at  $z=0$  implies  $V_3(y)$  be vanishing, while the fixed point at  $z=\pi$  necessitates a negative tension, viz.

$$V_3(y) = 0, \quad V_4(y) = \frac{-8M^4 k}{r_z} \tanh(k\pi). \quad (7)$$

With the warping in the  $y$ -direction being similar to that in the 5D RS model, the two 4-branes sitting at  $y=0$  and  $y=\pi$  have equal and opposite energy densities. However, the  $z$ -warping dictates that, rather than being constants, these energy densities must be  $z$ -dependent, viz.

$$V_1(z) = -V_2(z) = 8M^2 \sqrt{\frac{-\Lambda}{10}} \operatorname{sech}(kz). \quad (8)$$

Such a  $z$ -dependence can arise from a scalar field distribution confined on the brane. For a detailed discussion on this we refer our

reader to Section III of [28]. The (derived) 4-dimensional Planck scale can be related to the fundamental scale  $M$  through

$$M_p^2 \sim \frac{M_6^4 r_z R_y}{2ck} \left(1 - e^{-2c\pi}\right) \left[ \frac{\tanh k\pi}{\cosh^2 k\pi} + \frac{\tanh^3 k\pi}{3} \right]. \quad (9)$$

If there exists no other brane with an energy scale lower than ours, we must identify the SM brane with the one at  $y = \pi$ ,  $z = 0$ . This immediately gives the required hierarchy factor (i.e. the mass rescaling due to warping) to be

$$w = \frac{e^{-c\pi}}{\cosh k\pi}. \quad (10)$$

For the large hierarchy that we need to explain, this equation, along with the relation between  $c$  and  $k$  (Eq. (6)) demands that, unless there is a very large hierarchy between the moduli, the warping is substantial in only one of the two directions, and rather subdominant in the other. In other words, we can have either (i) a large ( $\sim 10$ ) value for  $k$  accompanied by an infinitesimally small  $c$  or (ii) a large ( $\sim 10$ ) value for  $c$  with a moderately small ( $\lesssim 0.3$ )  $k$ . The issue of moduli stabilization in such multiple moduli scenario is yet to be addressed. However, in view of the essential similarity of the warp factors to the RS case, we believe that an analogue of the Goldberger–Wise stabilization mechanism [2], using either a bulk six-dimensional scalar field, or a combination of 4-brane localized scalars would fit the bill. This is currently under investigation.

In summary, we are dealing with a brane world which is doubly warped, with the warping being large along one direction and small in the other. The very structure of the theory typically requires a small hierarchy between the two moduli, both of which remain comparable to the fundamental length scale in the theory. The stability issues in such models have been studied along with the effects of bulk gauge field or higher form anti-symmetric tensor field [29–32].

Apart from the gauge hierarchy problem, such a model can offer a possible resolution of the observed fermion mass hierarchy [33]. Furthermore, we can achieve a consistent description of a bulk Higgs and gauge fields with spontaneous symmetry breaking in the bulk, along with proper  $W$  and  $Z$  boson masses on the visible brane [34]. Given these successes of the model, it is interesting to consider the graviton sector of the theory and, in particular, to investigate whether it is consistent with the LHC bounds.

### 3. The graviton KK modes

To obtain the KK modes, one needs to consider the fluctuations of the metric,

$$g_{MN} = \bar{g}_{MN} + \Delta_{MN} \quad (11)$$

where  $\bar{g}_{MN}$  denotes the background (classical) metric corresponding to the line element of Eq. (4). We focus our attention on the relevant (four-dimensional) tensor fluctuations  $\Delta_{\mu\kappa}$  which, for the sake of convenience, are parametrized as

$$\Delta_{\mu\kappa} = b^2(z) a^2(y) \tilde{\Delta}_{\mu\kappa}(x_\mu, y, z) \quad (12)$$

The corresponding equation of motion is,

$$R_{\mu\kappa} = \frac{-\Lambda}{2} g_{\mu\kappa} \quad (13)$$

The gauge conditions

$$\Delta_{\mu}^{\mu} = 0, \quad \partial^{\mu} \Delta_{\mu\kappa} = 0,$$

in turn, imply

$$\tilde{\Delta}_{\mu}^{\mu} = 0, \quad \partial^{\mu} \tilde{\Delta}_{\mu\kappa} = 0. \quad (14)$$

The KK mode expansion, in terms of the four-dimensional fields  $h_{\mu\nu}^{(n,p)}(x)$  can now be written in terms of the two winding numbers as

$$\tilde{\Delta}_{\mu\nu}(x^{\mu}, y, z) = \frac{1}{\sqrt{R_y r_z}} \sum_{n,p} h_{\mu\nu}^{(n,p)}(x) \psi_{np}(y) \chi_p(z). \quad (15)$$

This, then, yields the equations of motion, viz.

$$\begin{aligned} 0 &= (\square + m_{np}^2) h_{\mu\nu}^{(n,p)}(x) \\ 0 &= R_y^{-2} \frac{d}{dy} \left( a^4 \frac{d\psi_{np}}{dy} \right) - m_p^2 a^4 \psi_{np} + m_{np}^2 a^2 \psi_{np} \\ 0 &= r_z^{-2} \frac{d}{dz} \left( b^5 \frac{d\chi_p}{dz} \right) + m_p^2 b^3 \chi_p \end{aligned} \quad (16)$$

To obtain the spectrum, we need to solve the equations for the modes  $\chi_p(z)$  and  $\psi_{np}(y)$ , which we now proceed to do.

#### 3.1. The $z$ equation

For the zeroth mode, we have

$$\partial_z \left( b^5 \partial_z \chi_0 \right) = 0$$

which has the particularly simple solution

$$\begin{aligned} \chi_0 &= c_0^{(0)} + \frac{c_1^{(0)}}{8k} \left[ 6 \tan^{-1} \left( \tanh \frac{kz}{2} \right) \right. \\ &\quad \left. + \left( 3 + \operatorname{sech}^2(kz) \right) \operatorname{sech}(kz) \tanh(kz) \right]. \end{aligned} \quad (17)$$

The constants  $c_{0,1}^{(0)}$  are determined from the boundary conditions and/or normalization of the wavefunction  $\chi_0(z)$ . The solution for the higher modes  $\chi_p$  is obtained in terms of associated Legendre polynomials of the first and second kinds, viz.

$$\begin{aligned} \chi_p(z) &= \tilde{\Xi}_p \operatorname{sech}^{5/2}(kz) \left[ \cos \theta_p P_{\nu_p}^{5/2}(\tanh(kz)) \right. \\ &\quad \left. + \sin \theta_p Q_{\nu_p}^{5/2}(\tanh(kz)) \right] \\ \nu_p &\equiv \sqrt{4 + \frac{m_p^2 r_z^2 \cosh^2(k\pi)}{k^2}} - \frac{1}{2} = \sqrt{4 + \frac{m_p^2 R_y^2}{c^2}} - \frac{1}{2} \\ &\equiv \sqrt{4 + x_p^2 \cosh^2(k\pi)} - \frac{1}{2} \end{aligned} \quad (18)$$

where  $\theta_p$  determines the relative weight of the two independent solutions and  $\tilde{\Xi}_p$  is the normalization constant obtained from

$$\delta_{pp'} = \int_{-\pi}^{\pi} dz b^3(z) \chi_p(z) \chi_{p'}(z). \quad (19)$$

That the above solution reduces to the aforementioned  $\chi_0(p)$  for  $m_p = 0$  (i.e.,  $\nu_p = 3/2$ ) is easy to see.

It should be noted that  $\nu_p$  is not necessarily integral (or, even half-integral). The presence of the associated Legendre functions renders the analysis much more complicated than is the case for the 5D analogue. This, in turn, introduces interesting new features.

It has been argued in the literature [35] that the  $z$ -equation can be simplified to a great extent by approximating the warp factor  $1/\cosh(kz)$  by an exponential, which ought to be valid for large  $kz$ . Indeed, thus simplified equation of motion has solutions in terms of Bessel and Neumann functions, and the corresponding analysis has exact parallels with the 5D case. The approximation

however would not work for the small  $k$  regime. Moreover even for large  $k$ , such approximation is invalid for  $z \sim 0$ , precisely the region where we are supposed to be located. And since the values of the graviton wavefunctions would determine the strength of their couplings to the SM fields, we should expect that such an approximation would lead to some inaccuracies. Moreover, such an approximation changes the differentiability of the warp factors, thereby changing the boundary conditions on the graviton wavefunctions. As we shall see later, the consequences of such an approximation are really profound and, hence, we desist from adopting it.

### 3.2. The $y$ equation

The equation for the  $y$ -mode function can be simplified by making the transformations

$$\begin{aligned} \psi_{np}(y) &= e^{2c|y|} \bar{\psi}_{np}(\theta) \\ \theta &= \frac{m_{np} R_y}{c} e^{c|y|}, \end{aligned} \quad (20)$$

leading to

$$\theta \frac{d\bar{\psi}_{np}}{d\theta} + \theta^2 \frac{d^2\bar{\psi}_{np}}{d\theta^2} - \left(4 + \frac{m_p^2 R_y^2}{c^2}\right) \bar{\psi}_{np} + \theta^2 \bar{\psi}_{np} = 0$$

This, again, leads to a solution in terms of Bessel functions of the first and second kinds, viz.

$$\psi_{np}(y) = \Xi_{np} e^{2c|y|} \left[ J_{\nu_p + \frac{1}{2}}(\theta) + \zeta_{np} Y_{\nu_p + \frac{1}{2}}(\theta) \right], \quad (21)$$

where  $\nu_p$  has been defined earlier. Once again, the constants  $\Xi_{np}$  and  $\zeta_{np}$  are to be determined by using the orthonormality conditions, viz.

$$\delta_{nn'} = \int_{-\pi}^{\pi} dy a^2(y) \psi_{np}(y) \psi_{n'p}(y) \quad (22)$$

The parallel with the 5D case is very apparent and, thus, all the analyses for the original RS case can be trivially transported to this sector. However, it should be appreciated that  $\psi_{np}$  are crucially dependent on the eigenspectrum of the  $z$ -equation operator. Indeed, the very order of the Bessel functions ( $\nu_p + 1/2$ ) is determined entirely by it. While this may, at first, seem to imply that the spectrum is determined by a single parameter  $\nu_p$ , note that it is not so, for the others enter through  $\theta$ . A further issue needs to be clarified here. It has been argued in the literature ([35] as well as in the context of a different system with close parallels to the current discussion) that, for  $p \neq 0$  modes such as  $\psi_{0p}$  would not exist. We shall explicitly show below that this is not the case.

### 3.3. Mass spectrum for the KK graviton

Our aim, now, is to compute the allowed values of  $m_{np}$  (i.e., the KK graviton masses). We first obtain these in terms of  $m_p$ , the eigenvalues of the  $z$ -direction differential operator, and, then, determine  $m_p$ . Either exercise is crucially dependent on the differentiability structure of the wavefunctions.

The self-adjoint nature of the  $y$ -direction operator implies that the derivatives  $\psi'_{np}(y)$  must be continuous at either boundary. Note that the presence of the brane tension has, in essence, been factored out by the inclusion of the warp factors in the definition of  $\tilde{\Delta}_{\mu\kappa}$  (see Eq. (12)). This, then, leads to

$$\begin{aligned} \zeta_{np} &= - \frac{x_{np} e^{c(|y|-\pi)} J_{\nu_p - \frac{1}{2}}(x_{np} e^{c(|y|-\pi)}) + (\frac{3}{2} - \nu_p) J_{\nu_p + \frac{1}{2}}(x_{np} e^{c(|y|-\pi)})}{x_{np} e^{c(|y|-\pi)} Y_{\nu_p - \frac{1}{2}}(x_{np} e^{c(|y|-\pi)}) + (\frac{3}{2} - \nu_p) Y_{\nu_p + \frac{1}{2}}(x_{np} e^{c(|y|-\pi)})} \Big|_{y=0, \pi} \end{aligned} \quad (23)$$

where

$$x_{np} \equiv m_{np} \frac{R_y}{c} e^{c\pi}, \quad (24)$$

and the two conditions summarised in Eq. (23) reflect the boundary conditions at  $y = 0, \pi$  respectively. Once  $\nu_p$  is known, these two, together, determine  $\psi_n(y)$  as well as serve to quantize  $x_{np}$  (and, hence,  $m_{np}$ ).

We now turn our attention to  $\chi_p(z)$ . As these have to be even functions of  $z$ , we have  $\chi'_p(z=0) = 0$ . This is identically satisfied by  $\chi_0(z)$  as  $\nu_p(m_p=0) = 3/2$  and the corresponding functions satisfy  $P_{3/2}^{5/2}(x) \propto (1-x^2)^{-5/4}$  and  $Q_{3/2}^{5/2}(x) = 0$ . For  $p \neq 0$ , we may use the identities

$$\begin{aligned} \left( \frac{dP_N^M(x)}{dx} \right)_{x=0} &= \frac{2^{M+1}}{\sqrt{\pi}} \sin\left(\frac{\pi(N+M)}{2}\right) \frac{\Gamma(1+(N+M)/2)}{\Gamma((N-M+1)/2)} \\ \left( \frac{dQ_N^M(x)}{dx} \right)_{x=0} &= 2^M \sqrt{\pi} \cos\left(\frac{\pi(N+M)}{2}\right) \frac{\Gamma(1+(N+M)/2)}{\Gamma((N-M+1)/2)} \end{aligned}$$

leading to

$$\cot\theta_p = \frac{-\pi}{2} \cot\frac{\pi(\nu_p + 5/2)}{2}. \quad (25)$$

To determine the mass spectra of the KK gravitons, we need to analyze the continuity condition at  $z = \pi$  which, for convenience, we separately consider in two distinct cases namely large and small  $k$ .

#### Large $k$ (small $c$ )

Denoting  $\tau = \tanh(kz)$ , we have

$$\chi_p(z) = \tilde{\Xi}_p (1 - \tau^2)^{5/4} \left[ \cot\theta_p P_{\nu_p}^{5/2}(\tau) + Q_{\nu_p}^{5/2}(\tau) \right].$$

As the orbifolding condition necessitates<sup>1</sup> that  $\chi'_p(z = \pi) = 0$ , we need to examine the derivative close to  $\tau = 1$ . For the zero mode ( $\nu_p = 3/2$  or  $m_p = 0$ ) this implies  $c_1^{(0)} = 0$  in Eq. (17), or in other words,  $\chi_0(z)$  is flat (as would be expected). For the others, we have

$$\begin{aligned} f(\tau) &\equiv \frac{d\chi_p}{d\tau} \\ &= \frac{\tilde{\Xi}_p}{2} (2\nu_p - 3) \sqrt[4]{1 - \tau^2} \left[ \cot\theta_p \tau P_{\nu_p}^{5/2}(\tau) - \cot\theta_p P_{\nu_p+1}^{5/2}(\tau) + \tau Q_{\nu_p}^{5/2}(\tau) - Q_{\nu_p+1}^{5/2}(\tau) \right]. \end{aligned}$$

In the infinitesimal neighbourhood of  $\tau = 1$ ,

$$\begin{aligned} f(\tau = 1 - \delta) &= \frac{\cot\theta_p (2\nu_p - 3)(2\nu_p + 5)}{2\sqrt{2\pi}} \\ &\times \left[ -1 + \delta \frac{(2\nu_p - 1)(2\nu_p + 3)}{4} \right] + \mathcal{O}(\delta^2). \end{aligned}$$

For the higher modes ( $\nu_p > 3/2$ ), the disappearance of  $\chi'_p(z = \pi)$ , thus, needs  $\cot\theta_p = 0$  or

<sup>1</sup> Since  $\chi_p(z)$  is even, its derivative  $f(\tau)$  is odd. On the other hand, the orbifolding and the continuity of the derivative imply  $\chi'_p(z = \pi_-) = \chi'_p(z = \pi_+) = \chi'_p(z = -\pi_-)$ .

$$\nu_p = 2n + \frac{1}{2} \quad n \in Z^+ \quad (26)$$

This result can be appreciated by noting that  $P_{\nu_p > 3/2}^{5/2}(\tau) \rightarrow \infty$  as  $\tau \rightarrow \pm 1$ . Since, for large  $k$ , the wavefunctions must extend close to  $\tau \approx \pm 1$ , normalizability of the same requires  $\cot \theta_p \rightarrow 0$ .

Using Eq. (26) in the second of Eqs. (18) would determine the allowed values of  $m_p$ . Substituting the latter in Eq. (23) would, then, yield the allowed values of  $m_{n,p}$ , or, in other words, the spectrum. However, since a large  $k$  implies a  $c$  that is almost infinitesimally small, there is virtually no warping in the  $y$ -direction and the latter is essentially flat. This would immediately imply that  $m_{np}^2 \approx m_p^2 + n^2 R_y^{-2}$ . With  $R_y$  being very small,  $h^{(n>0,p)}$  are too heavy to be of any relevance, and we effectively have but a single tower  $h^{(0,p)}$  with masses  $m_{0p} \approx m_p$ .

### Small $k$ (large $c$ )

The boundary is now at  $\tau = \tau_\pi = \tanh(k\pi)$ , and somewhat away from  $\tau = 1$ . Being away from the singular points of the associated Legendre functions means one can numerically calculate the functions, and the vanishing of  $f(\tau_\pi)$  dictates that

$$\begin{aligned} \cot \theta_p \tau_\pi P_{\nu_p}^{5/2}(\tau_\pi) - \cot \theta_p P_{\nu_p+1}^{5/2}(\tau_\pi) \\ + \tau_\pi Q_{\nu_p}^{5/2}(\tau_\pi) - Q_{\nu_p+1}^{5/2}(\tau_\pi) = 0. \end{aligned} \quad (27)$$

This equation has to be solved numerically to obtain the quantized values of  $\nu_p$ . To now obtain  $x_{np}$ , concentrate on Eq. (23). Since  $e^{c\pi} \gg 1$ , this relation is satisfied only if

$$2x_{np} J_{\nu_p - \frac{1}{2}}(x_{np}) + (3 - 2\nu_p) J_{\nu_p + \frac{1}{2}}(x_{np}) = 0. \quad (28)$$

Finally, for large  $c$ , the graviton spectrum will be given by the solutions of the above equation. It is worth remembering that, in this case, there is a non-negligible warping in the  $z$ -direction, and thus, the  $h^{(n,p>0)}$  are not necessarily superheavy. The two branches (large  $k$  and large  $c$ ) are, thus, not quite symmetrical.

### 3.4. Couplings with brane fields

The interaction term of a graviton with any brane field is given by

$$L_{\text{int}} = \frac{1}{M_6^2} T^{\mu\nu} h_{\mu\nu}(x_\mu, y = \pi, z = 0), \quad (29)$$

where  $T^{\mu\nu}$  is the energy-momentum tensor of the field. The coupling of brane-localized matter with the  $(n, p)$ th graviton mode is, thus, determined by the value of the latter's wavefunction on the brane location. In other words,

$$C_{np} = \frac{1}{M_6^2 \sqrt{R_y r_z}} \Psi_{np}(\pi) \chi_p(0). \quad (30)$$

Once again, we examine the two cases separately.

### Large $k$ (small $c$ )

In this case, as argued earlier, the lowest mass modes correspond to the  $\psi_{0p}$  states. From the solutions of  $\psi_{np}(y)$  and  $\chi_p(z)$ , we have

$$\begin{aligned} \psi_{0p}(\pi) = \Xi_{0p}, \quad \chi_{p=0}(0) = \tilde{\Xi}_0, \\ \chi_{p \neq 0}(0) = \tilde{\Xi}_p \left[ Q_{\nu_p}^{5/2}(0) \right], \end{aligned}$$

where  $\Xi_{0p}$  and  $\tilde{\Xi}_p$  are to be determined from the orthonormality conditions of the mode functions. From Eq. (30) we then have

$$\begin{aligned} C_{00} &= \frac{1}{M_6^2 \sqrt{2\pi R_y r_z}} B_0^{-1/2} \cosh^{3/2}(k\pi) \\ C_{0p} &= \frac{1}{M_6^2 \sqrt{2\pi R_y r_z}} B_p^{-1/2} \cosh^{3/2}(k\pi) \left[ Q_{\nu_p}^{5/2}(0) \right], \end{aligned} \quad (31)$$

where

$$\begin{aligned} B_{p=0} &\equiv \int_{-\pi}^{\pi} \cosh^3(kz) dz \\ B_{p \neq 0} &\equiv \int_{-\pi}^{\pi} \text{sech}^2(kz) \left[ Q_{\nu_p}^{5/2}(\tanh(kz)) \right]^2 dz. \end{aligned} \quad (32)$$

In the above, terms subleading in  $c$  have been dropped as  $c \ll 1$ .

### Small $k$ (large $c$ )

In this case, the wavefunctions on our brane are given by

$$\begin{aligned} \psi_{np}(\pi) = \Xi_{np} e^{2c\pi} J_{\nu_p + \frac{1}{2}}(\theta_\pi), \quad \chi_{p=0}(0) = \tilde{\Xi}_0, \\ \chi_{p>0}(0) = \tilde{\Xi}_p \left[ \cot \theta_p P_{\nu_p}^{5/2}(0) + Q_{\nu_p}^{5/2}(0) \right]. \end{aligned}$$

As before,  $\Xi_{np}$  and  $\tilde{\Xi}_p$  are to be determined from the normalizations. Once again, to determine the couplings we refer to Eq. (30) which yields

$$\begin{aligned} C_{n0} &= \frac{1}{M_6^2 r_z} \cosh(k\pi) e^{c\pi} \sqrt{\frac{k}{2 A_{n0} B_0}} \left[ J_2(\theta_\pi) \right] \\ C_{n,p \neq 0} &= \frac{1}{M_6^2 r_z} \cosh(k\pi) e^{c\pi} \sqrt{\frac{k}{2 A_{np} B_p}} \left[ J_{\nu_p + \frac{1}{2}}(\theta_\pi) \right] \\ &\quad \times \left[ \cot \theta_p P_{\nu_p}^{5/2}(0) + Q_{\nu_p}^{5/2}(0) \right], \end{aligned} \quad (33)$$

where,

$$\begin{aligned} A_{np} &= \int_0^1 r \left[ J_{\nu_p + \frac{1}{2}}(x_{np} r) \right]^2 dr \\ B_{p=0} &= \int_{-\pi}^{\pi} \cosh^3(kz) dz \\ B_{p \neq 0} &= \int_{-\pi}^{\pi} \text{sech}(kz)^2 \left[ \cot \theta_p P_{\nu_p}^{5/2}(\tanh(kz)) \right. \\ &\quad \left. + Q_{\nu_p}^{5/2}(\tanh(kz)) \right]^2 dz. \end{aligned} \quad (34)$$

Several points need to be noted at this point.

- Unlike in the previous case, the KK-modes in the  $y$ -direction are now relatively light and visible. This is but a consequence of the fact that the  $y$ -direction warping is dominant.
- Although the  $z$ -warping is subdominant, it is not entirely negligible. (This is quite contrary to the other case, where the  $y$ -warping was virtually nonexistent.) Thus, there is hope that some of the  $z$ -direction modes might be visible.
- In addition, the wave function in  $y$ -direction is dependent on  $p$  (the momentum in the  $z$ -direction).



- For  $p = 0$ , the levels  $h^{(n,0)}$  have almost the same coupling with the SM fields for  $n > 0$ . While this may seem counter-intuitive given that the normalizations  $A_{n0}$  depend on  $n$ , the same is essentially cancelled by the  $n$ -dependence in  $J_2(\theta_\pi)$ . Indeed, this result is exactly analogous to that for the (five-dimensional) RS model, and was to be expected given that the  $h^{(n,0)}$  wavefunctions are flat in the  $z$ -direction. On the other hand, for a given  $p > 0$ , increasing  $n$  results in the suppression of the corresponding couplings. Understandably, the extent of this suppression increases with  $k$  (which is a measure of the subdominant warping).
- For the very same reason, increasing  $p$ , while keeping  $n$  constant leads to an enhancement of the couplings.

#### 4. Numerical values for masses and couplings

In exploring the parameter space of the model, it is useful to consider two dimensionless quantities

$$\epsilon \equiv \frac{k}{r_z M_6}, \quad \alpha \equiv \frac{R_y}{r_z}. \quad (35)$$

Quite analogous to the 5D case, here too the applicability of the classical solutions can be related to the issue of the bulk curvature being small sufficiently small compared to  $M_6$ . To this end, we shall demand that  $\epsilon < 0.1$ . On the other hand, we would not like to introduce a new hierarchy (between moduli) in our efforts to ameliorate the SM hierarchy problem. Thus, the ratio  $\alpha$  should neither be too large nor too small.

We can, then, explore the parameter space of the theory in terms of  $\epsilon$ ,  $\alpha$  and any one other, say  $M_6$  (or, equivalently  $k$ ), relating all the rest through Eqs. (6) and (9). A very important distinction from the original RS scenario is that  $M_6$  need not be nearly the same as the four-dimensional Planck mass  $M_P$ . This freedom accrues from the larger parameter space of the model. In fact,  $M_6$  can be significantly smaller than  $M_P$  without any fine tuning. Indeed, the large  $c$  branch needs  $\alpha \gtrsim 50$  and with

$$M_P^2 \sim \frac{M_6^4 r_z R_y}{2ck} = \frac{M_6^4 r_z^2 \alpha}{2ck}$$

even the largest allowed value of  $k$  ( $\lesssim 0.3$ ) would lead to  $M_6 \lesssim M_P/2$ . Smaller (larger) values of  $k$  ( $\alpha$ ) would lead to even smaller  $M_6$ .

Furthermore, in this scenario, the cutoff for a four-dimensional quantum field theory is set not by  $M_6$ , but by  $\min(R_y^{-1}, r_z^{-1})$ . At such a scale, the higher-dimensional nature of the theory becomes apparent, and the four-dimensional effective theory (including the graviton modes) is no longer an apt language to describe physics.<sup>2</sup> Indeed, while the mechanism of compactification cannot be addressed in our theory (or within the RS mechanism), the physics responsible for it must be taken into account in any description that reaches beyond this scale. In other words, the quantity  $w^{-1}$  as defined in Eq. (10) refers to the ratio of the Higgs mass and this cutoff scale and is no longer constrained to be  $\gtrsim 10^{16}$ . Indeed, it can be significantly smaller. Once again, this freedom (absent in the 5D analogue) is but a consequence of the larger parameter space of the present theory.

At this stage, we wish to clarify an issue regarding effective theories that, often, leads to miscommunication. The cutoff scale of an effective theory is often described as the scale at which the loop contributions (often very large) are to be cutoff, for the new

**Table 1**

Four sample spectra for the small  $k$  case for a particular bulk curvature ( $\epsilon = 0.0775$ ).

$k = 0.05, \alpha = 211, w = 6.14 \times 10^{-15}$			$k = 0.1, \alpha = 108, w = 8.75 \times 10^{-15}$		
$(n, p)$	$m_{np}$ (TeV)	$C_{np} \times 10^3$ (TeV <sup>-1</sup> )	$(n, p)$	$m_{np}$ (TeV)	$C_{np} \times 10^3$ (TeV <sup>-1</sup> )
(1, 0)	5.07	8.04	(1, 0)	5.20	5.44
(2, 0)	9.29	8.04	(2, 0)	9.53	-5.44
(3, 0)	13.5	8.04	(3, 0)	13.8	5.44
(0, 1)	30.2	-24.1	(0, 1)	17.1	13.4
(1, 1)	37.1	16.4	(1, 1)	23.0	-9.99
(2, 1)	42.7	-14.7	(2, 1)	28.1	9.20

$k = 0.2, \alpha = 60.9, w = 1.31 \times 10^{-14}$			$k = 0.3, \alpha = 49.3, w = 1.81 \times 10^{-14}$		
$(n, p)$	$m_{np}$ (TeV)	$C_{np} \times 10^3$ (TeV <sup>-1</sup> )	$(n, p)$	$m_{np}$ (TeV)	$C_{np} \times 10^3$ (TeV <sup>-1</sup> )
(1, 0)	5.87	3.16	(1, 0)	7.07	1.87
(2, 0)	10.7	-3.16	(2, 0)	12.9	-1.87
(3, 0)	15.6	3.16	(3, 0)	18.8	1.87
(0, 1)	11.6	7.19	(0, 1)	11.3	-4.74
(1, 1)	17.4	-5.93	(1, 1)	17.8	4.13
(2, 1)	22.7	5.64	(2, 1)	24.0	-3.99

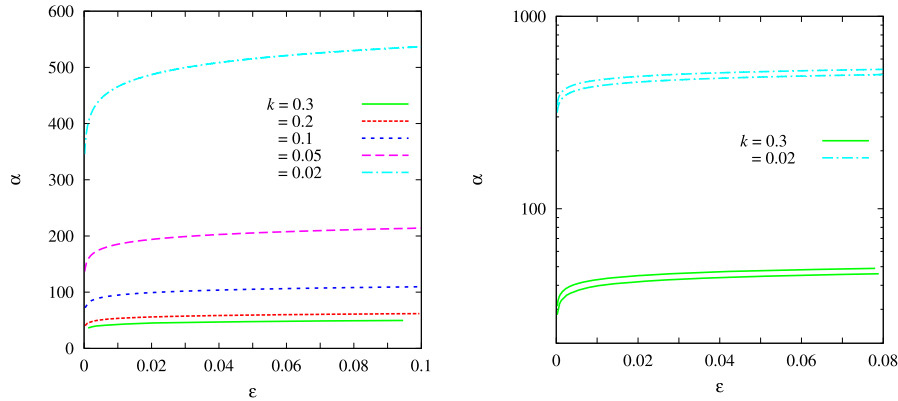
physics beyond this scale would naturally regulate them (i.e. cancel unwanted divergences). However, for this cancellation to be demonstrated, the said ultraviolet completion has to be known exactly. This is certainly not the case here (quite unlike, say the MSSM or gauge-Higgs unification scenarios, wherein the amelioration of the large corrections can be shown explicitly). On the contrary, *sans* a reliable theory of quantum gravity, no such calculation is possible. It has been argued that, within the five-dimensional context, the addition of the Planck-brane and/or the TeV-brane allows a holographic interpretation [37], with the former acting as a regulator leading to a UV cutoff ( $\lesssim r_c^{-1}$ ) on the corresponding CFT [38–40]. Similar analyses have also been made for theories with gauge fields extended in to the warped bulk [24,41,42]. Although no such duality has been constructed for the six-dimensional case, it is quite conceivable that one such would exist (for the large  $k$  case, the bulk is indeed AdS<sub>6</sub>-like). Consequently, even on this count, the branes are expected to provide a regulator with a cutoff  $\lesssim \min(R_y^{-1}, r_z^{-1})$ . In particular, let us concentrate on the situation  $R_y > r_z$ , which is mostly the case (with exceptions to this generically being bad phenomenologically). Remembering that the space is orbifolded on  $S^1/Z_2 \otimes S^1/Z_2$ , let us concentrate on the 4-brane at  $z = 0$  (with us being localized at the  $z = 0, y = \pi$  intersection). This 4-brane, thus, reflects an AdS<sub>5</sub> geometry in the bulk. Indeed, viewed in isolation, it is but a perturbation of the RS1 scenario with a corresponding CFT cutoff of  $R_y^{-1}$ . Thus, this part of the parameter space is manifestly consistent with our assertion about the cutoff.

We now examine the allowed parameter space in the light of the discussion above, considering, in turn, the large  $c$  and large  $k$  cases.

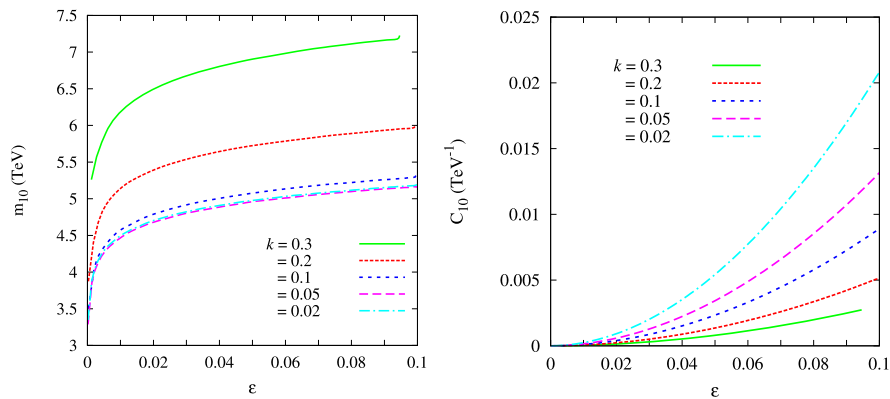
##### 4.1. Small $k$ (large $c$ )

In Table 1, we present part of the spectra for four representative points in the parameter space, each corresponding to a particular value of the ratio of the bulk curvature and the quantum gravity scale, namely  $\epsilon = 0.0775$ . Once  $\epsilon$  is fixed, for this branch of the solution,  $c$  has only a very subdominant dependence on  $k$  (see Eq. (10)). The relation  $c = \alpha k / \cosh(k\pi)$  would, then, imply that a larger  $k$  needs a smaller  $\alpha$ , as is demonstrated by Table 1. On the other hand, since the modes  $h^{(n,0)}$  are flat in the  $z$ -direction, the masses  $m_{n0}$  are essentially free of  $k$ , xs with the small difference

<sup>2</sup> A parallel is provided by an ADD-like [36] model with unequal radii of compactification. In fact, in the bulk, the large- $k$  branch is conformal to RS<sub>5</sub>  $\otimes$  ADD, with the correspondence broken only by the brane tensions.



**Fig. 1.** (Left panel.) Contour plots in the  $(\epsilon, \alpha)$  plane for fixed values of  $k$ . The curves are constrained to satisfy  $w R_y^{-1} = m_h$ . (Right panel.) The dependence of the contours on the value of the ratio  $w/R_y$ . In each case, the upper and lower curves correspond to  $m_h$  and 1 TeV respectively.



**Fig. 2.** The mass  $m_{10}$  (left panel) and matter coupling  $C_{10}$  (right panel) for the first graviton mode as a function of  $\epsilon$  for a fixed  $k$ . The parameter  $\alpha$  has been constrained to satisfy  $w R_y^{-1} = m_h$ .

in Table 1 accruing from the difference in the values of the other parameters.

The masses  $m_{n1}$ , on the other hand, do exhibit considerable dependence on  $k$ . Moreover, these modes are considerably heavier than several of the  $h^{(n,0)}$ . As can be expected, these masses grow very fast as  $k$  becomes smaller, a consequence of the decreasing severity of the  $z$ -warping.

What is of particular significance in each case is that the masses are much larger than what has been probed at the LHC. Indeed, masses such as these were practically out of reach of the runs at  $\sqrt{s} = 7, 8$  TeV, and would be accessible only in the next run. However, with the couplings to the SM fields being much smaller than those for the original RS gravitons, the production rates would continue to be highly suppressed even at the future runs at  $\sqrt{s} = 13, 14$  TeV. Indeed, as Table 1 suggests, for the large  $c$  branch of the solution, discovering even the first graviton mode at the LHC will remain a dream unless  $k$  is very small indeed, when the system becomes RS-like with the graviton couplings increasing appropriately. On the other hand, such values of  $k$  typically necessitate a somewhat large value of  $\alpha$ .

Such conclusions are brought into focus by Fig. 1 where we have depicted the relation between the parameters  $(\alpha, \epsilon)$  that, for a given choice of  $k$ , leads to the correct hierarchy (with the ultraviolet cutoff being given by  $R_y^{-1}$ ). The modulus ratio  $\alpha$  is a monotonically increasing (decreasing) function of  $\epsilon$  ( $k$ ), with the dependence on  $k$  being much more pronounced. In the left panel of the figure (as also in the subsequent numerical analysis), we hold  $m_h = w M_{\text{cutoff}}$  with the cutoff scale being defined by the larger of the two compactification radii. While this choice of the

hierarchy factor  $w$  is certainly as good as any other, the numerical results are not greatly sensitive to the exact value. This is borne out by the right panel of the same figure, which demonstrates (for the two extreme choices of  $k$  in the left panel) that the values remain qualitatively the same even if we change  $w$  by a factor of 8.

In Fig. 2, we depict the corresponding mass and SM-coupling strength of the lowest non-trivial graviton, viz.  $h^{(1,0)}$ . As has been argued above, decreasing  $k$  not only makes this graviton lighter, but also strengthens its couplings, thereby making it more amenable to discovery at the LHC. This trend holds for the other modes too. The existence of the double tower is another interesting point to note, especially for not too small values of  $k$ . As Table 1 shows, one can have a clustering of the KK modes, each of which has an enhanced coupling to the SM fields, and are likely to be seen in future experiments as a series of relatively closely lying resonances, with almost identical decay patterns. This proliferation of KK modes will be further enhanced if the number of extra dimensions increases.

#### 4.2. Large $k$ (small $c$ )

The situation changes considerably now when compared to the preceding case. With  $c$  being very small, the low-lying spectrum is essentially independent of it. And, as already stated, with the  $y$ -direction suffering virtually no warping, all  $h^{(n,p)}$  are superheavy ( $m_{np} > R_y^{-1}$ ) for  $n > 0$ , and, henceforth, we shall concentrate only on  $h^{(0,p)}$ .

As Table 2 shows,  $\alpha$  can be much smaller now (even smaller than one), and a large hierarchy between the moduli is no longer

**Table 2**Four sample spectra for the large  $k$  case.

$k = 8.2, \alpha = 9.87, \epsilon = 0.027$			$k = 8.5, \alpha = 9.87, \epsilon = 0.044$		
$w = 1.3 \times 10^{-11}$			$w = 5.06 \times 10^{-12}$		
$(n, p)$	$m_{np}$ (TeV)	$C_{np}$ (TeV $^{-1}$ )	$(n, p)$	$m_{np}$ (TeV)	$C_{np}$ (TeV $^{-1}$ )
(0, 1)	22.98	-0.881	(0, 1)	23.35	-3.62
(0, 2)	47.09	0.745	(0, 2)	47.86	3.06
(0, 3)	68.94	-0.720	(0, 3)	70.07	-2.96
(0, 4)	90.17	0.710	(0, 4)	91.65	2.92

$k = 8.2, \alpha = 1.56, \epsilon = 0.00675$			$k = 8.5, \alpha = 1.56, \epsilon = 0.0111$		
$w = 1.3 \times 10^{-11}$			$w = 5.06 \times 10^{-12}$		
$(n, p)$	$m_{np}$ (TeV)	$C_{np}$ (TeV $^{-1}$ )	$(n, p)$	$m_{np}$ (TeV)	$C_{np}$ (TeV $^{-1}$ )
(0, 1)	3.61	-0.881	(0, 1)	3.74	-3.62
(0, 2)	7.40	0.745	(0, 2)	7.66	3.06
(0, 3)	10.8	-0.720	(0, 3)	11.2	-2.96
(0, 4)	14.2	0.710	(0, 4)	14.7	2.92

necessary. Indeed, the smaller  $\alpha$  is, the lighter the graviton excitations are. The dependence of the masses on  $k$  is subdominant, though. These two features can be understood by recalling that the masses, in this case, are essentially given by  $m_p$ , the eigenvalues of the  $z$ -equation of motion. If we had a flat  $z$ -direction, the eigenvalues would have been evenly separated, namely  $m_p = p/r_z$ . In the current scenario, this is tempered by the warping. Since, for large  $k$ , the hierarchy is almost uniquely determined ( $w \approx \text{sech}(k\pi)$ ), so is the cutoff scale  $R_y^{-1}$ . Consequently, a smaller  $\alpha$  implies a smaller  $r_z^{-1}$  and, hence, a lighter spectrum. If  $M_6$  were to be held constant, this would also translate to a smaller  $\epsilon$ , as hinted at by Table 2. The dependence of the masses on  $k$ , thus, accrues, only through the warping and unless the latter changes by a great degree, the former remain relatively stable.

The arguments above also tell us why the couplings  $C_{0p}$  are insensitive to  $\alpha$ . With the  $h^{(0,p)}$  wavefunctions being independent of  $y$ , any dependence of the couplings on the parameters of the  $y$ -equation must disappear. Note, though, that the couplings of the gravitons to the SM fields are much larger now than was the case for the small  $k$  branch of the theory. In fact,  $C_{0p}$  for the two  $k = 8.2$  points listed in Table 2 are of the same order of magnitude as those for the RS model as investigated by the ATLAS Collaboration [4]. Consequently, the gravitons for  $k = 8.2, \alpha = 1.56$  should definitely be visible as resonances in the next run of the LHC, while those corresponding to  $k = 8.2, \alpha = 9.87$  may leave behind some indications through virtual diagrams (at least in the high luminosity run).

Things take a more interesting turn for larger  $k$  values, as the couplings increase substantially (the entries on the right column of Table 2). While the  $k = 8.5, \alpha = 1.56$  gravitons would be seen as very prominent resonances, even the large contact interactions generated by the  $k = 8.5, \alpha = 9.87$  would alter the continuum spectrum for the associated processes to a significant degree. If we increase  $k$  even further (see Table 3), the couplings rise very fast and quickly cross over to the nonperturbative regime. This is but a consequence of the fact that the wavefunctions  $\chi_p(z)$  are highly concentrated near  $z = 0$  with the extent of peaking increasing with  $k$ . This hitherto undiscovered strongly-coupled sector of the theory is potentially of great theoretical interest. The strong coupling, though, does not manifest itself for  $k \lesssim 9.0$  and a perturbative treatment does make sense. In summary, the parameter region corresponding to  $k \lesssim 8.5$  is still far from being ruled out and admits very interesting phenomenology.

In Fig. 3, we present the interrelationship between the couplings for various choices of  $k$ . As in small  $k$  sector, here too

**Table 3**Two additional sample spectra for the large  $k$  case.

$k = 8.9, \alpha = 1.56, \epsilon = 0.021$			$k = 11, \alpha = 0.002, \epsilon = 0.1$		
$w = 1.44 \times 10^{-12}$			$w = 1.96 \times 10^{-15}$		
$(n, p)$	$m_{np}$ (TeV)	$C_{np}$ (TeV $^{-1}$ )	$(n, p)$	$m_{np}$ (TeV)	$C_{np}$ (TeV $^{-1}$ )
(0, 1)	3.87	-23.9	(0, 1)	3.20	$-4.29 \times 10^5$
(0, 2)	7.92	20.2	(0, 2)	6.56	$3.62 \times 10^5$
(0, 3)	11.59	-19.5	(0, 3)	9.59	$-3.50 \times 10^5$
(0, 4)	15.16	19.2	(0, 4)	12.54	$3.45 \times 10^5$

$\alpha$  increases (decreases) monotonically with  $\epsilon$  ( $k$ ) with the  $k$ -dependence being much stronger. As was expected from the tables, the typical values of the modulus ratio  $\alpha$  tends to be smaller for this sector. The bend in the curves (see the left panel) at  $\alpha = 1$  are a consequence of our assertion that the cutoff of the four-dimensional theory is given by  $\min(R_y^{-1}, r_z^{-1})$ , thereby changing the parametric dependence of the hierarchy at this point.<sup>3</sup> Naturally, this change is also manifested in the relation between  $\alpha$  and  $\epsilon$  in the shape of very sharp bends (with the position of the bend being given by  $\alpha(\epsilon, k) = 1$ ). Below this point, the mass of the first KK mode,  $h^{(0,1)}$  in the case, is almost independent of  $\epsilon$ , and is given essentially in terms of  $R_y^{-1}$ , which, of course, is determined once the Higgs mass and the hierarchy determinant  $k$  are fixed. A further feature of this sector is that the coupling  $C_{01}$  is essentially fixed by  $k$  alone with only a very subdominant dependence on  $\epsilon$ .

## 5. Discussion and summary

Within the original (five-dimensional) RS scenario, the masses and couplings of the graviton KK modes are determined in terms of very few tunable parameters. Exploiting this, the ATLAS group searched for the existence of a graviton resonance in the dilepton mode, and has ruled out the existence of any such mode below  $\sim 2.2$  TeV as long as it couples to the SM fields with a strength of the order of an inverse TeV. This negative result is in direct conflict with the RS mechanism's resolution of the mass hierarchy problem. Thus, one is forced to accept at least a partial hierarchy, whether it be applicable to the low energy theory or whether it appears in the guise of an ad hoc introduction of a scale (for the four-dimensional theory) at least two orders lower than the natural scale of the problem.

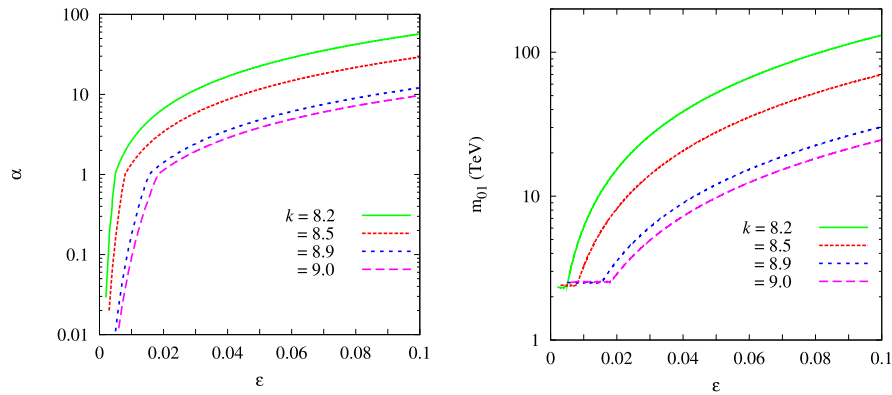
Since neither of these solutions are particularly attractive given the great promise of the RS paradigm, we have striven here to offer an alternative and natural solution. The key is the generalization to dimensions larger than five and admit multiple warping. Such a situation, of course, is not unexpected within, say, a string theoretic framework.

While the number of extra dimensions (and independent warpings) can be arbitrary [28], we have restricted ourselves, for reasons of simplicity, to the six-dimensional theory with two subsequent warpings and orbifoldings. This immediately introduces some extra tunable parameters in the shape of moduli and/or extent of warping. Further generalization is straightforward and only serves to increase the parameters. It should be noted at this stage that the reconciliation of the ATLAS bounds with the resolution of the hierarchy problem does not need any extreme tuning of these parameters. Rather, the natural values of the parameters serve to resolve the conflict.

In a multiple moduli warped model, such as the one under discussion, it would be advisable to restrict the hierarchy between

<sup>3</sup> Note that  $\alpha < 1$  was impossible to obtain in the small  $k$  sector.





**Fig. 3.** Left panel: Contour plots in the  $(\epsilon, \alpha)$  plane for fixed values of  $k$ . Right panel: The mass  $m_{01}$  for the first graviton mode as a function of  $\epsilon$  for a fixed  $k$ . The curves are constrained to satisfy  $\min(R_Y^{-1}, r_Z^{-1}) = m_h/w$ .

them to as small a value as possible. This is over and above maintaining the smallest of them to be close to the fundamental length scale of the problem. This serves to maximize the stability of the ratios against radiative corrections, or, in other words, prevents the reappearance of the hierarchy problem in a different guise. Such a requirement forces us to have large warping in only one direction. In other words, we can have either a large  $c$  ( $\sim 10$ ) and a small  $k$  ( $\lesssim 0.3$ ) or large  $k$  ( $\gtrsim 8$ ) and an almost infinitesimally small  $c$ .

The first scenario (large  $c$ ) requires a moderately large ( $\gtrsim 40$ ) hierarchy between the moduli. This small hierarchy is minimized by assuming the largest possible ratio between the bulk curvature and the fundamental mass scale (i.e., the largest  $k$ ). While, at first sight, this scenario might seem to be a small perturbation of the 5-dimensional RS model, it is not really so. For one, the graviton masses are typically larger than those in the RS model, and, simultaneously have much smaller couplings. Thus, it is almost straightforward to evade the ATLAS bounds. However, the next run of the LHC should be able to find them. Even more interestingly, we now have a double tower of gravitons. In other words, there is a cluster of relatively closely placed resonances, each with enhanced (to at least the same level as the first KK mode) couplings waiting to be discovered at the forthcoming runs of the LHC. And, increasing the number of warped directions would only serve to increase the density of these excitations, thereby making the situation quite lively. Indeed, if we admit as many as 6 extra dimensions, it is conceivable that these modes can, in the collider environment, start to mimic a pseudo-continuum of resonances.

The second branch (large  $k$ ) is potentially even more interesting. For one, it can admit essentially no hierarchy between the moduli. Essentially only one tower is germane to low energy physics, and the spacing between the levels is minimized by minimizing the moduli hierarchy. Even though the modes tend to be somewhat heavier than those in the RS (thereby largely escaping the ATLAS bounds), the couplings are no longer suppressed. Thus, a reanalysis of even the present data can serve to rule out part of the parameter space.

This branch, thus, seems to be an even smaller perturbation of the RS, or more correctly, a marriage of the RS with a very small ADD-like direction. However, the extremely tiny warping has a profound role to play. For one, it is this that allows the 4-brane at  $z=0$  (on which our 3-brane is located) to be tensionless. (Compare this to the negative tension that the visible brane must have in the RS model.) At a phenomenological level, this also serves to bring down the fundamental (six-dimensional) mass scale to the GUT scale or even below. This is likely to have profound implications for model building. Indeed, if we aim to push the fundamental scale close to the Planck scale, we enter a strongly coupled

phase of the theory! This feature is a stark departure from the usual RS scenario.

In summary, we have shown that augmenting the RS scenario by incorporating even a single slightly warped extra dimension can lead to profound implications. Not only are the current collider bounds avoided (though, with the promise of very interesting physics in the next run of the LHC), but a host of new and exciting features emerge.

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