



On the thermo-mechanical properties of unsaturated soils

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Abstract: The establishment of energy balance equation is necessary to study the thermo-mechanical properties of unsaturated soils. To solve this equation, the determination of two fundamental parameters as volumetric specific parameter and thermal conductivity coefficient is essential. In this paper, the effective thermal conductivity coefficient of dry soil grain is analyzed for soils with different compositions, and the thermo-mechanical properties of porous media with water and gas are studied by considering the soil water retention curve (SWRC). Different methods, i.e. volumetric average method, self-consistent method, Hashin-Strikman method, are employed to calculate thermal conductivity coefficients, and a new method is proposed to determine the thermo-mechanical parameters. Comparison of the results obtained by different methods shows that the proposed method is in a good agreement with the experimental results and is suitable for describing the main properties of the thermo-mechanical behaviors of soils. The relationship between the SWRC and the seepage curve is further studied by the natural proportional rule. The characteristics of the SWRC, its differential coefficient and the seepage curve, are investigated by considering the physico-mechanical mechanism; the limit scopes of the indices of the SWRC and the seepage curve are also given.

Key words: unsaturated soil; soil water retention curve (SWRC); seepage curve; thermal conductivity coefficient; comparative analysis

1 Introduction

The strength of unsaturated soils changes with water evaporation and infiltration: the strength increases as the soils are dried and it decreases when the soils are wetted. Under the effect of decreasing evaporation groundwater level, the soil above the water table becomes in general unsaturated and shrinkage occurs. As a result, cracking on the soil surface can take place, enhancing further evaporation. When there are cracks on the ground surface, organic matters may infiltrate in the soil along the cracks, leading to changes in soils composition as well as their hydro-mechanical properties. In general, the porosity of soils becomes larger and their strength becomes lower [1–5].

The mechanism of the migrations of moisture and gas in soils is complex [1–12]. It is influenced by

many factors, for instance, gas pressure, stress history, ion concentration and temperature gradient. Therefore, it is quite difficult to describe the fundamental rule by mathematical formulae.

It is generally admitted that the migrations of moisture and gas in soils are governed by four potentials: hydraulic gradient, matrix gradient, osmotic gradient and gas pressure gradient. The osmotic gradient is neglected in most cases, only hydraulic gradient and matrix gradient are considered in the unsaturated seepage theory. In this paper, the fundamental seepage theory is developed by considering the above influential gradients, and different models are evaluated.

SWRC represents the relationship between suction and water content, and it is related to the unsaturated seepage properties of soils [1–11]. For the determination of this curve, we can control the suction and measure the corresponding water content, and also change the water content and measure the corresponding suction using various methods.

Many researches show that after a long-term drought, a heavy rainfall can cause serious water seepage in the soil because of the surface dried-up

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plants, surface cracks and human activities [12]. This will increase the degree of saturation drastically and result in reduction of the soil strength and stiffness. Obviously, the reduction of soil mechanical strength could lead to serious damage of structures and/or buildings. Such kind of damages can be the subgrade and pavement failures, buildings and concrete cracking, bridge destruction, etc.

To deal with this kind of problems, it is necessary to study the heat transfer in the soil and the changes in water content. One of the important factors that affect the heat transfer and the changes in water content in unsaturated soil is the thermal conductivity coefficient because the temperature gradient in soil affects both the humidity and the temperature distributions. In this paper, the determination of thermodynamic conductivity coefficient of unsaturated soil is studied, the SWRC is investigated, and the comparison between theoretical and experimental results is conducted.

2 Thermal conductivity coefficients of dry soils

There are some conventional methods often used to calculate the thermal parameters of dry soils such as the volumetric average method, the self-consistent method and the Hashin-Strikman method. For dry soils, the basic formulae for calculating specific heat coefficient (C) and/or thermal conductivity coefficient (λ) according to the volumetric average method, the self-consistent method and the Hashin-Strikman method are listed as follows.

(1) Volumetric average method for C and λ :

$$C = \sum_{i=1}^n n_i C_i \quad (1)$$

$$\lambda = \sum_{i=1}^n n_i \lambda_i \quad (2)$$

(2) Self-consistent method for λ :

$$\frac{1}{\lambda} = \sum_{i=1}^n \frac{n_i}{\lambda_i} \quad (3)$$

(3) Hashin-Strikman method for λ :

$$\lambda_{\text{upp}} = \lambda_1 - (1 - n_1) / [n_1 / (3\lambda_1) + 1 / (\lambda_1 - \lambda_2)] \quad (4a)$$

$$\lambda_{\text{low}} = \lambda_2 + n_1 / [1 / (\lambda_1 - \lambda_2) + (1 - n_1) / (3\lambda_2)] \quad (4b)$$

where n_i is the volume ratios of different components; C_i and λ_i are the specific heat and the thermal

conductivity coefficients of components, respectively; and i is the number of soil components. For the capacity of heat storage, the volumetric average method can meet the requirement of calculation. But the thermal conductivity coefficient is used to describe the rule of heat transfer, and usually the volumetric average method is only suitable for parallel and uniform heat transfer in different soil layers.

(4) Method of depolarization factor

The method of depolarization factor is often used for normal heat transfer in layered soil. As a result, the upper and lower limit theory is put forward by Hashin-Strikman (Eqs.4(a) and 4(b)). For this kind of soils, the method involving a depolarization factor can be used. Based on this method, the macroscopic thermal conductivity coefficient can be expressed as follows:

$$\lambda = \frac{n_1 \lambda_1 + \sum_{i=2}^n k_i n_i \lambda_i}{n_1 + \sum_{i=2}^n k_i n_i} \quad (5)$$

$$k_i = \frac{1}{3} \sum_j \left[1 + \left(\frac{\lambda_i}{\lambda_1} - 1 \right) g_j \right]^{-1} \quad (j = a, b, c)$$

$$g_a = \frac{1}{2} abc \int_0^\infty \frac{du}{(a^2 + u)^{3/2} (b^2 + u)^{1/2} (c^2 + u)^{1/2}} \quad (\text{in axial } a\text{-direction}) \quad (6a)$$

$$g_b = \frac{1}{2} abc \int_0^\infty \frac{du}{(b^2 + u)^{3/2} (a^2 + u)^{1/2} (c^2 + u)^{1/2}} \quad (\text{in axial } b\text{-direction}) \quad (6b)$$

$$g_c = \frac{1}{2} abc \int_0^\infty \frac{du}{(c^2 + u)^{3/2} (a^2 + u)^{1/2} (b^2 + u)^{1/2}} \quad (\text{in axial } c\text{-direction}) \quad (6c)$$

In order to determine k_i , it is assumed that: (1) the particle shapes of dry soils are elliptical; and (2) the particles are separated and uncorrelated with each other. Under this condition, the value of k_i is just defined by the particles shape, the axial direction and the ratio of λ_i to λ_1 .

Parameters of g_a , g_b and g_c have the following relation:

$$g_a + g_b + g_c = 1 \quad (7)$$

When $g_a = g_b = g_c = 1/3$, the shape of particles is spherical; if $a = b = mc$ (here m is a constant), the shape of particles is elliptical; and if $g_a = g_b$, Eq.6(c) can be satisfied. If the cross-section of the cylindrical particle is elliptical, $a = mb$, $c \rightarrow \infty$, then $g_a = 1/(m+1)$, $g_b = m/(m+1)$, and $g_c = 0$. For some very thin tabular particles, if $b = c \rightarrow \infty$, then $g_a = 1$, $g_b = g_c = 0$.

3 Thermal conductivity coefficients of unsaturated soil

Heat flow through conductivity, convection, dissolution and diffusion in unsaturated soil is a complicated process. In unsaturated state, latent heat flow can take place through the voids full of dry air and water vapor, thus the thermal conductivity coefficient for these voids, λ_a , consists of two parts: thermal conductivity of dry air, λ_g , and thermal conductivity of vapor, λ_v :

$$\lambda_a = \lambda_g + \lambda_v \quad (8)$$

The thermal conductivity of vapor, λ_v , can be expressed as follows:

$$\lambda_v = \lambda_v^s h_r \quad (9a)$$

where h_r is the relative humidity, and λ_v^s is the thermal conductivity of saturated vapor under the saturated vapor pressure:

$$\lambda_v^s = \frac{LDP}{RT(P - P_w^s)} \frac{dP_w^s}{dT} \quad (9b)$$

where L is the latent thermal conductivity for water evaporation, $L = 4.156 \times 10^3 \times (607 - 0.68T)$; R is the gas constant, $R = 8.31 \text{ J}/(\text{mol} \cdot \text{K})$; D is the vapor diffusion coefficient; P is the total pressure, $P = P_a + P_v$, where P_a is the atmospheric pressure and P_v is the vapor pressure; P_w^s is the saturated vapor pressure; and T is the temperature.

The vapor diffusion coefficient can be expressed as a function of temperature as follows:

$$D = \frac{17.6}{P} \left(\frac{T}{273} \right)^{2.3} \quad (10)$$

The relative humidity, h_r , is related to the total suction ψ :

$$\psi = - \frac{R(T + 273.16)}{M_v g} \ln h_r \quad (11)$$

where M_v is the mole mass of water vapor (0.018 kg/mol), and g is the gravitational acceleration (m/s^2).

The total suction potential includes gravitational potential, matric potential and osmotic potential. The temperature gradient is mainly related to the matric suction gradient.

4 SWRC and seepage

4.1 Natural proportional rule

The soil permeability is strongly related to the ability of water retention of soil. The water molecule movement in unsaturated soil is governed essentially by two processes: convection and diffusion. Basically, the permeability function can be simply related to the suction state. When the suction increases, the permeability coefficient decreases and the quantity of moving water is reduced. At the same time, the diffusion in unsaturated soil results in the decrease of soil moisture. To establish the relationship between permeability and suction, it is possible to use the natural proportional rule based on the following fundamental assumption: the permeability coefficient of unsaturated soil is only dependent on the suction; the range of suction is from 0 to ∞ and the corresponding permeability coefficient changes from k_s (saturated permeability coefficient) to 0. Assuming that a representative function f ranges from 0 to ∞ and it depends solely on the suction, the following expression can be proposed according to the natural proportional rule:

$$f = 1/k_w - 1/k_s \quad (12)$$

$$df/f = \xi d\psi/\psi \quad (13)$$

where k_w is the unsaturated permeability coefficient, and ξ is the natural proportional coefficient.

4.2 Permeability function

Integration of Eqs.(12) and (13) by neglecting the residual permeability coefficient gives

$$k_w = \frac{k_s}{1 + (k_s/k_{w1} - 1)(\psi/\psi_1^s)^\rho} \quad (14)$$

where k_{w1} and ψ_1^s are the unsaturated permeability coefficient and suction in a reference unsaturated state, respectively; and ρ is the natural proportional coefficient. Often, the following simplified formula is used:

$$k_w = \frac{k_s}{1 + (\psi/\psi^*)^\rho} \quad (15)$$

where ψ^* is the value of suction when the unsaturated permeability coefficient equals the half of the saturated permeability coefficient.

For further analysis, the second order derivative of Eq.(14) can be conducted and the following equation is obtained:

$$\frac{\rho - 1}{\psi_1^s} \left(\frac{\psi}{\psi_1^s} \right)^{\rho - 2} \left[1 + \left(\frac{k_s}{k_{w1}} - 1 \right) \left(\frac{\psi}{\psi_1^s} \right)^\rho \right]^2 - 2 \left(\frac{\psi}{\psi_1^s} \right)^{\rho - 1} \cdot \left[1 + \left(\frac{k_s}{k_{w1}} - 1 \right) \left(\frac{\psi}{\psi_1^s} \right)^\rho \right] \frac{\rho}{S_1} \left(\frac{k_s}{k_{w1}} - 1 \right) \left(\frac{\psi}{\psi_1^s} \right)^{\rho - 1} \right] = 0 \quad (16)$$

The simplified form is as follows:

$$\left(\frac{\psi}{\psi_1^s}\right)^\rho = \frac{\rho - 1}{1 + \rho} \frac{k_{wl}}{k_s - k_{wl}} \quad (17a)$$

or

$$\psi = \psi_1^s \left(\frac{\rho - 1}{1 + \rho} \frac{k_{wl}}{k_s - k_{wl}}\right)^{1/\rho} \quad (17b)$$

According to Eqs.(17a) and (17b), the natural proportional coefficient ρ is larger than 1.

4.3 SWRC

When the residual volumetric water content is considered, the SWRC can be also obtained using the natural proportional rule:

$$\theta_w = \frac{\theta_{ws} - \theta_r}{1 + \left(\frac{\theta_{ws} - \theta_r}{\theta_{wl} - \theta_r} - 1\right) \left(\frac{\psi}{\psi_1^t}\right)^\zeta} + \theta_r \quad (18)$$

where θ_r is the residual volumetric water content; θ_w , θ_{ws} , θ_{wl} , ψ_1^t are the volumetric water content, saturated volumetric water content, unsaturated volumetric water content and suction in a reference unsaturated state, respectively; ζ is the natural proportional coefficient of SWRC. Similar to the permeability function, Eq.(18) is differentiated with respect to suction:

$$\left(\frac{\theta_{ws} - \theta_{wl}}{\theta_{wl} - \theta_r}\right) \left(\frac{\psi}{\psi_1^t}\right)^\zeta = \frac{\zeta - 1}{1 + \zeta} \quad (19a)$$

or

$$\psi = \psi_1^t \left(\frac{\zeta - 1}{\zeta + 1} \frac{\theta_{wl} - \theta_r}{\theta_{ws} - \theta_{wl}}\right)^{1/\zeta} \quad (19b)$$

As for Eqs.(17a) and (17b), the natural proportional coefficient in Eqs.(19a) and (19b), ζ , is larger than 1.

4.4 Relationship between permeability function and SWRC

The permeability function and SWRC have been obtained by using the natural proportional rule. If the residual unsaturated permeability coefficient is considered, the forms of permeability function and SWRC are the same. As the relationship between them can be used to describe the hydraulic behavior of unsaturated soil, they may be correlated to each other. Basically, the permeability function and SWRC should be related to the same suction state. When the second order derivative of permeability function and SWRC with respect to suction are equal to zero in the same suction state, the natural proportional coefficients of permeability function and SWRC, ρ and ζ , respectively, are related through the following expression:

$$\psi = \psi_1^s \left(\frac{\rho - 1}{\rho + 1} \frac{k_{wl}}{k_s - k_{wl}}\right)^{\frac{1}{\rho}} = \psi_1^t \left(\frac{\zeta - 1}{\zeta + 1} \frac{\theta_{wl} - \theta_r}{\theta_{ws} - \theta_{wl}}\right)^{\frac{1}{\zeta}} \quad (20)$$

If one natural proportional coefficient is given, the other can be obtained using Eq.(20). Several special cases are presented as follows:

(1) Case 1

If $\psi_1^s = \psi_1^t$, Eq.(20) becomes

$$\left(\frac{\zeta - 1}{\zeta + 1} \frac{\theta_{wl} - \theta_r}{\theta_{ws} - \theta_{wl}}\right)^{\frac{1}{\zeta}} = \left(\frac{\rho - 1}{\rho + 1} \frac{k_{wl}}{k_s - k_{wl}}\right)^{\frac{1}{\rho}} \quad (21)$$

(2) Case 2

If $\psi_1^s = \psi_1^t$ and $\zeta = \rho$, k_{wl} and θ_{wl} are related by

$$k_{wl} = \frac{\theta_{wl} - \theta_r}{\theta_{ws} - \theta_r} k_s \quad (22)$$

(3) Case 3

If $\zeta = \rho$, the natural proportional coefficient ρ or ζ can be expressed as follows:

$$\rho = \frac{\lg\left(\frac{\theta_{wl} - \theta_r}{\theta_{ws} - \theta_{wl}}\right) + \lg\left(\frac{k_s - k_{wl}}{k_{wl}}\right)}{\lg\psi_1^s - \lg\psi_1^t} \quad (23)$$

5 Numerical simulation of soil heat conductivity

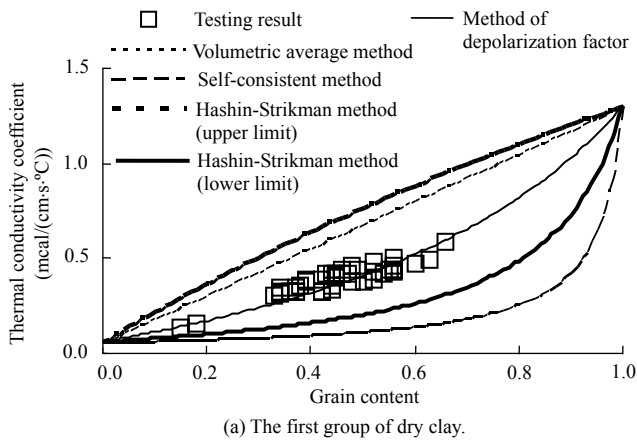
Generally, the heat conductivity of soil particles does not change significantly with temperature. Thermal conductivity coefficients of some typical matters are listed in Table 1 [1, 3, 9, 10, 13–17].

Table 1 Thermal conductivity coefficients of different materials [1, 3, 9, 10, 13–17].

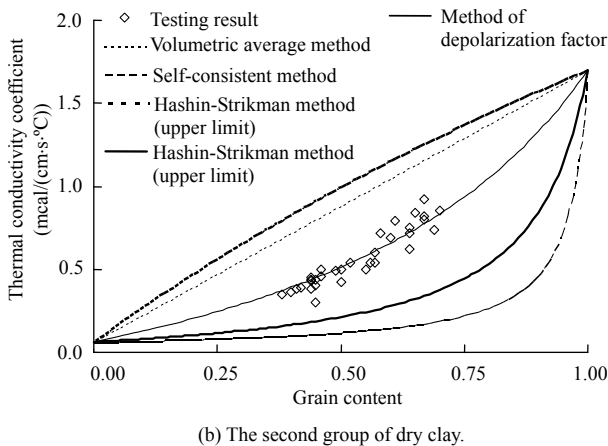
Matter	λ (mcal/(cm · s · °C))	C (cal/(cm ³ · °C))	ρ_0 (g/cm ³)
Quartz	21	0.48	2.66
Clay mineral	7	0.48	2.65
Organic composites	0.6	0.6	1.3
Water	1.37	1.00	1.00
Ice	5.2	0.45	0.92
Atmosphere	0.06	0.000 3	0.001 25

The thermal conductivity coefficient, λ , storage coefficient, C , and density, ρ_0 , of different matters in Table 1 are measured at a temperature of 10 °C, except ice whose values are measured at 0 °C. By using the four calculation methods presented before (volumetric average method, self-consistent method, Hashin-Strikman method, method of depolarization factor), the thermal conductivity coefficient is calculated at different volumetric grain contents for

two dry soils. The calculation is then compared with the measurement in Fig.1. The porosity of both samples is comprised between 32% and 81%. The relative humidity is controlled at 10%. Note that in the calculation using the method of depolarization factor, the following condition is considered: $g_a = g_b = 0.5$.



(a) The first group of dry clay.



(b) The second group of dry clay.

Fig.1 Curves of thermal conductivity coefficients vs. grain contents of two kinds of dry clays.

From Fig.1 it can be seen that for both dry soils, the upper limit of the Hashin-Strikman method and the numerical results from the self-consistent method are the upper and lower limits of thermal conductivity coefficient, respectively. The lower limit of the Hashin-Strikman method locates between the upper and lower limits. The method of depolarization factor seems to be in good agreement with the experimental results. According to Ref.[3], the numerical results are consistent with the experimental ones only if the volumetric average method and the self-consistent method are used to simulate parallel heat flow and heat flows in series.

For further analysis, the Hashin-Strikman method in Ref.[11] is used to simulate Wilson’s experimental results obtained from tests on a sand at various temperatures: 15 °C, 30 °C, 45 °C, 60 °C and

75 °C. The thermal conductivity coefficient and the porosity of the dry sand are 1.82 mcal/(cm·s·°C) and 45%, respectively. Figure 2 presents the comparison between experiment and calculation. Good agreement is obtained, showing the relevance of the method used.

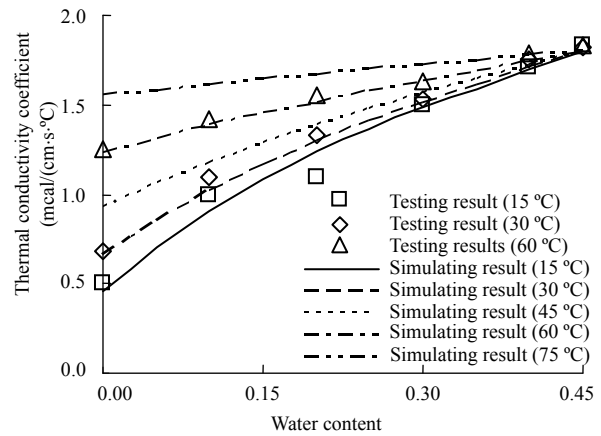


Fig.2 Relationships between thermal conductivity coefficient and water content of sand.

6 Conclusions

The existing methods for thermal conductivity coefficient determination are evaluated. Four methods are considered for this purpose: volumetric average method, self-consistent method, Hashin-Strikman method, and method of depolarization factor. The following conclusions can be drawn from this study.

(1) The natural proportional rule can be used to describe both the permeability function and the SWRC.

(2) The self-consistent method and the Hashin-Strikman method allow the determination of the lower and upper limits of thermal conductivity coefficients, respectively. However, the method of depolarization factor seems to be the best one since it is in good agreement with the experimental results.

(3) Soil thermal conductivity coefficient becomes higher when the water content increases. Furthermore, with increasing temperature, the thermal conductivity coefficient also increases. However, the thermal conductivity coefficient of dry soil changes little with temperature.

(4) The diffusion also increases with the temperature rise. With this diffusion increase, the humidity distribution in soil is changed, which tends to change the temperature gradient in soil and vice versa. Therefore, the balance state of unsaturated soil is transient and it changes with time.

(5) In the analysis of thermal conductivity, it is necessary to consider dry air and water vapor separately. When taking water and vapor as two distinct phases, it is helpful to use the method of depolarization factor.

References

- [1] Blight G E. Interaction between the atmosphere and the earth. *Geotechnique*, 1997, 47 (4): 715–767.
- [2] Lu Yingfa, Cui Yujun, Yao Hailin, et al. Seepage properties of non-saturated soils in vadose zones and soil-surface model. *Rock and Soil Mechanics*, 2003, 24 (Supp.2): 8–12 (in Chinese).
- [3] Farouki O T. Thermal properties of soils. [S.l.]: Trans. Tech. Publications, 1986.
- [4] Cui Y J, Lu Y F, Delage P, et al. Field simulation of in-situ water content and temperature changes due to ground-atmospheric interaction. *Geotechnique*, 2005, 55 (7): 557–567.
- [5] Lu Yingfa, Cui Yujun, Zheng Junjie. Variation of suction of soil under atmosphere. *Chinese Journal of Geotechnical Engineering*, 2005, 27 (10): 1 111–1 115 (in Chinese).
- [6] Peng Hua, Chen Shenghong. Finite element analysis of unstable seepage of saturated-unsaturated. *Journal of Wuhan University of Hydraulic and Electric Engineering*, 2001, 34 (3): 9–12 (in Chinese).
- [7] Wilson W, Fredlund D G, Barbour S L. Coupled soil-atmosphere modeling for soil evaporation. *Canadian Geotechnical Journal*, 1994, 31 (2): 151–161.
- [8] Haijink B. Gas generation and gas migration in deep geological repositories for radioactive waste. In: *Proceedings of the 6th Spectrum International Conference on Nuclear Hazardous Waste*. Seattle: [s.n.], 1996: 18–23.
- [9] Van Wijk W R. *Physics of plant environment*. Amsterdam: North-Holland Publishing Company, 1963: 211–235.
- [10] Wilson G W. Soil evaporative fluxes for geotechnical engineering problems. PhD Thesis. Saskatoon, Canada: University of Saskatchewan, 1990.
- [11] Lu Yingfa, Wu Yanchun, Luo Xianqi. Mechanical characteristics of gas in non-saturated soil mass. In: *Proceeding of the 9th Conference on Rock Mechanics and Engineering*. Wuhan: [s.n.], 2006: 432–440 (in Chinese).
- [12] Blight G E. The vadose zone soil-water balance and transpiration rates of vegetation. *Geotechnique*, 2003, 53 (1): 55–64.
- [13] Sultan N, Delage P, Cui Y J. Temperature effects on the volume change behavior of Boom clay. *Engineering Geology*, 2002, 64 (2): 135–145.
- [14] Dakshanamurthy V, Fredlund D G. A mathematical model for predicting moisture flow in an unsaturated soil under hydraulic and temperature gradients. *Water Resources Research*, 1981, 17 (3): 714–722.
- [15] Cui Y J, Yahia-Aissa M, Delage P. A model for the volume change behaviour of heavily compacted swelling clay. *Engineering Geology*, 2002, 64 (2/3): 233–250.
- [16] Ye Weimin, Cui Yunjun, Huang Yu, et al. Collapsibility of loess and its discrimination criteria. *Chinese Journal of Rock Mechanics and Engineering*, 2006, 25 (3): 551–556 (in Chinese).
- [17] Ng C W W, Pang Y W. Influence of stress state on soil-water characteristics and slope stability. *Journal of Geotechnical and Geoenvironmental Engineering*, 2000, 126 (2): 157–166.