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# Reliability Analysis in Performance-Based Earthquake Engineering

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# Abstract

The performance-based engineering approach, as opposed to prescriptive rules of code-based design, is based on simulation of real structural behavior. Reliability of the expected performance state is assessed by using various methodologies based on finite element nonlinear static pushover analysis and specialized reliability software package.

Reliability approaches that were considered included full coupling with an external finite element code based methods in conjunction with either first order reliability method or importance sampling method. The building considered in the actual study has been designed against seismic hazard according to the Moroccan code RPS2000.

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# 1. Introduction

The finite element method is currently the dominating tool for simulating structural behavior. A coupling of this method with reliability analysis algorithms leads to the finite element reliability method described in this work. The first coupling between FORM (first-order reliability method) reliability analysis and the finite element method is found in Der Kiureghian and Taylor [10]. Since then, a number of advances have been reported, including those by Liu and Der Kiureghian [11], Gutierrez et al.[12], Zhang and Der Kiureghian [13], Der Kiureghian and Zhang [14], Sudret and Der Kiureghian [15], Imai and Frangopol [16], Haldar and Mahadevan [17], and Frier and Sorensen [18]. Such methods address the key issue in performance-based engineering. Based on performance thresholds are computed. In addition, sensitivity and importance measures for the model parameters are available.

Finite element reliability analysis (FERA) has been developed to account for uncertainties in structural analysis. The input parameters of the finite element model are provided as random variables to account for uncertainty in the material, geometry, and loading parameters. Subsequently, the probability of response events is computed. This is achieved by defining limit-state functions (also referred to as performance functions) in terms of response quantities of the finite element analysis.

Finite element reliability analysis using full coupling between a finite element code and reliability methods such as the FORM and Monte Carlo methods tend to be computational time consuming for practical problems which could include a large number of random variables. At each iteration, the limit-state function and its derivatives are to be evaluated through finite element computations. An effective method which combines FORM and subsequent importance sampling around the most probable failure point has been proposed by (Haukaas and Der Kiureghian [19]. The Importance Sampling Method (ISM) requires only a limited number of evaluations of the limit-state function (and its gradient with respect to the random variables) to find the approximation point, followed by efficient importance sampling analysis centered at this point. In this last reference, Haukaas and Der Kiureghian [19] have presented numerical examples involving comprehensive nonlinear finite element models with approximately 500 random variables were presented.

#### 2. Finite element seismic reliability analysis of buildings

The performance-based engineering approach is based on simulation of real structural behavior. This feature sets it apart from classical prescriptive rules associated to code-based design. The client or government regulations prescribe desired performance objectives, which are translated into decision variables or functions and serve defining performance criteria. Applying reliability analysis methods in the context of performance-based engineering uses the specified performance functions as performance criteria or limit-states. The term failure denotes then the event of not meeting a given performance criterion. It is then important to know how to translate such performance requirements into explicit performance functions attainable by finite element structural analysis. In this work reference is made to the performance level introduced by the Moroccan seismic code RPS 2000, [20], which intends limiting the building roof displacement ratio. Other performance criteria introduced to distinguish performance-based engineering states with regards to earthquake events could be for example those defined according to the Federal Emergency Management Agency [21]. In this case, the performance states include: operational performance for which the event does not affect the occupants or functioning of the building; immediate occupancy performance for which the occupants can immediately return to the building after the seismic event; life safety performance and collapse prevention performance. The client or code regulations determine in general an acceptable hazard level for each of these performance requirements. For an earthquake event with probability, say, 50% in 50 years, immediate occupancy performance may be demanded. On the other hand, for an earthquake event with probability 2% in 50 years only life safety performance may be desirable. A formal similarity could be stated between the limit-state in the present work according to RPS 2000 and the collapse prevention limit-state according to FEMA.

# 3. Estimation of performance probabilities and response statistics

The primary concern in structural reliability analysis is to estimate probabilities of failure to achieve predefined performance. In the simplest case of one performance function, the component reliability problem is formulated as

$$p_f = \int_{g(x) \le 0} f(x) dx \tag{1}$$

Where  $P_f$  is the probability of failure, x is the vector of random finite element model parameters, g(x) is the performance function and f(x) is the joint PDF of x.Note that the integration is over the set of random variables x,which in finite element reliability analysis can be large. Closed-form solutions of Eq. (3.1) are unavailable except for a few special cases. For this reason, a number of methods have been developed for the purpose of solving the

integral approximately. These include the first- and second-order reliability methods, FORM and SORM, sampling analysis, response surface methods and numerical integration schemes. The latter method is usually not a feasible alternative when the number of random variables is greater than 3 or 4. Furthermore, in finite element reliability analysis, it is desirable to limit the number of evaluations of g and its gradient. This makes methods such as FORM, SORM and importance sampling (IS) tractable, while it excludes the crude Monte Carlo sampling scheme for problems with small failure probabilities. A common aspect of FORM, SORM and IS is that they all employ the so-called design point. This is the most likely point in the failure domain, when the variables are transformed to the standard normal space. As such, this is the ideal point for approximating the limit-state surface separating the safe and failure domains. FORM analysis estimates the failure probability by approximating the limit-state surface by the tangent hyper-plane at the design point. SORM analysis estimates the failure probability by approximating the limit-state surface by a quadratic surface tangent at the design point. An IS analysis may subsequently use the design point as the center of sampling to obtain an improved estimate of the failure probability. These conclusions could not be generalized without precautions to other problems dealing with reliability analysis and thorough analysis is needed to assess performance of the various methodologies.

#### 4. Reliability analysis of retrofitted building

Let's consider a pre-code regular 4 story reinforced concrete building having the plane view shown in Fig 1 and the elevation in the most severe seismic direction shown in Fig 2. The selected structure has four stories and lays on a horizontal surface of  $192 \text{ m}^2$ . The inter-story height is 3m.

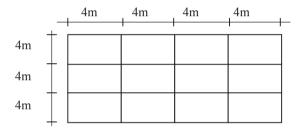


Fig 1: Plane view of the considering 4 story RC building.

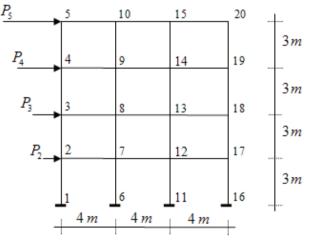


Fig 2: Elevation of the considering 4 story RC building.

Design of this building had been performed by using the French reinforced concrete code BAEL91 [26], in conjunction with the Moroccan seismic code RPS2000 [20], under the following assumptions: steel yield stress 500MPa; concrete resistance 25MPa; priority class 2; amplification factor 2.5; coefficient of soil 1.6; seismic acceleration 0.16g (g = 9.81m.s<sup>-2</sup>); damping coefficient 0.05 and ductility coefficient 2. Table 1 displays the obtained dimensions of beams and columns as well as their reinforcements.

Table 1: Beams and columns sections dimensions with their steel reinforcement bars

	Section width (cm)	Section depth (cm)	Reinforcements at section bottom	Reinforcements at section top	Reinforcements at mid-section
Exterior columns	40	40	4HA14	4HA14	4HA14
Interior columns	40	40	4HA14	4HA14	4HA14
Beams	30	40	3HA14	3HA14	0

This building does not comply with the Moroccan RPS2000 seismic code requirements as to the roof displacement u5 limitation under the action of the design seismic load. The performance function associated to this limit state writes

(2)

$$g(x)=0.004x12000-u_5$$

where H the total height of the building,  $H=12m . u_5$  is function of the applied seismic load and of the design variables including geometry and material properties of the building. It is desired then to retrofit this building in order to satisfy the performance function defined by equation g(x)<0.Due to variations affecting material characteristics and geometry, it is desired also to guarantee this criterion with a given reliability.

The building is assumed to behave elastically under the action of the seismic design load. Geometric and material properties are assumed to be random variables. There are those which are not modified by rehabilitation such the seismic lateral loads, homogenized reinforced concrete modulus of elasticity, E, and beam area and inertia respectively A and I. Table 2 summarizes the building random variables.

Table 2: Uncertainty modeling of the random variables of the reinforced concrete buildings

Variable	Mean value	Distribution
E side column	32551.4MPa	Lognormal
E center column	32551.4MPa	Lognormal
E beam	32678.4MPa	Lognormal
A side column	1600cm <sup>2</sup>	Lognormal
A center column	$1600 \text{ cm}^2$	Lognormal
A beam	$1200 \text{ cm}^2$	Lognormal
I side column	$160000 \text{ cm}^4$	Lognormal
I center column	213333.333 cm <sup>4</sup>	Lognormal
I beam	213333.333 cm <sup>4</sup>	Lognormal
Load seismic P2	46.75kN	Lognormal
Load seismic P3	84.96kN	Lognormal
Load seismic P4	127.44kN	Lognormal
Load seismic P5	165.67kN	Lognormal
Loads W10 and W15	176.99kN	Lognormal
Loads W5 and W20	88.5kN	Lognormal

When the mechanical problem could be assumed to have a linear elastic behaviour and quantities entering in the

Haukaas and Der Kiureghian [19]. Version 3.1 dating of 2003 was used in this work. Fig 3 shows the synoptic structure of this code. FERUM is the main program that orchestrates various sub-programs to perform reliability analysis according to the methods: First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and Monte Carlo Importance Sampling (ISM). Required information for reliability analysis by

means of FERUM (random variables and their marginal distributions of probability, eventually correlations, performance functions, parameters for the optimisation algorithm) is introduced by the input file. FORM is the most important reliability analysis method. It enables determining reliability index through Nataf iso-probability transformation and the improved Hasofer-Lind-RackwitzFiessler algorithm for solution of the minimization problem. This module is also used in the other reliability methods: SORMs and Monte Carlo Importance Sampling. For SORM analysis three variants are provided: Breitung formula (SORM1), Hohenbichler / Rackwitz formula (SORM2) and Tvedt Exact Integral (SORM3). Using the ISM method needs specifying the convergence criterion in terms of the coefficient of variance (COV). Using random variables defined by tables 2 and 3, FERUM software is used to perform reliability analysis.

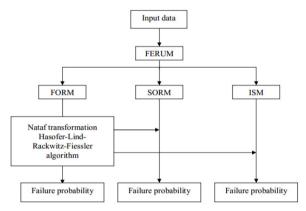


Fig 3: Synoptic diagram of code FERUM.

# 5. Reliability analysis of retrofitted building

Tables 3, 4 and 5 show that results obtained by Monte Carlo based Sampling Analysis method, SORM and FORM method are quite different as function of the Deviation ratio in case of Lognormal distributions of probability.

Table 3: Reliability results obtained by FORM as function of Deviation ratio

Deviation ratio	Time to complete the analysis	Reliability index	Failure probability x10 <sup>-2</sup>
0.10	0.931	2.3214	1.01337
0.15	0.962	1.4942	6.75582
0.20	0.814	1.066	14.3222
0.25	0.831	0.79594	21.3032
0.30	0.891	0.60598	2.72264

Table 4. Reliability results obtained by SORM as function of Deviation ratio

Deviation ratio	SORM method	Time to complete the analysis	Reliability index	Failure probability x10 <sup>-2</sup>
0.1	SORM1	52.393	2.3031	1.06374
	SORM2	52.393	2.3002	1.07189
	SORM3	52.393	2.3005	1.07103
	SORM1	50.702	1.4699	7.07979
0.15	SORM2	50.702	1.4621	7.18620 7.18620e
	SORM3	50.702	1.4629	7.17426
	SORM1	52.349	0.98412	16.2529
0.20	SORM2	52.349	0.94211	17.3068
	SORM3	52.349	0.9476	17.1666
	SORM1	55.863	0.72372	23.4617
0.25	SORM2	55.863	0.66784	25.2118
0.30	SORM3	55.863	0.67503	24.9827
	SORM1	55.445	0.54533	29.2763
	SORM2	55.445	0.47777	31.6406
	SORM3	55.445	0.48654	31.3292

Derviction actio	Number of Reliability index		Failure	
Deviation ratio	iterations		probability x10 <sup>-2</sup>	
0.10	6553	2.3006	1.07057	
0.15	4555	1.4572	7.25359	
0.20	3738	1.041	14.8931	
0.25	3095	0.74379	22.8501	
0.30	2981	0.57002	28.4331	

# 6. Conclusions

Use was made of the software FERUM which couples finite element method with specialized reliability toolbox. It was assessed that FORM method is sufficient to perform reliability analysis with minimum computational time. Within the framework of the same methodology of reliability analysis (either full coupling), the approximate method FORM does not give the same results than the more precise modified Monte Carlo ISM Method. In general, FORM overestimates the probability of failure. These conclusions could not be generalized without precautions to other problems dealing with reliability analysis and thorough analysis is needed to assess performance of the various methodologies.

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