Finite element analysis and optimization of flexure bearing for linear motor compressor

Maruti Khot¹,* , Bajirao Gawali¹

¹Mechanical Engineering DepartmentWalchand College of EngineeringSangli, Maharashtra, 416415, India

Abstract

Nowadays linear motor compressors are commonly used in miniature cryocoolers instead of rotary compressors because rotary compressors apply large radial forces to the piston, which provide no useful work, cause large amount of wear and usually require lubrication. Recent trends favour flexure supported configurations for long life. The present work aims at designing and geometrical optimization of flexure bearings using finite element analysis and the development of design charts for selection purposes. The work also covers the manufacturing of flexures using different materials and the validation of the experimental finite element analysis results.

Keywords: cryocooler; linear compressor; flexure bearing; FE analysis; strain and stiffness measurement

1. Introduction

Lubrication is a common problem in miniature cryocoolers which contaminating the regenerators. Small capacity cryocoolers for space borne application generally use linear electromagnetic drives for long life, maintenance free operation and high reliability. Flexure bearings are used for dynamic support in linear motor compressors. A typical flexure supported electromagnetic drive system is shown in Fig. 1. For eliminating rubbing contact between the piston and the cylinder wall, flexure bearing is used to support the piston inside the cylinder. A
radial clearance gap of 10 to 20 μm between piston and cylinder provides the necessary flow impedance to serve as
dynamic seal. The Flexure bearings provide a stiff support in radial direction and act as a weak spring in axial
direction. In linear motor compressors two types of flexures are used; spiral and linear. The schematic of these
flexures is shown in Fig. 2.

Gifford and Longsworth [1] used the Pulse Tube cryocooler to achieve the low temperature. They have reported
that Pulse Tube cryocoolers make it possible to get any temperature with no low temperature moving parts and
almost at any gas pressure in small, simple devices with small gas flows. Gawali et al. [2] developed an integral
Stirling type Pulse Tube cryocooler of 15W at 70K, using a linear motor driven dual opposed piston compressor.
The cryocooler was also used for liquefaction of nitrogen gas. Van der Walt and Nicholas [3] have reported that the
efficiency of linear motors is 15% to 25% more than of rotary compressors. A cryocooler using flexure bearings was
reported by the Oxford University in 1980s [4]. Wong et al. [5] analyzed the spiral flexure bearing for static and
dynamic stress. Chen et al. [6] have used finite element methods for designing and analyzing flexure bearings. In the
present work geometrical parameters of spiral and linear flexures are considered. The effect of these geometrical
parameters on the design of flexures is studied. The geometrical parameters are optimized using Taguchi and Grey
relational analysis method. By using the optimized geometry the design charts are developed for selection of spiral
and linear flexure bearings for miniature cryocoolers. The work also covers the manufacturing of spiral and linear
type flexures by photo etching method using beryllium copper and steel material. The experimental setups are
developed for axial stiffness, static and dynamic strain measurement. The axial stiffness and Von Mises strain is
experimentally validated. The photoelastic models of spiral and linear flexures are developed for the experimental
validation of the finite element analysis results. Close agreement between finite element analysis and experimental
results is observed.

2. Finite element analysis of flexure bearing

Since the exact analysis of the flexure is not possible as it is subjected to combined tensile, shear and
bending stresses, the Finite Element Method (FEM) for analysis was chosen. The geometrical nonlinear, static
structural analysis is carried out using ANSYS software.

2.1 Spiral flexure bearing

The three arm spiral profile is selected. Fig 3 shows the geometrical parameters of single spiral arm. Where r₁ is
the inner radius, r₂ is the outer radius, θ is the spiral angle, s is the slot width; the effective diameter Dₑ is equals to
2 (r₂) and thickness t. Table 1 shows the range and levels of geometrical parameters considered for analysis purpose.

Design of Experiments (DOE) theory was used. By using an orthogonal array selector given by the Taguchi
method for five parameters and five levels, an L25 array is to be used. Thus the number of combinations of
parameters reduces to 25. So that only 25 geometries need to be analyzed. The finite element analysis consists of
three main steps such as pre-processing, solution and post processing. These three steps for spiral flexure bearings
are carried out and from this analysis it is found that the model having an effective diameter 60 mm, spiral angle
480°, slot width 0.2 mm, thickness 0.4 mm and starting radius 9 mm has the most optimum geometrical parameters
to satisfy design objectives of minimum stress level, minimum axial stiffness and maximum radial stiffness. The
design charts for the spiral flexure are developed by using the optimum geometry.
Table 1. Parameters for FE analysis of spiral flexure bearing

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Parameter</th>
<th>Range</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Effective diameter (mm)</td>
<td>40-80</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Thickness (mm)</td>
<td>0.1-0.5</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Swept angle</td>
<td>360°-600°</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>Slot Width (mm)</td>
<td>0.2-1</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>Starting radius (mm)</td>
<td>5-9</td>
<td>5</td>
</tr>
</tbody>
</table>

2.2 Linear flexure bearing

The methodology developed for analysis of the spiral flexure bearing is also used for the analysis of linear flexure bearings. Fig. 4 shows the geometrical parameters of the linear flexure bearing. The geometrical parameters and their levels identified for linear flexure analysis are given in Table 2.

Table 2. Parameters for FE analysis of linear flexure bearing

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Parameter</th>
<th>Range</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Effective diameter (mm)</td>
<td>50-80</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Arm width (mm)</td>
<td>5-8</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Arm angle</td>
<td>15°-22.5°</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>Clamp angle</td>
<td>15°-30°</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>Thickness (mm)</td>
<td>0.1-0.4</td>
<td>4</td>
</tr>
</tbody>
</table>

The linear flexure bearing consists of various geometrical parameters like effective diameter (D), arm width (W), clamp angle (β) and arm angle (θ). Different models of flexure bearings are to be analyzed by varying geometrical parameters. The L16 array was selected using the Taguchi method. The finite element analysis was carried out. The most optimum linear flexure bearing for the present application is one having geometry, effective diameter (mm) = 70, thickness (mm) = 0.2, arm width (mm) = 5, arm angle = 20° and clamp angle = 30°. The design charts for the linear flexure are developed by using the optimum geometry.

3. Experimental validation of FEA results

In order to carry out the experimental verification of the finite element analysis (FEA) results; the displacement and strain need to be measured accurately. For this purpose a linear variable differential transducer (LVDT) and strain gauges are used with static and dynamic loading test setups. The experimental testing was conducted to validate the FEA results for axial stiffness and Von Mises strain.

3.1 Experimental setup for static loading

A simple dead weight method is used to measure the axial stiffness. In this method a pan is attached to the shaft connected at center of the flexure bearing. The weights are gradually added and displacements are measured using the LVDT. The graph of force vs. axial displacement (z) is plotted. The slope of this line gives the axial stiffness. Fig. 5 shows a photograph of the static loading experimental setup. The experimental data obtained for the spiral flexure is shown in Figs. 6 (a) and (b). The graph shows the relation between the FE analysis and the experimental
results. The maximum deviation between the results found is below 10 %. The measurement of strain has been carried out using a rectangular rosette type strain gauge. The strain gauge rosette is mounted at the position where the maximum strain was observed during FE analysis. The displacement was measured with the LVDT and the corresponding strain with strain gauges. A close match between the FE analysis and experimental results is found with a maximum deviation of 9 %.

Fig. 5. Static loading experimental setup   Fig. 6. FEA Results (a) Axial stiffness ; (b) Von Mises strain

3.2 Experimental setup for dynamic loading

Fig. 7 shows the photograph of the dynamic loading experimental setup. The linear motor with ±7 mm axial displacement is designed with 50 W output power. The frequency of operation of linear motor is equal to the input current frequency. The rectangular rosette type strain gauge was used with data acquisition system DEWE 43 V for dynamic strain measurement.

The flexure stack under test is assembled on the flexure support plate shown in Fig. 8. The support plate is provided with six holes at different pitch circle diameters so that the flexures in diameter range of 40 mm to 100 mm can be assembled and tested. The dynamic average values of $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are found to be 407, 123 and 7 $\mu$-strains, respectively. The dynamic Von Mises strain is calculated as 402.5 $\mu$-strains which are comparable with the FEM static strain of 374 $\mu$-strains. The experimental value of the static Von Mises strain is 393 $\mu$-strains. The dynamic strain is found to be 8 % more than the observed static strain. This dynamic strain should be considered before selecting the flexure for the compressor.

3.3 Stress analysis by photoelasticity

The experimental stress analysis method is based on the property of birefringence or double refraction exhibited by certain transparent materials. The birefringence is a property of material where a ray of light passing through a birefringent experiences two refractive indices. Upon the application of stress, photoelastic materials exhibit the property of birefringence and the magnitude of the refractive indices at each point in the material is directly related to the state of stress at that point. Information such as maximum stress and its orientation is available by analyzing the birefringence with an instrument called polariscope. The magnitude of the relative retardation is given by the stress-optic law [7]:

![stress-optic law diagram]
\[ \Delta = \frac{2\pi t}{\lambda} C_o (\sigma_1 - \sigma_2) \]  

(1)

Where \( \Delta \) is the induced retardation, \( C_o \) is the stress-optic coefficient, \( t \) is the specimen thickness, \( \sigma_1 \) and \( \sigma_2 \) are the first and second principal stresses, respectively. It may be written as,

\[ \sigma_1 - \sigma_2 = \left( \frac{f_o}{t} \right) N \]  

(2)

Where \( \sigma_1 - \sigma_2 \) is difference of principal stresses, \( f_o = (\lambda/C_o) \), depends upon the model material and the wavelength of light employed, and \( N = (\Delta/2\pi) \), is the relative retardation of rays forming the pattern. The term \( N \) is also known as the isochromatic fringe order. Fig. 9 shows the photoelastic model for spiral and linear flexure bearings. The solid circular disc was made from photoelastic material. With the help of diametric compression method the material fringe value was calculated. The material fringe value for the model is \( f_o = 14.2 \).

![Fig. 9. Photoelastic model](image)

![Fig. 10. Flexure model loading](image)

(a) Radial Loading; (b) Axial Loading

Fig. 10 shows the flexure model loaded in the polariscope arrangement. Radial and axial loading was applied on the spiral and linear flexures. The combined radial and axial loading was also applied and the stress distribution within the model was studied and high stress location points were identified.

Fig. 11 shows the isoclinic and isochromatic fringes in the flexure bearing. Isoclinic fringes are the loci of the points in the model along which the principal stresses are in the same direction. Isochromatic fringes are the loci of the points in the model along which the difference in the first and second principal stress is constant. They are the lines which join the points with equal maximum shear stress in the model.

![Fig. 11. Isoclinic and isochromatic fringes.](image)

Table 3. Principal stress comparison (linear flexure)

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Point of Interest</th>
<th>ESA stress (N/mm²)</th>
<th>FEM stress (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>26N</td>
<td>A</td>
<td>13.608</td>
<td>15.26</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>14.896</td>
<td>16.11</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>15.288</td>
<td>17.23</td>
</tr>
</tbody>
</table>

The photoelastic model of the spiral and the linear flexure was loaded in the polariscope arrangement. The points of interest were identified based on the finite element analysis. With the help of the plane polariscope arrangement the isoclinic pattern, the principal stress directions were identified. A radial load of 26 N was applied and with the help of the circular polariscope arrangement with polychromatic light the lower and higher fringe value at the point of interest was noted. By using the Tardy’s method the fractional fringe value at the point of interest was calculated. By knowing the fractional fringe value the principal stresses at the points of interest were calculated in the photoelastic model. The principal stresses at the points of interest were calculated in prototype by applying the model to the prototype transition formula.
Table 3 shows the comparison between FE and experimental stress analysis results. The deviation between these results is about 10%.

4. Development of linear motor compressor

The compressor with specifications; swept volume: 5 cc, PV power: 100 W, charge pressure: 16 bars, pressure ratio: 1.2 to 1.4, piston stroke: ±5 mm, frequency: 50 Hz and efficiency goal: 70 to 80% was developed. Fig. 12 shows a photograph of the fully assembled dual opposed piston, moving coil type linear motor compressor.

![Photograph of the fully assembled linear motor compressor](image)

The flexure stack is assembled on the support ring. The support ring, coil former and outer pole are fastened together with the main body. The support ring is made from aluminium to reduce weight. It is provided with holes at different pitch circle diameters so that flexures with diameters 40, 50 and 60 mm can be tested on this compressor. The number of flexures in the stack was selected from the developed design charts. The required stroke, frequency and pressure ratio was achieved by using spiral as well as linear flexures.

5. Conclusion

A design methodology is developed to analyze the effects of geometrical parameters of spiral and linear flexures on design parameters. In case of spiral flexure bearings, as the diameter, spiral angle and the starting radius increase the Von Mises stress, axial and radial stiffness decrease. As the thickness increases the Von Mises stress, the axial and the radial stiffness increase. From design requirement point of view, radial stiffness should be high and stress and axial stiffness should be low to achieve the optimum value of geometric parameters of the flexure bearing. The optimum values of geometric parameters are obtained using the Taguchi method. In case of linear flexure bearings, as the diameter increases the Von Mises stress decreases. As the thickness increases the Von Mises stress, the axial and the radial stiffness increase. The effects of arm angle, clamp angle and arm width are negligible on design parameters. From the theoretical analysis of spiral and linear flexure bearings, it is observed that in both cases the diameter, the thickness and the geometry are significant parameters. Theoretical results of spiral and linear flexures were compared and it is observed that for the same thickness and diameter, the Von Mises strain in the linear flexure is three times greater, the axial stiffness five times greater and the radial stiffness is ten times greater. Thus the linear flexure bearing is suitable for compact size and long life application but for smaller axial strokes. Close agreement between theoretical and experimental results is found.

Acknowledgement

The research work is funded under the Fast Track Scheme by the Department of Science and Technology India.

References