

Mass and magnetic moments of the heavy flavored baryons with $J = \frac{3}{2}$ in light cone QCD sum rules

T.M. Aliev¹, K. Azizi^{*}, A. Ozpineci

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

Received 22 July 2008; received in revised form 14 August 2008; accepted 10 September 2008

Available online 19 September 2008

Abstract

Inspired by the results of recent experimental discoveries for charm and bottom baryons, the masses and magnetic moments of the heavy baryons with $J^P = 3/2^+$ containing a single heavy quark are studied within light cone QCD sum rules method. Our results on the masses of heavy baryons are in good agreement with predictions of other approaches, as well as with the existing experimental values. Our predictions on the masses of the states, which are not yet discovered in the experiments, can be tested in the future experiments. A comparison of our results on the magnetic moments of these baryons and the hyper central model predictions is presented.

© 2008 Elsevier B.V. All rights reserved.

PACS: 11.55.Hx; 13.40.Em; 14.20.Lq; 14.20.Mr

1. Introduction

Recently, considerable experimental progress has been made in the spectroscopy of baryons containing a single heavy quark. The CDF Collaboration has observed four bottom baryons Σ_b^{\pm} and $\Sigma_b^{*\pm}$ [1]. The DO [2] and CDF [3] Collaborations have seen the Ξ_b . The BaBar Collaboration discovered the Ω_c^* state [4].

The CDF sensitivity appears adequate to observe new heavy baryons. Study of the electromagnetic properties of baryons can give noteworthy information on their internal structure. One of

^{*} Corresponding author.

E-mail addresses: taliev@metu.edu.tr (T.M. Aliev), e146342@metu.edu.tr (K. Azizi), ozpineci@metu.edu.tr (A. Ozpineci).

¹ Permanent address: Institute of Physics, Baku, Azerbaijan.

the main static electromagnetic parameters of the baryons is their magnetic moments. Magnetic moments of the heavy baryons in the framework of different approaches are widely discussed in the literature. In [5–7] the magnetic moments of heavy baryons containing c-quark have been calculated using the naive quark model. In [8–10], the magnetic moments of charm and bottom baryons are computed in quark model and in [11,12] heavy baryon magnetic moments are investigated within soliton type approaches. Calculation of the magnetic moments of heavy baryons in the framework of relativistic quark model is done in [13]. In [14] the magnetic moments of Σ_c and Λ_c baryons are estimated in QCD sum rules with external electromagnetic field. The magnetic moments of the Λ_Q ($Q = c, b$) and $\Sigma_Q \Lambda_Q$ transition magnetic moments in light cone QCD sum rules are calculated in [15,16]. The magnetic moments of Ξ_Q ($Q = c, b$) in the same framework is studied in [17] (a detailed description of this method can be found in [18] and references therein).

In the present work, we study the magnetic moments and masses of the ground state baryons with total angular momentum $3/2$ and containing one heavy quark within light cone QCD sum rules. The paper is organized as follows. In Section 2, the light cone QCD sum rules for mass and magnetic moments of heavy baryons are calculated. Section 3 is devoted to the numerical analysis of the mass and magnetic moment sum rules and discussion.

2. Light cone QCD sum rules for the mass and magnetic moments of the heavy flavored baryons

Before giving detailed calculation of the magnetic moments and masses of the heavy baryons, few words about their classifications are in order. The baryons with single heavy quarks belongs to either antisymmetric $\bar{3}_F$ or symmetric 6_F flavor representation. For the S-wave heavy baryons, total flavor-spin wave function of the two light quarks must be symmetric, since their color wave function is antisymmetric. Hence the total spin of the two light quarks is either $S = 1$ for 6_F or $S = 0$ for $\bar{3}_F$. The total angular momentum and parity of the S-wave heavy baryons are $J^P = 1/2^+$ or $3/2^+$ for 6_F and $J^P = 1/2^+$ for $\bar{3}_F$. The antitriplet $\bar{3}$ of baryons contain isosinglet Λ_Q with quark content $Q[u, d]$ and the isodoublet Ξ_Q with quark content $Q[u, s]$ and $Q[d, s]$. The sextet 6_F of baryons involves isotriplet Σ_Q with quark content $Q\{u, u\}$, $Q\{u, d\}$ and $Q\{d, d\}$ and the isodoublet Ξ'_Q with quark content $Q\{u, s\}$ and $Q\{d, s\}$. Here $[\dots]$ and $\{\dots\}$ denote the antisymmetric and symmetric flavor wave functions.

After these preliminary remarks let us start calculation of the magnetic moments of the heavy flavored hadrons. The basic objects in LCSR method is the correlation function where hadrons are represented by the interpolating quark currents. For this aim, we consider the correlator

$$T_{\mu\nu} = i \int d^4x e^{ipx} \langle 0 | T \{ \eta_\mu(x) \bar{\eta}_\nu(0) \} | 0 \rangle_\gamma, \quad (1)$$

where η_μ is the interpolating current of the heavy baryon and γ means the electromagnetic field. In QCD sum rules method, this correlation function is calculated in two different approaches: On the quark level, it describes a hadron as quarks and gluons interacting in QCD vacuum. In the phenomenological side, it is saturated by a tower of hadrons with the same flavor quantum numbers. The magnetic moments are determined by matching two different representations of the correlation function, i.e., theoretical and phenomenological forms, using the dispersion relations.

From Eq. (1), it follows that to calculate the correlation function from QCD side, we need the explicit expressions of the interpolating currents of heavy baryons with the angular momentum $J^P = 3/2^+$. The main condition for constructing the interpolating currents from quark field is

Table 1

The value of A and quark fields q_1 and q_2 for the corresponding baryons.

	A	q_1	q_2
$\Sigma_{b(c)}^{*++(++)}$	$1/\sqrt{3}$	u	u
$\Sigma_{b(c)}^{*0(+)}$	$\sqrt{2/3}$	u	d
$\Sigma_{b(c)}^{*- (0)}$	$1/\sqrt{3}$	d	d
$\Xi_{b(c)}^{*0(+)}$	$\sqrt{2/3}$	s	u
$\Xi_{b(c)}^{*- (0)}$	$\sqrt{2/3}$	s	d
$\Omega_{b(c)}^{*- (0)}$	$1/\sqrt{3}$	s	s

that they should have the same quantum numbers of the baryons under consideration. For the heavy baryons with $J^P = 3/2^+$, the interpolating current is chosen in the following general form

$$\eta_\mu = A\epsilon_{abc}\{(q_1^{aT} C \gamma_\mu q_2^b) Q^c + (q_2^{aT} C \gamma_\mu Q^b) q_1^c + (Q^{aT} C \gamma_\mu q_1^b) q_2^c\}, \quad (2)$$

where C is the charge conjugation operator and a, b and c are color indices. The value of A and quark fields q_1 and q_2 for each heavy baryon is given in Table 1.

The phenomenological part of the correlation function can be obtained by inserting the complete set of states between the interpolating currents in (1) with quantum numbers of heavy baryons.

$$T_{\mu\nu} = \frac{\langle 0|\eta_\mu|B(p_2)\rangle\langle B(p_2)|B(p_1)\rangle_\gamma}{p_2^2 - m_B^2} \frac{\langle B(p_1)|\bar{\eta}_\nu|0\rangle}{p_1^2 - m_B^2}, \quad (3)$$

where $p_1 = p + q, p_2 = p$ and q is the photon momentum. The vacuum to baryon matrix element of the interpolating current is defined as

$$\langle 0|\eta_\mu(0)|B(p, s)\rangle = \lambda_B u_\mu(p, s), \quad (4)$$

where λ_B is the residue and $u_\mu(p, s)$ is the Rarita–Schwinger spinor. The matrix element $\langle B(p_2)|B(p_1)\rangle_\gamma$ entering Eq. (3) can be parameterized in the following way

$$\begin{aligned} \langle B(p_2)|B(p_1)\rangle_\gamma &= \epsilon_\rho \bar{u}_\mu(p_2) \left\{ -g_{\mu\nu} \left[\gamma_\rho (f_1 + f_2) + \frac{(p_1 + p_2)_\rho}{2m_B} f_2 + q_\rho f_3 \right] \right. \\ &\quad \left. - \frac{q_\mu q_\nu}{(2m_B)^2} \left[\gamma_\rho (G_1 + G_2) + \frac{(p_1 + p_2)_\rho}{2m_B} G_2 + q_\rho G_3 \right] \right\} \bar{u}_\nu(p_1), \end{aligned} \quad (5)$$

where ϵ_ρ is the photon polarization vector, f_i and G_i are the form factors and they are functions of $q^2 = (p_1 - p_2)^2$. In order to obtain the explicit expressions of the correlation function, we perform summation over spins of the spin 3/2 particles using

$$\sum_s u_\mu(p, s) \bar{u}_\nu(p, s) = \frac{(\not{p} + m)}{2m} \left\{ -g_{\mu\nu} + \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2p_\mu p_\nu}{3m^2} - \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3m} \right\}. \quad (6)$$

Using Eqs. (3)–(6) in principle one can write down the phenomenological part of the correlator. But, the following two drawbacks appear: (a) all Lorentz structures are not independent; (b) not

only spin 3/2, but spin 1/2 states also contribute to the correlation function. Indeed the matrix element of the current η_μ between vacuum and spin 1/2 states is nonzero and is determined as

$$\langle 0 | \eta_\mu(0) | B(p, s = 1/2) \rangle = \alpha(4p_\mu - m\gamma_\mu)u(p, s = 1/2), \tag{7}$$

where the condition $\gamma_\mu \eta^\mu = 0$ is imposed.

There are two different ways to remove the unwanted spin 1/2 contribution and retain only independent structures in the correlation function: (1) Introduce projection operators for the spin 3/2 states, which kill the spin 1/2 contribution; (2) Ordering Dirac matrices in a specific order and eliminate the structures that receive contributions from spin 1/2 states. In this work, we will follow the second method and choose the ordering for Dirac matrices as $\gamma_\mu \not{p} \not{q} \not{q} \gamma_\nu$. With this ordering for the correlator, we get

$$T_{\mu\nu} = \lambda_B^2 \frac{1}{(p_1^2 - m_B^2)(p_2^2 - m_B^2)} \left[g_{\mu\nu} \not{p} \not{q} \not{q} \frac{g_M}{3} + \text{other structures with } \gamma_\mu \text{ at the beginning and } \gamma_\nu \text{ at the end or which are proportional to } p_{2\mu} \text{ or } p_{1\nu} \right], \tag{8}$$

where $g_M/3 = f_1 + f_2$ and at $q^2 = 0$, g_M is the magnetic moment of the baryon in units of its natural magneton, $e\hbar/2m_{BC}$. The factor 3 is due the fact that in the non-relativistic limit the interaction Hamiltonian with magnetic field is equal to $g_M B = 3(f_1 + f_2)B$.

On QCD side, the correlation function (1) can be evaluated using operator product expansion. After contracting out the quark pairs in Eq. (1) using the Wick’s theorem, we get the following expression for the correlation function in terms of quark propagators

$$\begin{aligned} \Pi_{\mu\nu} = & -iA^2 \epsilon_{abc} \epsilon_{a'b'c'} \int d^4x e^{ipx} \langle 0 | [\gamma(q)] | \{ S_Q^{ca'} \gamma_\nu S_{q_2}^{bb'} \gamma_\mu S_{q_1}^{ac'} \\ & + S_Q^{cb'} \gamma_\nu S_{q_1}^{aa'} \gamma_\mu S_{q_2}^{bc'} + S_{q_2}^{ca'} \gamma_\nu S_{q_1}^{bb'} \gamma_\mu S_Q^{ac'} + S_{q_2}^{cb'} \gamma_\nu S_Q^{aa'} \gamma_\mu S_{q_1}^{bc'} \\ & + S_{q_1}^{cb'} \gamma_\nu S_{q_2}^{aa'} \gamma_\mu S_Q^{bc'} + S_{q_1}^{ca'} \gamma_\nu S_Q^{bb'} \gamma_\mu S_{q_2}^{ac'} + \text{Tr}(\gamma_\mu S_{q_1}^{ab'} \gamma_\nu S_{q_2}^{ba'}) S_Q^{cc'} \\ & + \text{Tr}(\gamma_\mu S_Q^{ab'} \gamma_\nu S_{q_1}^{ba'}) S_{q_2}^{cc'} + \text{Tr}(\gamma_\mu S_{q_2}^{ab'} \gamma_\nu S_Q^{ba'}) S_{q_1}^{cc'} \} | 0 \rangle, \end{aligned} \tag{9}$$

where $S' = CS^T C$ and $S_Q(S_q)$ is the full heavy (light) quark propagator. In calculation of the correlation function from QCD side, we take into account terms linear in m_q and neglect quadratic terms. The correlator contains three different contributions: (1) Perturbative contributions; (2) Mixed contributions, i.e., the photon is radiated from freely propagating quarks at short distance and at least one of quark pairs interact with QCD vacuum non-perturbatively. The last interaction is parameterized in terms of quark condensates. (3) Non-perturbative contributions, i.e., when photon is radiated at long distances. In order to get expressions of the contributions when the photon is radiated at short distance, it is enough to replace the propagator of the quark that emits the photon by

$$S_{\alpha\beta}^{ab} \Rightarrow -\frac{1}{2} \left\{ \int d^4y F^{\mu\nu} y_\nu S^{\text{free}}(x-y) \gamma_\mu S^{\text{free}}(y) \right\}_{\alpha\beta}^{ab}, \tag{10}$$

where the Fock–Schwinger gauge, $x_\mu A^\mu(x) = 0$ has been used. The expressions of the free light and heavy quark propagators in x representation are:

$$\begin{aligned}
 S_q^{\text{free}} &= \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2}, \\
 S_Q^{\text{free}} &= \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} - i \frac{m_Q^2 \not{x}}{4\pi^2 x^2} K_2(m_Q \sqrt{-x^2}),
 \end{aligned}
 \tag{11}$$

where K_i are Bessel functions. The non-perturbative contributions to the correlation function can be obtained from Eq. (9) by replacing one of the light quark propagators (the quark that emits the photon) by

$$S_{\alpha\beta}^{ab} \rightarrow -\frac{1}{4} \bar{q}^a \Gamma_j q^b (\Gamma_j)_{\alpha\beta},
 \tag{12}$$

where Γ is the full set of Dirac matrices $\Gamma_j = \{1, \gamma_5, \gamma_\alpha, i\gamma_5\gamma_\alpha, \sigma_{\alpha\beta}/\sqrt{2}\}$ and sum over index j is implied. Under this procedure, two remaining quark propagators are full propagators involving perturbative as well as non-perturbative contributions. Therefore, for the calculation of the correlation function from QCD side, the expressions of the heavy and light quark propagators in the presence of external field are needed.

The light cone expansion of the quark propagator in external field is done in [19]. The propagator receives contributions from $\bar{q}Gq$, $\bar{q}GGq$ and $\bar{q}q\bar{q}q$, where G is the gluon field strength tensor. In present work, we neglect terms with two gluons as well as four quarks operators due to the fact that their contributions are small [20]. In this approximation the heavy and light quark propagators have the following expressions:

$$\begin{aligned}
 S_Q(x) &= S_Q^{\text{free}}(x) - i g_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \left[\frac{\not{k} + m_Q}{(m_Q^2 - k^2)^2} G^{\mu\nu}(vx) \sigma_{\mu\nu} \right. \\
 &\quad \left. + \frac{1}{m_Q^2 - k^2} vx_\mu G^{\mu\nu} \gamma_\nu \right], \\
 S_q(x) &= S_q^{\text{free}}(x) - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} \left(1 - i \frac{m_q}{4} \not{x} \right) - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left(1 - i \frac{m_q}{6} \not{x} \right) \\
 &\quad - i g_s \int_0^1 du \left[\frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(ux) \sigma_{\mu\nu} - ux^\mu G_{\mu\nu}(ux) \gamma^\nu \frac{i}{4\pi^2 x^2} \right. \\
 &\quad \left. - i \frac{m_q}{32\pi^2} G_{\mu\nu} \sigma^{\mu\nu} \left(\ln \left(\frac{-x^2 \Lambda^2}{4} \right) + 2\gamma_E \right) \right].
 \end{aligned}
 \tag{13}$$

Here, we would like to make the following remark about the parameter Λ . In order to achieve a factorization of large and small scales in the OPE, all infrared logarithms should be removed from coefficient functions and absorbed in the matrix elements of operators. In our case, this means that the $\ln \Lambda$ must be included in the condensates of different operator or distribution amplitude. A more detailed discussion on this point can be found in [21]. For this reason, we will choose the scale parameter Λ as a factorization scale, i.e., $\Lambda = 1 \text{ GeV}$.

In order to calculate the contributions of the photon emission from large distances, the matrix elements of nonlocal operators $\bar{q}\Gamma_i q$ between the photon and vacuum states are needed, $\langle \gamma(q) | \bar{q}\Gamma_i q | 0 \rangle$. These matrix elements are determined in terms of the photon distribution amplitudes (DA's) as follows [22].

$$\begin{aligned}
& \langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle \\
&= -i e_q \bar{q} q (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left(\chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right) \\
&\quad - \frac{i}{2(qx)} e_q \langle \bar{q} q \rangle \left[x_\nu \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u), \\
& \langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle = e_q f_{3\gamma} \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u), \\
& \langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle \\
&= -\frac{1}{4} e_q f_{3\gamma} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u), \\
& \langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle \\
&= -i e_q \langle \bar{q} q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i), \\
& \langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu} i\gamma_5(vx) q(0) | 0 \rangle \\
&= -i e_q \langle \bar{q} q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \tilde{\mathcal{S}}(\alpha_i), \\
& \langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle \\
&= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i), \\
& \langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i\gamma_\alpha q(0) | 0 \rangle \\
&= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i), \\
& \langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle \\
&= e_q \langle \bar{q} q \rangle \left\{ \left[\left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\nu} - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) \right) q_\beta \right. \right. \\
&\quad - \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\nu} - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha \\
&\quad - \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\mu} - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta \\
&\quad \left. \left. + \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\mu} - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) \right) q_\alpha \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_1(\alpha_i) \right. \\
&\quad \left. + \left[\left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) q_\nu \right. \right. \\
&\quad \left. \left. - \left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) q_\mu \right] \right\}
\end{aligned}$$

$$\begin{aligned}
 & - \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) q_\nu \\
 & + \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) q_\mu \Big] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + \nu\alpha_g)qx} \mathcal{T}_2(\alpha_i) \\
 & + \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + \nu\alpha_g)qx} \mathcal{T}_3(\alpha_i) \\
 & + \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + \nu\alpha_g)qx} \mathcal{T}_4(\alpha_i) \Big\}, \tag{14}
 \end{aligned}$$

where χ is the magnetic susceptibility of the quarks, $\varphi_\gamma(u)$ is the leading twist 2, $\psi^v(u)$, $\psi^a(u)$, \mathcal{A} and \mathcal{V} are the twist 3 and $h_\gamma(u)$, \mathbb{A} , \mathcal{T}_i ($i = 1, 2, 3, 4$) are the twist 4 photon DA's, respectively. The photon DA's is calculated in [22] and for completeness their explicit expressions are presented in the numerical analysis section. The measure $\mathcal{D}\alpha_i$ is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g). \tag{15}$$

Note that, in definition of photon DA's with leading twist there are terms proportional to the strange quark mass. In our calculations, we neglect these terms. Using the expressions of the light and heavy full propagators and the photon DA's and separating the coefficient of the structure $g_{\mu\nu} \not{p} \not{q}$, the expression of the correlation function from QCD side is obtained. Separating the coefficient of the same structure from phenomenological part and equating these representations of the correlator, sum rules for the magnetic moments of the $J^P = 3/2^+$ heavy baryons is obtained. In order to suppress the contribution of higher states and continuum, Borel transformation with respect to the variables $p_2^2 = p^2$ and $p_1^2 = (p + q)^2$ is applied.

Before presenting the sum rules for the magnetic moments, few words about the relations between the correlation functions are in order. Note that, the coefficient of any structure in the correlation function can be written in the form

$$\Pi(q_1, q_2, Q) = e_{q_1} \Pi_1(q_1, q_2, Q) + e_{q_2} \Pi'_1(q_1, q_2, Q) + e_Q \Pi_2(q_1, q_2, Q), \tag{16}$$

where Π_1 corresponds to the emission of the photon from the quark q_1 , Π'_1 to emission from q_2 and Π_2 to emission from the heavy quark Q . Note that the functions $\Pi_i(q_1, q_2, q_3)$ ($i = 1, 2$) do not depend on the quark charges in the limit where we neglect electromagnetic corrections.

From the explicit form of the interpolating current it follows that it is symmetric under the exchange of the light quarks, i.e. $q_1 \longleftrightarrow q_2$, and for this reason $\Pi_1(q_1, q_2, Q) = \Pi'_1(q_2, q_1, Q)$.

Due to the symmetries between the interpolating currents, any interpolating current can be obtained from the interpolating current for $\Sigma_b^{*0(+)}$. This also leads to relations between their correlation functions. In terms of the defined $\Pi_1(q_1, q_2, Q)$ and $\Pi_2(q_1, q_2, Q)$, all the correlation functions can be written as:

$$\begin{aligned}
 \Pi^{\Sigma_b^{*0}} &= e_u \Pi_1(u, d, b) + e_d \Pi_1(d, u, b) + e_b \Pi_2(u, d, b), \\
 \Pi^{\Sigma_b^{*+}} &= \Pi^{\Sigma_b^{*0}}(d \rightarrow u) = 2e_u \Pi_1(u, u, b) + e_b \Pi_2(u, u, b), \\
 \Pi^{\Sigma_b^{*-}} &= \Pi^{\Sigma_b^{*0}}(u \rightarrow d) = 2e_d \Pi_1(d, d, b) + e_b \Pi_2(d, d, b), \\
 \Pi^{\Xi_b^{*0}} &= \Pi^{\Sigma_b^{*0}}(d \rightarrow s) = e_u \Pi_1(u, s, b) + e_s \Pi_1(s, u, b) + e_b \Pi_2(u, s, b), \\
 \Pi^{\Xi_b^{*-}} &= \Pi^{\Sigma_b^{*0}}(u \rightarrow s) = e_s \Pi_1(s, d, b) + e_d \Pi_1(d, s, b) + e_b \Pi_2(s, d, b), \\
 \Pi^{\Omega_b^{*-}} &= \Pi^{\Sigma_b^{*0}}(u \rightarrow s, d \rightarrow s) = 2e_s \Pi_1(s, s, b) + e_b \Pi_2(s, s, b), \tag{17}
 \end{aligned}$$

and the relations for the charmed baryons can simply be obtained from Eq. (17), by the replacement $b \rightarrow c$. Note that in the $SU(3)_f$ limit, the correlation functions can be expressed as:

$$\begin{aligned}
 \Pi^{\Sigma_b^{*0}} &= \frac{1}{3}\Pi_1(b) - \frac{1}{3}\Pi_2(b), \\
 \Pi^{\Sigma_b^{*+}} &= \frac{4}{3}\Pi_1(b) - \frac{1}{3}\Pi_2(b), \\
 \Pi^{\Sigma_b^{*-}} &= -\frac{2}{3}\Pi_1(b) - \frac{1}{3}\Pi_2(b), \\
 \Pi^{\Xi_b^{*0}} &= \frac{1}{3}\Pi_1(b) - \frac{1}{3}\Pi_2(b), \\
 \Pi^{\Xi_b^{*-}} &= -\frac{2}{3}\Pi_1(b) - \frac{1}{3}\Pi_2(b), \\
 \Pi^{\Omega_b^{*-}} &= -\frac{2}{3}\Pi_1(b) - \frac{1}{3}\Pi_2(b),
 \end{aligned} \tag{18}$$

where, for simplicity, we have used the short hand notation $\Pi_i(q_1, q_2, Q) \equiv \Pi_i(Q)$ ($i = 1, 2$) since in the assumed $SU(3)_f$ limit, all the light quarks have the same mass, condensates, etc.

From Eq. (18) follows the well known $SU(3)_f$ relations between the correlation functions, and hence the magnetic moments (part of these relations were derived in [23]):

$$\begin{aligned}
 \Pi^{\Sigma_b^{*+}} + \Pi^{\Sigma_b^{*-}} &= 2\Pi^{\Sigma_b^{*0}}, \\
 \Pi^{\Sigma_b^{*-}} &= \Pi^{\Xi_b^{*-}} = \Pi^{\Omega_b^{*-}}, \\
 \Pi^{\Sigma_b^{*+}} + \Pi^{\Omega_b^{*-}} &= 2\Pi^{\Xi_b^{*0}}, \\
 \Pi^{\Sigma_b^{*+}} + 2\Pi^{\Xi_b^{*-}} &= \Pi^{\Sigma_b^{*-}} + 2\Pi^{\Xi_b^{*0}}, \\
 \Pi^{\Sigma_b^{*0}} &= \Pi^{\Xi_b^{*0}},
 \end{aligned} \tag{19}$$

and the corresponding expressions for the charmed baryons. Any deviation from these relations in the magnetic moments is a sign of $SU(3)_f$ violation. Note that the first relation in Eq. (19) is a direct consequence of the isospin subgroup of $SU(3)_f$. Since in this work, we set $m_u = m_d$ and $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle$, i.e. we assume isospin symmetry, our results satisfy this relation exactly. And hence, in the following, we will not show numerical results for the magnetic moments of $\Sigma_{b(c)}^{*0(+)}$.

Once the explicit forms of the functions $\Pi_i(q_1, q_2, Q)$ ($i = 1, 2$) are known, the sum rules for the magnetic moments can be obtained using Eq. (17) and

$$-\lambda_B^2 \frac{\mu_{BQ}}{3} e^{-\frac{m_{BQ}^2}{M^2}} = A^2 \Pi^{BQ}. \tag{20}$$

The functions $\Pi_i(q_1, q_2, Q)$ can be written as:

$$\Pi_i = \int_{m_Q^2}^{s_0} e^{-\frac{s}{M^2}} \rho_i(s) ds + e^{-\frac{m_Q^2}{M^2}} \Gamma_i, \tag{21}$$

where

$$\begin{aligned}
 \rho_1(s) = & \langle q_1 q_1 \rangle \langle q_2 q_2 \rangle \left[-\frac{1}{6} \chi_i \varphi_\gamma(u_0) \right] \\
 & + m_0^2 \langle q_2 q_2 \rangle \left[\left\{ -\frac{m_Q^2}{16\pi^2 M^4} - \frac{19}{288\pi^2 M^2} \right\} m_{q_1} \ln\left(\frac{s - m_Q^2}{\Lambda^2}\right) \right. \\
 & + \frac{1}{288\pi^2 m_Q^2} \left\{ 8(1 + \psi_{11})m_Q - [-9 + 4\psi_{02} + 5\psi_{10} + 8\psi_{12} - 9\psi_{21} + 8\psi_{22} \right. \\
 & + 4\gamma_E(23 + \psi_{02} + 2\psi_{12} + \psi_{22}) + 4\psi_{32} \} m_{q_1} - 6(-1 + 2\psi_{12} + \psi_{22})m_{q_2} \\
 & + 6(16 + 2\psi_{02} - 18\psi_{03} - 50\psi_{10} + 9\psi_{21} + 93\psi_{23} - 4\psi_{32} + 120\psi_{33} + 45\psi_{43}) \\
 & \left. \times m_{q_1} \ln\left(\frac{s - m_Q^2}{\Lambda^2}\right) \right\} \left. \right] \\
 & + \langle q_2 q_2 \rangle \frac{1}{16\pi^2} \left[4(\psi_{10} - \psi_{21})m_Q - \{-1 + 4\psi_{01} - \psi_{02} + 2\psi_{11} + \psi_{22} \right. \\
 & + 2\gamma_E(1 + \psi_{02} + 2\psi_{12} + \psi_{22}) \} m_{q_1} - (1 + \psi_{02})m_{q_2} + 2m_{q_1} \left\{ \ln\left(\frac{\Lambda^2}{m_Q^2}\right) \right. \\
 & \left. + (2 - \psi_{02} - 2\psi_{10} + 3\psi_{22} + 2\psi_{32}) \ln\left(\frac{s - m_Q^2}{\Lambda^2}\right) + \ln\left(\frac{m_Q^2(s - m_Q^2)}{\Lambda^2 s}\right) \right\} \left. \right] \\
 & + \langle q_1 q_1 \rangle \frac{1}{96\pi^2} \left[-3\{(\psi_{10} - \psi_{21})m_Q + m_{q_2}\} \mathbb{A}(u_0) \right. \\
 & + 2\left(\{-1 + 4\psi_{01} - \psi_{02} + 2\psi_{11} + \psi_{22} + 2\gamma_E(1 + \psi_{02} + 2\psi_{12} + \psi_{22}) \} \right. \\
 & \times m_{q_2}(\eta_1 - 3\eta_2) + 2(\psi_{21} - \psi_{10})\{m_Q(\eta_1 + 3\eta_2) + 2m_{q_2}\eta_1 \\
 & - (m_Q + q_2)(\eta_4 - \eta_3)\} - 2(\eta_1 - 3\eta_2) \left\{ m_Q \ln\left(\frac{s}{m_Q^2}\right) + m_{q_2} \left\{ \ln\left(\frac{\Lambda^2}{m_Q^2}\right) \right. \right. \\
 & \left. \left. + (2 - \psi_{02} - 2\psi_{10} + 3\psi_{22} + 2\psi_{32}) \ln\left(\frac{s - m_Q^2}{\Lambda^2}\right) + \ln\left(\frac{m_Q^2(s - m_Q^2)}{\Lambda^2 s}\right) \right\} \right\} \left. \right] \\
 & + 3m_Q^2 \chi_i \left\{ 2m_Q \psi_{10} - (m_Q - m_{q_2})(\psi_{20} - \psi_{31}) + 2m_Q \ln\left(\frac{m_Q^2}{s}\right) \right\} \varphi_\gamma(u_0) \left. \right] \\
 & + \frac{m_Q^3}{256\pi^4} \left[m_Q \left(12\psi_{10} - 6\psi_{20} + 4\psi_{30} - 5\psi_{42} - 4\psi_{52} + m_{q_1}(18 - 18\psi_{01} \right. \right. \\
 & + 114\psi_{10} - 66\psi_{20} + 12\psi_{31}) - 24m_{q_2}(2\psi_{10} - \psi_{20} + \psi_{31}) \\
 & - 12 \left\{ 6 + \psi_{10} + 12 \ln\left(\frac{\Lambda^2}{m_Q^2}\right) \right\} m_{q_1} \ln\left(\frac{\Lambda^2}{m_Q^2}\right) - 12m_Q \ln\left(\frac{s}{m_Q^2}\right) \\
 & + 12 \left(\left\{ 2 + \psi_{01} - \ln\left(\frac{\Lambda^2}{s}\right) \right\} m_{q_1} \ln\left(\frac{\Lambda^2}{s}\right) \right. \\
 & \left. + \ln\left(\frac{m_Q^2}{s}\right) \left\{ 2 \left(1 - 4 \ln\left(\frac{s - m_Q^2}{\Lambda^2}\right) \right) m_{q_1} - 4m_{q_2} \right\} \right.
 \end{aligned}$$

$$\begin{aligned}
& + m_{q_1} \left\{ \left(1 + \psi_{01} - 7\psi_{10} + 3\psi_{20} + 2 \ln \left(\frac{s}{m_Q^2} \right) \right) \ln \left(\frac{s - m_Q^2}{\Lambda^2} \right) \right. \\
& - \left. (3 + \psi_{10}) \ln \left(\frac{m_Q^2 (s - m_Q^2)}{\Lambda^2 s} \right) + \psi_{01} \ln \left(\frac{s(s - m_Q^2)}{\Lambda^2 m_Q^2} \right) \right\} \\
& - 24 \left[4 \text{Li}_2 \left(1 - \frac{m_Q^2}{s} \right) - \text{Li}_2 \left(1 - \frac{s}{m_Q^2} \right) \right] m_{q_1} \\
& + \frac{f_{3\gamma} m_Q^2}{48\pi^2} \left[-2\eta' \left\{ \psi_{10} + \ln \left(\frac{m_Q^2}{s} \right) \right\} + (3\psi_{31} + 4\psi_{32} + 3\psi_{42}) \psi^a(u_0) \right], \quad (22)
\end{aligned}$$

$$\begin{aligned}
\rho_2(s) = & m_0^2 \langle q_1 q_1 \rangle \left[-\frac{5(1 + \psi_{11})}{288\pi^2 m_Q} + \frac{m_{q_2}}{18\pi^2 M^2} \ln \left(\frac{s - m_Q^2}{\Lambda^2} \right) - \frac{m_{q_2}}{36\pi^2 m_Q^2} \left\{ \psi_{02} - \psi_{10} \right. \right. \\
& + 2(\psi_{12} + \psi_{22}) + \gamma_E (-4 + \psi_{02} + 2\psi_{12} + \psi_{22}) \\
& + \left. \left. \psi_{32} + 3(1 - \psi_{02} - 2\psi_{10} + 3\psi_{22} + 2\psi_{32}) \ln \left(\frac{s - m_Q^2}{\Lambda^2} \right) \right\} \right] \\
& + m_0^2 \langle q_2 q_2 \rangle \left[-\frac{5(1 + \psi_{11})}{288\pi^2 m_Q} + \frac{m_{q_1}}{18\pi^2 M^2} \ln \left(\frac{s - m_Q^2}{\Lambda^2} \right) \right. \\
& - \frac{m_{q_1}}{36\pi^2 m_Q^2} \left\{ \psi_{02} - \psi_{10} + 2(\psi_{12} + \psi_{22}) + \gamma_E (-4 + \psi_{02} + 2\psi_{12} + \psi_{22}) + \psi_{32} \right. \\
& + \left. \left. 3(1 - \psi_{02} - 2\psi_{10} + 3\psi_{22} + 2\psi_{32}) \ln \left(\frac{s - m_Q^2}{\Lambda^2} \right) \right\} \right] \\
& + \langle q_1 q_1 \rangle \frac{1}{16\pi^2} \left[(\psi_{02} - 1)m_{q_1} - 2(\psi_{10} - \psi_{21})(m_Q - 2m_{q_2}) + 2m_Q \ln \left(\frac{s}{m_Q^2} \right) \right] \\
& + \langle q_2 q_2 \rangle \frac{1}{16\pi^2} \left[(\psi_{02} - 1)m_{q_2} - 2(\psi_{10} - \psi_{21})(m_Q - 2m_{q_1}) + 2m_Q \ln \left(\frac{s}{m_Q^2} \right) \right] \\
& + \frac{m_Q^3}{768\pi^4} \left[(-4\psi_{30} + 5\psi_{42} + 4\psi_{52})m_Q + 18\psi_{31}(m_{q_1} + m_{q_2}) \right. \\
& - 12\psi_{10} \{7m_Q - 9(m_{q_1} + m_{q_2})\} - 6\psi_{20} \{m_Q + 3(m_{q_1} + m_{q_2})\} \\
& - 24 \{2(1 + \psi_{10})m_Q - 3(m_{q_1} + m_{q_2})\} \ln \left(\frac{m_Q^2}{s} \right) \\
& \left. + 36 \{m_Q - (1 + \psi_{10})(m_{q_1} + m_{q_2})\} \ln \left(\frac{s}{m_Q^2} \right) \right], \quad (23)
\end{aligned}$$

$$\begin{aligned}
\Gamma_1 = & m_0^2 \langle q_1 q_1 \rangle \langle q_2 q_2 \rangle \left[\frac{m_Q^5 m_{q_2} \mathbb{A}(u_0)}{288M^8} - \frac{m_Q^3}{864M^6} \{ -4m_{q_2}(\eta_1 - 3\eta_2) \right. \\
& + (9m_Q + 10m_{q_2})\mathbb{A}(u_0) \} \\
& - \left. \frac{m_Q}{2164M^4} \{ m_{q_2}(\eta_1 - 3\eta_2) + 3m_Q(2\eta_1 + \eta_3 - 2\eta_4) \} \right]
\end{aligned}$$

$$\begin{aligned}
 & - (m_Q + m_{q_2})\mathbb{A}(u_0) + 3m_Q^2 m_{q_2} \chi_i \varphi_\gamma(u_0) \} \\
 & - \frac{m_Q}{216M^2} (9m_Q + 4m_{q_2}) \chi_i \varphi_\gamma(u_0) + \frac{5}{216} \chi_i \varphi_\gamma(u_0) \Big] \\
 & + \langle q_1 q_1 \rangle \langle q_2 q_2 \rangle \left[-\frac{m_Q^3 m_{q_2} \mathbb{A}(u_0)}{48M^4} + \frac{m_Q}{72M^2} \{ 2m_{q_2} (-\eta_1 + 3\eta_2) + 3m_Q \mathbb{A}(u_0) \} \right. \\
 & \left. + \frac{1}{72} \{ 2(4\eta_1 + 2\eta_3 - 4\eta_4) + 3\mathbb{A}(u_0) + 6m_Q m_{q_2} \chi_i \varphi_\gamma(u_0) \} \right] \\
 & + m_0^2 \langle q_2 q_2 \rangle \left[\frac{\gamma_E m_{q_1} M^2}{3\pi^2 m_Q^2} + \frac{m_Q^2}{96\pi^2 M^2} \left\{ 2m_{q_2} - 3m_{q_1} \ln\left(\frac{\Lambda^2}{m_Q^2}\right) \right\} \right. \\
 & \left. - \frac{1}{288\pi^2} \left\{ 18m_Q + 9m_{q_1} + 14m_{q_1} \left(2\gamma_E + \ln\left(\frac{\Lambda^2}{m_Q^2}\right) \right) + 4m_{q_2} \right\} \right. \\
 & \left. + \frac{f_{3\gamma} m_Q^2 m_{q_2} (\eta' + 4\psi^a(u_0))}{216M^4} \right] \\
 & + \langle q_2 q_2 \rangle \left[\frac{\gamma_E M^2 m_{q_1}}{4\pi^2} - \frac{1}{36} f_{3\gamma} m_{q_2} (\eta' + 3\psi^a(u_0)) \right] \\
 & - \langle q_1 q_1 \rangle \left[\frac{\gamma_E M^2 m_{q_2}}{12\pi^2} (\eta_1 - 3\eta_2) \right], \tag{24}
 \end{aligned}$$

$$\begin{aligned}
 \Gamma_2 = & m_0^2 \langle q_1 q_1 \rangle \langle q_2 q_2 \rangle \left[-\frac{5m_Q^3 (m_{q_1} + m_{q_2})}{144M^6} + \frac{m_Q (24m_Q + 5(m_{q_1} + m_{q_2}))}{144M^4} \right] \\
 & + \langle q_1 q_1 \rangle \langle q_2 q_2 \rangle \left[-\frac{1}{3} + \frac{m_Q (m_{q_1} + m_{q_2})}{12M^2} \right] \\
 & + m_0^2 \langle q_1 q_1 \rangle \left[-\frac{\gamma_E m_{q_2} M^2}{12\pi^2 m_Q^2} + \frac{1}{144\pi^2} \left\{ \left(-9 + 8\gamma_E + 4 \ln\left(\frac{\Lambda^2}{m_Q^2}\right) \right) m_{q_2} + m_{q_1} \right\} \right] \\
 & + m_0^2 \langle q_2 q_2 \rangle \left[-\frac{\gamma_E m_{q_1} M^2}{12\pi^2 m_Q^2} + \frac{1}{144\pi^2} \left\{ \left(-9 + 8\gamma_E + 4 \ln\left(\frac{\Lambda^2}{m_Q^2}\right) \right) m_{q_1} + m_{q_2} \right\} \right], \tag{25}
 \end{aligned}$$

where $\text{Li}_2(x)$ is the dilogarithm function. The functions also entering Eqs. (22)–(25) are given as

$$\begin{aligned}
 \eta_i &= \int \mathcal{D}\alpha_i \int_0^1 dv f_i(\alpha_i) \delta(\alpha_q + v\alpha_g - u_0), \\
 \eta' &= \int \mathcal{D}\alpha_i \int_0^1 dv \mathcal{V}(\alpha_i) \delta'(\alpha_q + v\alpha_g - u_0), \\
 \psi_{nm} &= \frac{(s - m_Q^2)^n}{s^m (m_Q^2)^{n-m}}, \tag{26}
 \end{aligned}$$

and $f_1(\alpha_i) = \mathcal{S}(\alpha_i)$, $f_2(\alpha_i) = \tilde{\mathcal{S}}(\alpha_i)$, $f_3(\alpha_i) = \mathcal{T}_4(\alpha_i)$ and $f_4(\alpha_i) = v\mathcal{T}_4(\alpha_i)$ are the photon distribution amplitudes. Note that in the above equations, the Borel parameter M^2 is defined as $M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$ and $u_0 = \frac{M_1^2}{M_1^2 + M_2^2}$. Since the masses of the initial and final baryons are the same, we will set $M_1^2 = M_2^2$ and $u_0 = 1/2$. The contributions of the terms $\sim \langle G^2 \rangle$ are also calculated, but their numerical values are very small and therefore for customary in the expressions these terms are omitted.

For calculation of the magnetic moments of the considered baryons, their residues λ_B as well as their masses are needed (see Eq. (20)). Note that many of the considered baryons are not discovered yet in the experiments. The residue is determined from two point sum rules. For the interpolating current given in Eq. (2), we obtain the following result for λ_B^2 :

$$\lambda_B^2 e^{-\frac{m_B^2}{M^2}} = A^2 [\Pi' + \Pi'(q_1 \longleftrightarrow q_2)], \quad (27)$$

where

$$\begin{aligned} \Pi' = & \int_{m_Q^2}^{s_0} ds e^{-s/M^2} \left\{ m_0^2 \langle q_1 q_1 \rangle \left[\frac{(m_{q_1} - 6m_Q)(\psi_{22} + 2\psi_{12} - 1)}{192m_Q^2 \pi^2} \right] \right. \\ & - \langle q_1 q_1 \rangle \left[\frac{2(\psi_{02} + 2\psi_{10} - 2\psi_{21} - 1)m_Q + (\psi_{02} - 1)(3m_{q_1} - 2m_{q_2})}{32\pi^2} \right] \\ & - \frac{m_Q^3}{512\pi^4} \left[-8(3\psi_{31} + 2\psi_{32})m_{q_1} \right. \\ & \left. + 3 \left\{ \left(2\psi_{30} - 4\psi_{41} + \frac{5}{2}\psi_{42} + 2\psi_{52} \right) m_Q - 4\psi_{42} m_{q_1} \right\} \right. \\ & \left. + 6(2\psi_{10} - \psi_{20}) \left(\frac{3}{2}m_Q - 2m_{q_1} \right) - 6(4m_{q_1} - 3m_Q) \ln \left(\frac{m_Q^2}{s} \right) \right] \left. \right\} \\ & + e^{-m_Q^2/M^2} \left\{ m_0^2 \langle q_1 q_1 \rangle \langle q_2 q_2 \rangle \left[\frac{-5m_Q^3 m_{q_1}}{144M^6} + \frac{m_Q(m_Q + 5m_{q_1})}{24M^4} - \frac{5}{24M^2} \right] \right. \\ & \left. + \langle q_1 q_1 \rangle \langle q_2 q_2 \rangle \left[\frac{1}{12} - \frac{m_Q m_{q_1}}{12M^2} \right] + m_0^2 \langle q_1 q_1 \rangle \left[\frac{6m_{q_2} - 7m_{q_1}}{192\pi^2} \right] \right\}. \quad (28) \end{aligned}$$

The masses of the considered baryons can be determined from the sum rules. For this aim, one can get the derivative from both side of Eq. (27) with respect to $-1/M^2$ and divide the obtained result to Eq. (27), i.e.,

$$m_{B_Q}^2 = \frac{-\frac{d}{d(1/M^2)} [\Pi' + \Pi'(q_1 \longleftrightarrow q_2)]}{[\Pi' + \Pi'(q_1 \longleftrightarrow q_2)]}. \quad (29)$$

3. Numerical analysis

In this section, we perform numerical analysis for the mass and magnetic moments of the heavy flavored baryons. Firstly, we present the input parameters used in the analysis of the sum rules: $\langle \bar{u}u \rangle(1 \text{ GeV}) = \langle \bar{d}d \rangle(1 \text{ GeV}) = -(0.243)^3 \text{ GeV}^3$, $\langle \bar{s}s \rangle(1 \text{ GeV}) = 0.8 \langle \bar{u}u \rangle(1 \text{ GeV})$, $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$ [24], $\Lambda = 1 \text{ GeV}$ and $f_{3\gamma} = -0.0039 \text{ GeV}^2$ [22]. The value of

the magnetic susceptibility was obtained in various papers as $\chi(1 \text{ GeV}) = -3.15 \pm 0.3 \text{ GeV}^{-2}$ [22], $\chi(1 \text{ GeV}) = -(2.85 \pm 0.5) \text{ GeV}^{-2}$ [25] and $\chi(1 \text{ GeV}) = -4.4 \text{ GeV}^{-2}$ [26].

Before proceeding to the results for the magnetic moments, we calculate the masses of heavy flavored baryons predicted from mass sum rule. Obviously, the masses should not depend on the Borel mass parameter M^2 in a complete theory. However, in sum rules method the operator product expansion (OPE) is truncated and as a result the dependency of the predictions of physical quantities on the auxiliary parameter M^2 appears. For this reason one should look for a region of M^2 such that the predictions for the physical quantities do not vary with respect to the Borel mass parameter. This region is the so-called the “working region” and within this region the truncation is reasonable and meaningful. The upper limit of M^2 is determined from condition that the continuum and higher states contributions should be small than the total dispersion integral. The lower limit is determined by demanding that in the truncated OPE the condensate term with highest dimension remains small than sum of all terms, i.e., convergence of OPE should be under control.

The aforementioned conditions for bottom (charmed) baryons are satisfied when M^2 varies in the interval $15 \text{ GeV}^2 < M^2 < 30 \text{ GeV}^2$ ($4 \text{ GeV}^2 < M^2 < 12 \text{ GeV}^2$). In Figs. 1–6, we presented the dependence of the mass of the heavy flavored baryons on M^2 . From these figures, we see very good stability with respect to M^2 . The sum rule predictions of the mass of the heavy flavored baryons are presented in Table 2 in comparison with some theoretical predictions and experimental results. Note that the masses of the heavy flavored baryons are calculated in the framework of heavy quark effective theory (HQET) using the QCD sum rules method in [27].

After determination of the mass as well as residue of the heavy flavored baryons our next task is the calculation of the numerical values of their magnetic moments. For this aim, from sum rules for the magnetic moments it follows that the photon DA’s are needed [22]:

$$\begin{aligned} \varphi_\gamma(u) &= 6u\bar{u}(1 + \varphi_2(\mu)C_2^{\frac{3}{2}}(u - \bar{u})), \\ \psi^v(u) &= 3(3(2u - 1)^2 - 1) + \frac{3}{64}(15w_\gamma^V - 5w_\gamma^A)(3 - 30(2u - 1)^2 + 35(2u - 1)^4), \\ \psi^a(u) &= (1 - (2u - 1)^2)(5(2u - 1)^2 - 1)\frac{5}{2}\left(1 + \frac{9}{16}w_\gamma^V - \frac{3}{16}w_\gamma^A\right), \end{aligned}$$

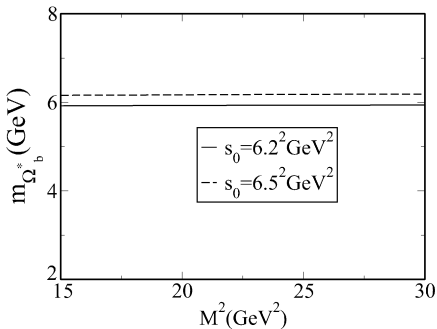


Fig. 1. The dependence of mass of the Ω_b^* on the Borel parameter M^2 for two fixed values of continuum threshold s_0 .

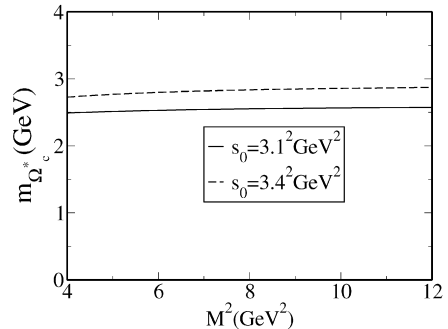


Fig. 2. The dependence of mass of the Ω_c^* on the Borel parameter M^2 for two fixed values of continuum threshold s_0 .

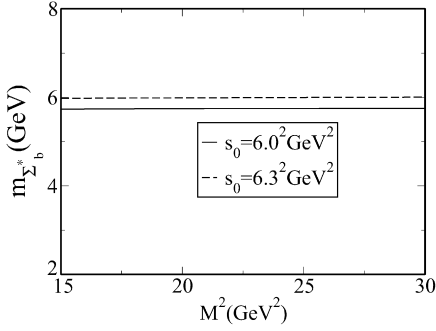
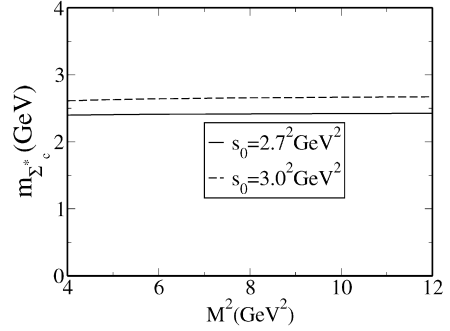
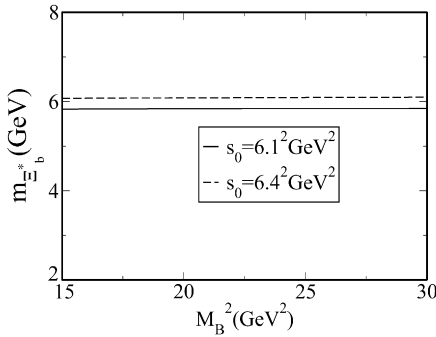
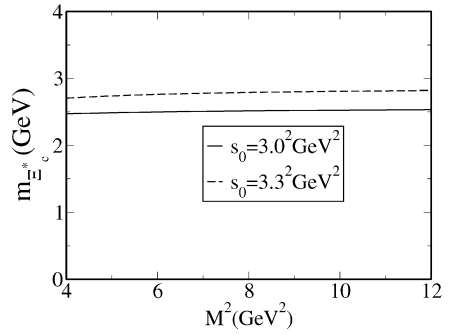
Fig. 3. The same as Fig. 1, but for Σ_b^* .Fig. 4. The same as Fig. 2, but for Σ_c^* .Fig. 5. The same as Fig. 1, but for Ξ_b^* .Fig. 6. The same as Fig. 2, but for Ξ_c^* .

Table 2

Comparison of mass of the heavy flavored baryons in GeV from present work and other approaches and with experiment.

	$m_{\Omega_b^*}$	$m_{\Omega_c^*}$	$m_{\Sigma_b^*}$	$m_{\Sigma_c^*}$	$m_{\Xi_b^*}$	$m_{\Xi_c^*}$
This work	6.08 ± 0.40	2.72 ± 0.20	5.85 ± 0.35	2.51 ± 0.15	5.97 ± 0.40	2.66 ± 0.18
[27]	$6.063^{+0.083}_{-0.082}$	$2.790^{+0.109}_{-0.105}$	$5.835^{+0.082}_{-0.077}$	$2.534^{+0.096}_{-0.081}$	$5.929^{+0.088}_{-0.079}$	$2.634^{+0.102}_{-0.094}$
[28]	6.088	2.768	5.834	2.518	5.963	2.654
[29]	–	–	5.805	2.495	–	–
[30]	6.090	2.770	5.850	2.520	5.980	2.650
[31]	–	2.768	–	2.518	–	–
[32]	6.083	2.760	5.840	–	5.966	–
[33]	6.060	2.752	5.871	2.5388	5.959	2.680
Exp [34]	–	2.770	5.836	2.520	–	2.645

$$\mathcal{A}(\alpha_i) = 360\alpha_q\alpha_{\bar{q}}\alpha_g^2 \left(1 + w_\gamma^A \frac{1}{2}(7\alpha_g - 3) \right),$$

$$\mathcal{V}(\alpha_i) = 540w_\gamma^V(\alpha_q - \alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2,$$

$$h_\gamma(u) = -10(1 + 2\kappa^+)C_2^{\frac{1}{2}}(u - \bar{u}),$$

$$\mathbb{A}(u) = 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2)\ln(u) + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})],$$

$$\begin{aligned}
 \mathcal{T}_1(\alpha_i) &= -120(3\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
 \mathcal{T}_2(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)((\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)), \\
 \mathcal{T}_3(\alpha_i) &= -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
 \mathcal{T}_4(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)((\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)), \\
 \mathcal{S}(\alpha_i) &= 30\alpha_g^2\{(\kappa + \kappa^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
 &\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}, \\
 \tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2\{(\kappa - \kappa^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
 &\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}.
 \end{aligned} \tag{30}$$

The constants appearing in the wave functions are given as [22] $\varphi_2(1 \text{ GeV}) = 0$, $w_\gamma^V = 3.8 \pm 1.8$, $w_\gamma^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$ and $\zeta_2^+ = 0$.

The sum rules for magnetic moments also contain the auxiliary parameters: Borel parameter M^2 and continuum threshold s_0 . Similar to mass sum rules, the magnetic moments should also be independent of these parameters. In the general case, the working region of M^2 and s_0 for the mass and magnetic moments should be different. To find the working region for M^2 , we proceed as follows. The upper bound is obtained requiring that the contribution of higher states and continuum should be less than the ground state contribution. The lower bound of M^2 is determined from condition that the highest power of $1/M^2$ be less than say 30% of the highest power of M^2 . These two conditions are both satisfied in the region $15 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$ and $4 \text{ GeV}^2 \leq M^2 \leq 12 \text{ GeV}^2$ for baryons containing b and c-quark, respectively. The working region for the Borel parameter for mass and magnetic moments practically coincide, but again we should stress that, this circumstance is accidental.

In Figs. 7–16, we present the dependence of the magnetic moment of heavy flavored baryons on M^2 at two fixed values of continuum threshold s_0 . From these figures, we see that the magnetic moments weakly depend on s_0 . The maximal change of results is about 10% with variation of s_0 . The magnetic moments also are practically insensitive to the variation of Borel mass parameter when it varies in the working region. We should also stress that our results practically do not change considering three values of χ which we presented at the beginning of this section. Our final results on the magnetic moments of heavy flavored baryons are presented in Table 3. For

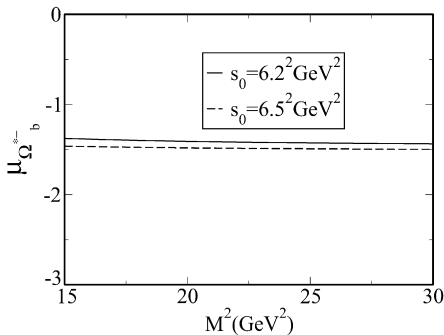


Fig. 7. The dependence of the magnetic moment of Ω_b^{*-} on Borel parameter M^2 (in units of nucleon magneton) at two fixed values of s_0 .

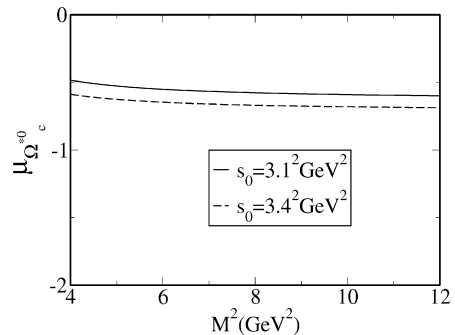


Fig. 8. The dependence of the magnetic moment of Ω_c^{*0} on Borel parameter M^2 (in units of nucleon magneton) at two fixed values of s_0 .

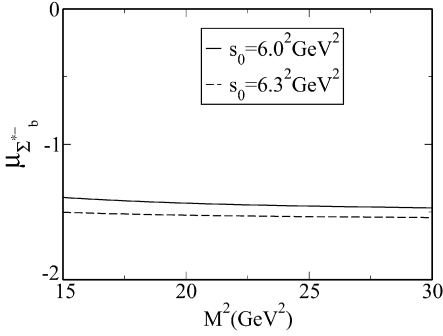


Fig. 9. The same as Fig. 7, but for Σ_b^{*-} .

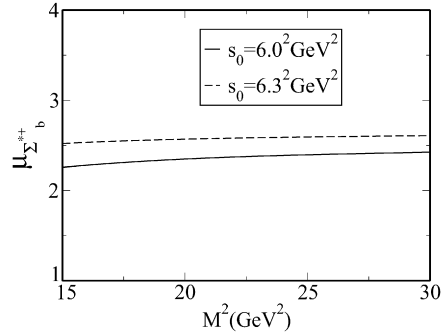


Fig. 10. The same as Fig. 7, but for Σ_b^{*+} .

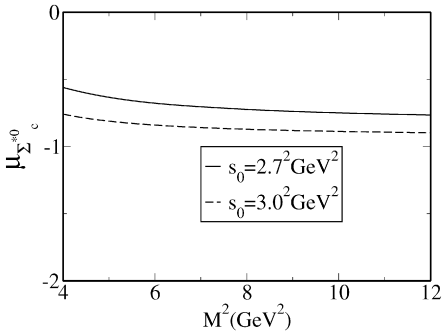


Fig. 11. The same as Fig. 8, but for Σ_c^{*0} .

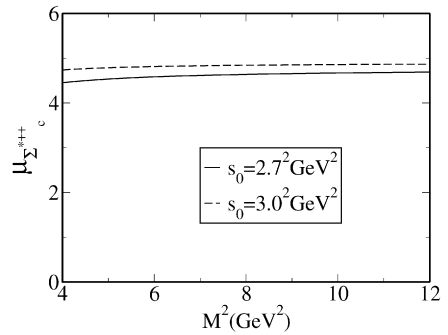


Fig. 12. The same as Fig. 8, but for Σ_c^{*++} .

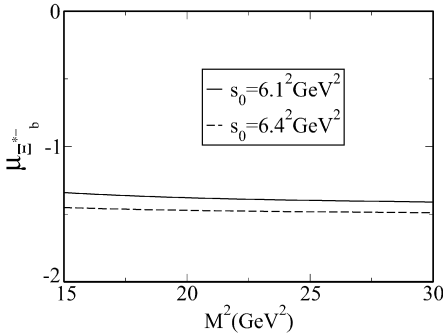


Fig. 13. The same as Fig. 7, but for Ξ_b^{*-} .

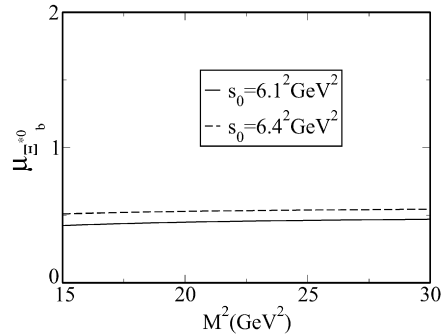


Fig. 14. The same as Fig. 7, but for Ξ_b^{*0} .

comparison, the predictions of hyper central model [35] are also presented. The quoted errors in Table 3 are due to the uncertainties in m_0^2 , variation of s_0 and M^2 as well as errors in the determination of the input parameters.

Although the $SU(3)_f$ breaking effects have been taken into account through a nonzero s -quark mass and different strange quark condensate, we predict that $SU(3)_f$ symmetry violation in the magnetic moments is very small, except the relations $\mu_{\Sigma_c^{*+}} = \mu_{\Sigma_c^{*++}}$ and $\Pi^{\Sigma_c^{*++}} + \Pi^{\Omega_c^{*0}} = 2\Pi^{\Xi_c^{*+}}$, where the $SU(3)_f$ symmetry violation is large. For the values of the magnetic moments,

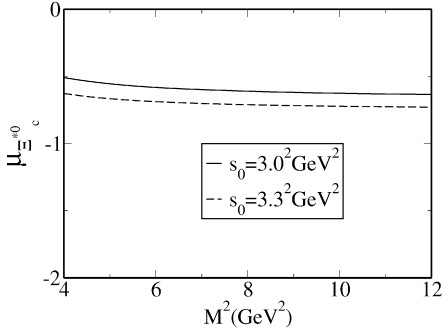


Fig. 15. The same as Fig. 8, but for Σ_c^{*0} .

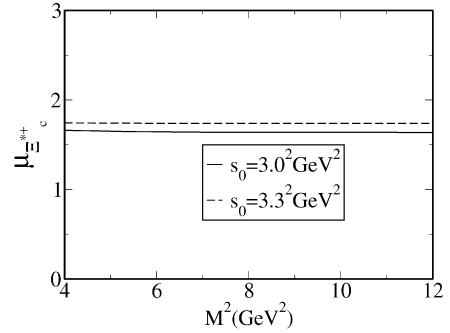


Fig. 16. The same as Fig. 8, but for Σ_c^{*+} .

Table 3

The magnetic moments of the heavy flavored baryons in units of nucleon magneton.

	Our results	Hyper central model [35]
$\mu_{\Omega_b^{*-}}$	-1.40 ± 0.35	$-1.178 - (-1.201)$
$\mu_{\Omega_c^{*0}}$	-0.62 ± 0.18	$-0.827 - (-0.867)$
$\mu_{\Sigma_b^{*-}}$	-1.50 ± 0.36	$-1.628 - (-1.657)$
$\mu_{\Sigma_b^{*0}}$	0.50 ± 0.15	$0.778 - 0.792$
$\mu_{\Sigma_b^{*+}}$	2.52 ± 0.50	$3.182 - 3.239$
$\mu_{\Sigma_c^{*0}}$	-0.81 ± 0.20	$-0.826 - (-0.850)$
$\mu_{\Sigma_c^{*+}}$	2.00 ± 0.46	$1.200 - 1.256$
$\mu_{\Sigma_c^{*++}}$	4.81 ± 1.22	$3.682 - 3.844$
$\mu_{\Xi_b^{*-}}$	-1.42 ± 0.35	$-1.048 - (-1.098)$
$\mu_{\Xi_b^{*0}}$	0.50 ± 0.15	$1.024 - 1.042$
$\mu_{\Xi_c^{*0}}$	-0.68 ± 0.18	$-0.671 - (-0.690)$
$\mu_{\Xi_c^{*+}}$	1.68 ± 0.42	$1.449 - 1.517$

our results are consistent with the results of [35] except for the $\mu_{\Omega_b^{*-}}$, $\mu_{\Xi_b^{*-}}$ and especially for the $\mu_{\Sigma_c^{*+}}$, $\mu_{\Xi_b^{*0}}$ which we see a big discrepancy between two predictions.

In summary, inspired by recent experimental discovery of the heavy and flavored baryons [1–3], the mass and magnetic moments of these baryons with $J^P = 3/2^+$ are calculated within the QCD sum rules. Our results on the masses are consistent with the experimental data as well as predictions of other approaches. Our results on the masses of the Ω_b^* , and Ξ_b^* can be tested in experiments which will be held in the near future. The predictions on the magnetic moments also can be verified in the future experiments.

Acknowledgements

Two of the authors (K.A. and A.O.), would like to thank TUBITAK, Turkish Scientific and Research Council, for their partial financial support through the project number 106T333 and also through the PhD scholarship program (K.A.).

References

- [1] T. Aaltonen, et al., CDF Collaboration, Phys. Rev. Lett. 99 (2007) 202001.
- [2] V. Abazov, et al., DO Collaboration, Phys. Rev. Lett. 99 (2007) 052001.
- [3] T. Aaltonen, et al., CDF Collaboration, Phys. Rev. Lett. 99 (2007) 052002.
- [4] B. Aubert, et al., BaBar Collaboration, Phys. Rev. Lett. 97 (2006) 232001.
- [5] A.L. Choudhury, V. Joshi, Phys. Rev. D 13 (1976) 3115.
- [6] D.B. Lichtenberg, Phys. Rev. D 15 (1977) 345.
- [7] R.J. Johnson, M. Shah-Jahan, Phys. Rev. D 15 (1977) 1400.
- [8] S.K. Bose, L.P. Singh, Phys. Rev. D 22 (1980) 773.
- [9] L.Y. Glozman, D.O. Riska, Nucl. Phys. A 603 (1996) 326.
- [10] B. Julia-Diaz, D.O. Riska, Nucl. Phys. A 739 (2004) 69.
- [11] S. Scholl, H. Weigel, Nucl. Phys. A 735 (2004) 163.
- [12] Y.S. Oh, B.Y. Park, Mod. Phys. Lett. A 11 (1996) 653.
- [13] A. Faessler, et al., Phys. Rev. D 73 (2006) 094013.
- [14] S.L. Zhu, W.Y. Hwang, Z.S. Yang, Phys. Rev. D 56 (1997) 7273.
- [15] T.M. Aliev, A. Ozpineci, M. Savci, Phys. Rev. D 65 (2002) 096004.
- [16] T.M. Aliev, A. Ozpineci, M. Savci, Phys. Rev. D 65 (2002) 056008.
- [17] T.M. Aliev, K. Azizi, A. Ozpineci, Phys. Rev. D 77 (2008) 114006.
- [18] P. Colangelo, A. Khodjamirian, in: M. Shifman (Ed.), At the Frontier of Particle Physics/Handbook of QCD, vol. 3, World Scientific, Singapore, 2001, p. 1495;
V.M. Braun, hep-ph/9801122;
I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, Nucl. Phys. B 312 (1989) 509.
- [19] I.I. Balitsky, V.M. Braun, Nucl. Phys. B 311 (1989) 541.
- [20] V.M. Braun, I.E. Filyanov, Z. Phys. C 48 (1990) 239.
- [21] K.G. Chetyrkin, A. Khodjamirian, A.A. Pivovarov, Phys. Lett. B 661 (2008) 250;
I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, Nucl. Phys. B 312 (1989) 509.
- [22] P. Ball, V.M. Braun, N. Kivel, Nucl. Phys. B 649 (2003) 263.
- [23] M.C. Banuls, I. Scimemi, J. Bernabeu, V. Gimenez, A. Pich, Phys. Rev. D 61 (2000) 074007.
- [24] V.M. Belyaev, B.L. Ioffe, JETP 56 (1982) 493.
- [25] J. Rohrwild, JHEP 0709 (2007) 073.
- [26] V.M. Belyaev, I.I. Kogan, Yad. Fiz. 40 (1984) 1035.
- [27] X. Liu, H.X. Chen, Y.R. Liu, A. Hosaka, S.L. Zhu, arXiv: 0710.0123 [hep-ph].
- [28] D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D 72 (2005) 034026.
- [29] S. Capstick, N. Isgur, Phys. Rev. D 34 (1986) 2809.
- [30] R. Ronaglia, D.B. Lichtenberg, E. Predazzi, Phys. Rev. D 52 (1995) 1722.
- [31] M.J. Savage, Phys. Lett. B 359 (1995) 189.
- [32] E. Jenkins, Phys. Rev. D 54 (1996) 4515;
E. Jenkins, Phys. Rev. D 55 (1997) R10.
- [33] N. Mathur, R. Lewis, R.M. Woloshyn, Phys. Rev. D 66 (2002) 014502.
- [34] C. Amsler, et al., Particle Data Group Collaboration, Phys. Lett. B 667 (2008) 1.
- [35] B. Patel, A.K. Rai, P.C. Vinodkumar, J. Phys. G 35 (2008) 065001.