Physics Letters B 748 (2015) 414–421

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Four dimensional supersymmetric theories in presence of a boundary

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ARTICLE INFO

Article history:
Received 4 May 2015
Received in revised form 25 June 2015
Accepted 14 July 2015
Available online 17 July 2015
Editor: M. Cvetič

ABSTRACT

In this paper, we study $\mathcal{N}=1$ supersymmetric theories in four dimensions in presence of a boundary. We demonstrate that it is possible to preserve half the supersymmetry of the original theory by suitably modifying it in presence of a boundary. This is done by adding new boundary terms to the original action, such that the supersymmetric variation of the new terms exactly cancels the boundary terms generated by the supersymmetric transformation of the original bulk action. We also analyze the boundary projections of such supercharges used in such a theory. We study super-Yang–Mills theories in presence of a boundary using these results. Finally, we study the Born–Infeld action in presence of a boundary. We analyze the boundary effects for the Born–Infeld action coupled to a background dilaton and an axion field. We also analyze the boundary effects for a non-abelian Born–Infeld action. We explicitly construct the actions for these systems in presence of a boundary. This action preserves half of the original supersymmetry.

1. Introduction

The action for most renormalizable quantum field theories in four dimensions, including supersymmetric theories, is at most quadratic in derivatives. So, variation of the action for such theories produces a bulk term as well as a total derivative term. For manifolds without a boundary, the total derivative terms vanish due to the absence of a boundary. However, in presence of a boundary, such total derivatives give rise to boundary contributions. The presence of a boundary breaks the translational invariance of the theory, and this in turn breaks supersymmetry. In fact, supersymmetric variation of a supersymmetric action is known to be a total derivative. Thus, in presence of a boundary, the supersymmetric variation of an action, which is supersymmetric in flat space, produces a nonvanishing boundary term, this in turn breaks supersymmetry.

It is possible to restore supersymmetry on-shell by imposing some boundary conditions [1,2]. There are various constraints generated from supersymmetry on the possible boundary conditions [3–7]. However, this does not resolve the problem with the surface terms off-shell, since these boundary conditions are only imposed on the on-shell fields, and the supersymmetry is still broken off-shell. Since most supersymmetric theories are quantized using path integral formalism which uses off-shell fields, it is important to try to construct actions which preserve some supersymmetry off-shell.

Here we show that it is possible to construct an action which preserves half the original supersymmetry off-shell. This can be done by modifying the original action through the addition of boundary terms. The new boundary terms added to the original action exactly cancel the boundary contribution generated from the supersymmetric variation of the original bulk action. This procedure has been applied in three dimensions for $\mathcal{N}=1$ supersymmetric theories [8]. The results thus obtained have been used for studying a system of multiple M2-branes ending on M5-brane [9–12]. As the gauge sector for the action of multiple M2-branes is comprised of Chern–Simons theories, and the gauge transformation of Chern–Simons theories in presence of a boundary also generates a boundary term, new boundary degrees of freedom had to be introduced on the boundary of the M2-branes. The gauge transformation of the action for these new boundary degrees of freedom exactly cancels the boundary contribution generated from the gauge transformation of the bulk action. A system of M2-branes intersecting with M5-branes has also been analyzed in the supergravity regime using a fuzzy funnel solution [13–18].

Apart from the M2-branes, the supersymmetric theory in presence of a boundary has also been used for analyzing non-

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http://dx.doi.org/10.1016/j.physletb.2015.07.024
0370-2693/

anticommutativity in presence of a boundary for a three dimensional theory with $\mathcal{N}=2$ supersymmetry [19]. By suitably combining the boundary effects with non-anticommutativity, a three dimensional theory with $\mathcal{N}=1/2$ supersymmetry has been constructed. In fact, the coupling of a three dimensional super-Yang–Mills theory to background flux has been studied on a manifold with a boundary [20]. The BRST symmetry for this system has also been analyzed. However, all this work has been done in three dimensions.

It may be noted that just like M2-branes can end on M5-branes, D3-branes can also end on other objects in string theory. Such systems can be studied using fuzzy funnel [21,22]. In fact, a system of D3-branes ending on other D3-branes has been analyzed using fuzzy funnel [23]. The fuzzy funnel has also been used to describe a system of D3-branes ending on D5-branes [24], and a system of D3-branes ending on D7-branes [25]. It would be interesting to apply to extend first develop a formalism for analyzing four dimensional supersymmetry in presence of a boundaries, and then using such a formalism for studying D3-branes ending on various objects in string theory. As the four dimensional super-Yang–Mills theory can be thought as a low energy limit of D3-brane action, we will analyze a four dimensional super-Yang–Mills theory in presence of a boundary. The construction of four dimensional supersymmetric theories in presence of a boundary can find several other applications, and we are going to mention some of them in conclusion section of this paper.

The remaining paper is organized as follows. In Section 2, we discuss the general formalism for analyzing $\mathcal{N}=1$ superfields in presence of a boundary, and construct a supersymmetric Lagrangian which preserves half of the original supersymmetry in presence of a boundary. In Section 3, we discuss the transformation of bulk and boundary superfields and supercharges in presence of a boundary. In Section 4, we apply this formalism to super-Yang–Mills theory. In Section 5, we will apply this formalism to Born–Infeld action. Finally, in Section 6, we summarize our results and discuss some possible applications of the results of this paper.

2. Boundary superfields

Let us start with a four dimensional theory in $\mathcal{N}=1$ superspace. This superspace can be parameterized by two supercharges, $Q_a = -i\partial_a - (\gamma^\mu \partial_\mu \bar\theta)_a$, and $\bar Q_a = i\bar\partial_a + (\gamma^\mu \partial_\mu \theta)_a$, which satisfy

$$\{Q_a, Q_b\} = 0, \qquad \{\bar Q_a, \bar Q_b\} = 0,$$
$$\{Q_a, \bar Q_b\} = -2i(\gamma^\mu \partial_\mu)_{ab}. \tag{1}$$

It is also possible to define superderivatives which commute with these generators of $\mathcal{N}=1$ supersymmetry, $\{D_a, \bar Q_b\} = \{D_a, \bar Q_b\} = \{\bar D_a, \bar Q_b\} = \{\bar D_a, Q_b\} = 0$. These superderivatives can be represented as $D_a = \partial_a + i(\gamma^\mu \partial_\mu \bar\theta)_a$, and $\bar D_a = \bar\partial_a + i(\gamma^\mu \partial_\mu \theta)_a$, and satisfy

$$\{D_a, D_b\} = 0, \qquad \{\bar D_a, \bar D_b\} = 0,$$
$$\{D_a, \bar D_b\} = 2i(\gamma^\mu \partial_\mu)_{ab}. \tag{2}$$

Now we can write the Lagrangian for a supersymmetric theory with $\mathcal{N}=1$ supersymmetry as

$$\mathcal{L} = D^2 \bar D^2 [\Phi(\theta, \bar\theta)]_{\theta=\bar\theta=0}. \tag{3}$$

It may be noted that a linear combination of θ_a and $\bar\theta_a$ can be used to represent the four dimensional supersymmetry,

$$\begin{pmatrix} \theta_{1a} \\ \theta_{2a} \end{pmatrix} = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} \begin{pmatrix} \theta_a \\ \bar\theta_a \end{pmatrix}, \tag{4}$$

where x_{ij} are complex numbers such that, $x_{11}x_{22} - x_{12}x_{21} \neq 0$. We can write the original Lagrangian using these new coordinates as

$$\mathcal{L} = D_1^2 D_2^2 \mathcal{J}[\Phi(\theta_1, \theta_2)]_{\theta_1=\theta_2=0}, \tag{5}$$

where $\mathcal{J}$ is the Jacobian for transformation. It is possible absorb the Jacobian for transformation, using field redefinition, $\Phi(\theta_1, \theta_2) = \mathcal{J}\tilde\Phi(\theta_1, \theta_2)$, if $\tilde\Phi(\theta_1, \theta_2)$ is the original superfield. We shall assume this to be this case and neglect the numerical factor coming from the Jacobian. Now we choose x_{ij}, such that in the new coordinates, the superderivatives take the form,

$$D_{1a} = \partial_{1a} + (\gamma^\mu \theta_1)_a \partial_\mu,$$
$$D_{2a} = \partial_{2a} + (\gamma^\mu \theta_2)_a \partial_\mu, \tag{6}$$

and satisfy

$$\{D_{1a}, D_{1b}\} = -2\gamma^\mu_{ab}\partial_\mu, \qquad \{D_{2a}, D_{2b}\} = -2\gamma^\mu_{ab}\partial_\mu,$$
$$\{D_{1a}, D_{2b}\} = 0. \tag{7}$$

The generators of $\mathcal{N}=1$ supersymmetry corresponding to these superderivatives are given by

$$Q_{1a} = \partial_{1a} - (\gamma^\mu \theta_1)_a \partial_\mu,$$
$$Q_{2a} = \partial_{2a} - (\gamma^\mu \theta_2)_a \partial_\mu, \tag{8}$$

and they also satisfy,

$$\{Q_{1a}, Q_{1b}\} = 2\gamma^\mu_{ab}\partial_\mu, \qquad \{Q_{2a}, Q_{2b}\} = 2\gamma^\mu_{ab}\partial_\mu,$$
$$\{Q_{1a}, Q_{2b}\} = 0. \tag{9}$$

These supercharges also commute with these superderivatives, $\{Q_{1a}, D_{1b}\} = \{Q_{1a}, D_{2b}\} = 0$ and $\{Q_{2a}, D_{1b}\} = \{Q_{2a}, D_{2b}\} = 0$.

We also define, $P_\pm = (1 \pm \gamma^3)/2$, so that $D_{1\pm a} = (P_\pm)_a^b D_{1b}$ and $D_{2\pm a} = (P_\pm)_a^b D_{2b}$. We can also define, and so, $Q_{1\pm a} = (P_\pm)_a^b Q_{1b}$ and $Q_{2\pm a} = (P_\pm)_a^b Q_{2b}$, we can write the bulk charges Q_{1a} and Q_{2a} as

$$\begin{aligned}\epsilon^{1a} Q_{1a} &= \epsilon^{1a}(P_- + P_+) Q_{1a} \\ &= \epsilon^{1+} Q_{1-} + \epsilon^{1-} Q_{1+}, \\ \epsilon^{2a} Q_{2a} &= \epsilon^{2a}(P_- + P_+) Q_{2a} \\ &= \epsilon^{2+} Q_{2-} + \epsilon^{2-} Q_{2+}.\end{aligned} \tag{10}$$

Using the superderivative which commutes with the generator of $\mathcal{N}=1$ supersymmetry, we can write a Lagrangian for a supersymmetric theory with $\mathcal{N}=1$ supersymmetry as

$$\begin{aligned}\mathcal{L} &= D_2^2 D_1^2 [\Phi(\theta_1, \theta_2)]_{\theta_1=\theta_2=0} \\ &= D_2^2 [r_2(\theta_2)]_{\theta_2=0} \\ &= D_1^2 [r_1(\theta_1)]_{\theta_1=0},\end{aligned} \tag{11}$$

where the $\mathcal{N}=1$ superfield has been decomposed as

$$\begin{aligned}\Phi(\theta_1, \theta_2) &= p_1(\theta_1) + q_1(\theta_1)\theta_2 + r_1(\theta_1)\theta_2^2 \\ &= p_2(\theta_2) + q_2(\theta_2)\theta_1 + r_2(\theta_2)\theta_1^2.\end{aligned} \tag{12}$$

It may be noted that $p_1(\theta_1), p_2(\theta_2), q_1(\theta_1), q_2(\theta_2), r_1(\theta_1), r_2(\theta_2)$ are superfields in their own right depending only on θ_2 or θ_1. Under the supersymmetric transformations generated by Q_{1a} and Q_{2a}, they transform as

$$\begin{aligned}
\epsilon^{1a} Q_{1a} p_1(\theta_1) &= \epsilon^{1a} q_{1a}(\theta_1), \\
\epsilon^{1a} Q_{1a} q_{1a}(\theta_1) &= -\epsilon_{1a} r_1(\theta_1) + (\gamma^\mu \epsilon_1)_a \partial_a p_1(\theta_1), \\
\epsilon^{1a} Q_{1a} r_1(\theta_1) &= \epsilon^{1a} (\gamma^\mu \partial_\mu)_a^b q_{1b}(\theta_1), \\
\epsilon^{2a} Q_{2a} p_2(\theta_2) &= \epsilon^{2a} q_{2a}(\theta_2), \\
\epsilon^{2a} Q_{2a} q_{2a}(\theta_2) &= -\epsilon_{2a} r_2(\theta_2) + (\gamma^\mu \epsilon_2)_a \partial_a p_2(\theta_2), \\
\epsilon^{2a} Q_{2a} r_2(\theta_2) &= \epsilon^{2a} (\gamma^\mu \partial_\mu)_a^b q_{2b}(\theta_2). \qquad (13)
\end{aligned}$$

Thus, under these supersymmetric transformations generated by Q_{1a} this Lagrangian transforms as $\epsilon^{1a} Q_{1a}\mathcal{L} = -\partial_\mu(\gamma^\mu \epsilon^1 q_1(\theta_1))$, and under these supersymmetric transformations generated by Q_{2a} this Lagrangian transforms as $\epsilon^{2a} Q_{2a}\mathcal{L} = -\partial_\mu(\gamma^\mu \epsilon^2 q_2(\theta_2))$. So, the action is invariant under the supersymmetric transformations generated by Q_{1a} and Q_{2a}, in absence of a boundary, $\epsilon^{1a} Q_{1a}\mathcal{L} = \epsilon^{2a} Q_{2a}\mathcal{L} = 0$. However, in presence of a boundary, the supersymmetric transformations generated by Q_{1a} and Q_{2a} produce boundary terms. Thus, if we assume that a boundary exists at $x_3 = 0$, then the supersymmetric transformations of the Lagrangian can be written as $\epsilon^{1a} Q_{1a}\mathcal{L} = -\gamma^3 \partial_3(\epsilon^{1a} q_{1a}(\theta_1))$ and $\epsilon^{2a} Q_{2a}\mathcal{L} = -\gamma^3 \partial_3(\epsilon^{2a} q_{2a}(\theta_2))$. The presence of these boundary terms will breaks the supersymmetry of the resultant theory.

We can perverse half the supersymmetry of the resultant theory by either adding or subtracting a boundary term to the original Lagrangian. Now if $\mathcal{L}_{1b}$ and $\mathcal{L}_{2b}$ is the boundary term added or subtracted from the bulk Lagrangian with $\mathcal{N} = 1$ supersymmetry, then we have

$$\begin{aligned}
\epsilon^1 Q_1[\mathcal{L} \pm \mathcal{L}_{1b}] &= \pm 2\partial_3 \epsilon^{1\pm} q_{1\mp}(\theta_1), \\
\epsilon^2 Q_2[\mathcal{L} \pm \mathcal{L}_{2b}] &= \pm 2\partial_3 \epsilon^{2\pm} q_{2\mp}(\theta_2), \qquad (14)
\end{aligned}$$

where $q_{1\pm}(\theta_1) = (1 \pm \gamma^3) q_1(\theta_1)/2$ and $q_{2\pm}(\theta_2) = (1 \pm \gamma^3) q_2(\theta_2)/2$. Hence, the Lagrangian $\mathcal{L} \pm \mathcal{L}_{1b}$ preserves the supersymmetry generated by $\epsilon^{1\mp} Q_{1\pm}$, and the Lagrangian $\mathcal{L} \pm \mathcal{L}_{2b}$ preserves the supersymmetry generated by $\epsilon^{2\mp} Q_{2\pm}$. It is not possible to simultaneously preserve both the supersymmetry generated by $\epsilon^{1-} Q_{1+}$ and $\epsilon^{1+} Q_{1-}$, or $\epsilon^{2-} Q_{2+}$ and $\epsilon^{2+} Q_{2-}$, in the presence of a boundary. However, in the presence of a boundary, we can construct the Lagrangian which preserves the supersymmetry generated by $\epsilon^{1\mp} Q_{1\pm}$ and $\epsilon^{2\mp} Q_{2\pm}$ as

$$\begin{aligned}
\mathcal{L}^{1-2-} &= (D_1^2 - \partial_3)(D_2^2 - \partial_3)[\Phi(\theta_1, \theta_2)]_{\theta_1=\theta_2=0}, \\
\mathcal{L}^{1-2+} &= (D_1^2 - \partial_3)(D_2^2 + \partial_3)[\Phi(\theta_1, \theta_2)]_{\theta_1=\theta_2=0}, \\
\mathcal{L}^{1+2-} &= (D_1^2 + \partial_3)(D_2^2 - \partial_3)[\Phi(\theta_1, \theta_2)]_{\theta_1=\theta_2=0}, \\
\mathcal{L}^{1+2+} &= (D_1^2 + \partial_3)(D_2^2 + \partial_3)[\Phi(\theta_1, \theta_2)]_{\theta_1=\theta_2=0}. \qquad (15)
\end{aligned}$$

It may be noted that this Lagrangian preserves only half of the supersymmetry of the original Lagrangian. This is because if we preserve the supersymmetry corresponding to $\epsilon^{1\mp} Q_{1\pm}$ and $\epsilon^{2\mp} Q_{2\pm}$, then we will break the supersymmetry corresponding to $\epsilon^{1\mp} Q_{1\mp}$ and $\epsilon^{2\mp} Q_{2\mp}$.

It may be noted that half the on-shell supersymmetry could also be preserved by using suitable boundary conditions. In fact, these on-shell boundary conditions can be motivated from this off-shell formalism. This is because the supersymmetric transformation of the original Lagrangian are given by

$$\begin{aligned}
\epsilon^1 Q_1[\mathcal{L} \pm \mathcal{L}_{1b}] &= \pm 2\epsilon^{1\pm'} q'_{1\mp}(\theta_1), \\
\epsilon^2 Q_2[\mathcal{L} \pm \mathcal{L}_{2b}] &= \pm 2\epsilon^{2\pm'} q'_{2\mp}(\theta_2), \qquad (16)
\end{aligned}$$

where $'$ means the quantity is evaluated at the boundary. As the supersymmetric transformation of $\mathcal{L} \pm \mathcal{L}_{1b}$ do not generate $\epsilon^{1\mp'} q'_{1\pm}(\theta_1)$, and the supersymmetric transformation of $\mathcal{L} \pm \mathcal{L}_{2b}$ do not generate $\epsilon^{2\ mp'} q'_{2\pm}(\theta_2)$, this Lagrangian is invariant under half the off-shell supersymmetry. However, half of the on-shell supersymmetry could also be preserved by imposing the following boundary conditions on the original Lagrangian,

$$q'_{1-}(\theta_1) = 0, \qquad q'_{2-}(\theta_2) = 0, \qquad (17)$$

$$q'_{1-}(\theta_1) = 0, \qquad q'_{2+}(\theta_2) = 0, \qquad (18)$$

$$q'_{1+}(\theta_1) = 0, \qquad q'_{2-}(\theta_2) = 0, \qquad (19)$$

$$q'_{1+}(\theta_1) = 0, \qquad q'_{2+}(\theta_2) = 0. \qquad (20)$$

As these terms would vanish on-shell by the imposition of the boundary conditions, this Lagrangian is also invariant under half of the on-shell supersymmetry of the original Lagrangian. As we have defined $\Phi(\theta_1, \theta_2) = p_1(\theta_1) + q_1(\theta_1)\theta_2 + r_1(\theta_1)\theta_2^2 = p_2(\theta_2) + q_2(\theta_2)\theta_1 + r_2(\theta_2)\theta_1^2$, we can write

$$\begin{aligned}
q_{1a}(\theta_1) &= [D_{2a}\Phi(\theta_1, \theta_2)]_{\theta_2=0}, \\
q_{2a}(\theta_2) &= [D_{1a}\Phi(\theta_1, \theta_2)]_{\theta_1=0}. \qquad (21)
\end{aligned}$$

Thus, on the boundary we can write

$$q'_{1a-}(\theta_1) = P^b_{a-}[D_{1b}\Phi(\theta_1, \theta_2)]'_{\theta_1=0}, \qquad (22)$$

$$q'_{2a-}(\theta_2) = P^b_{a-}[D_{2b}\Phi(\theta_1, \theta_2)]'_{\theta_2=0}, \qquad (23)$$

$$q'_{1a-}(\theta_1) = P^b_{a-}[D_{1b}\Phi(\theta_1, \theta_2)]'_{\theta_1=0}, \qquad (24)$$

$$q'_{2a+}(\theta_2) = P^b_{a+}[D_{2b}\Phi(\theta_1, \theta_2)]'_{\theta_2=0}, \qquad (25)$$

$$q'_{1a+}(\theta_1) = P^b_{a+}[D_{1b}\Phi(\theta_1, \theta_2)]'_{\theta_1=0}, \qquad (26)$$

$$q'_{2a-}(\theta_2) = P^b_{a-}[D_{2b}\Phi(\theta_1, \theta_2)]'_{\theta_2=0}, \qquad (27)$$

$$q'_{1a+}(\theta_1) = P^b_{a+}[D_{1b}\Phi(\theta_1, \theta_2)]'_{\theta_1=0}, \qquad (28)$$

$$q'_{2a+}(\theta_2) = P^b_{a+}[D_{2b}\Phi(\theta_1, \theta_2)]'_{\theta_2=0}, \qquad (29)$$

where the projection operator is defined by $P^b_{a\pm} = [\delta^b_a \pm (\gamma^3)^b_a]/2$ and $'$ indicates that only the boundary values are considered. Now half the on-shell supersymmetric can also be preserved by imposing the following boundary conditions on the superfield,

$$\begin{aligned}
P^b_{a+}[D_{1b}\Phi(\theta_1, \theta_2)]'_{\theta_1=0} &= 0, \\
P^b_{a+}[D_{2b}\Phi(\theta_1, \theta_2)]'_{\theta_2=0} &= 0, \qquad (30)
\end{aligned}$$

$$\begin{aligned}
P^b_{a+}[D_{1b}\Phi(\theta_1, \theta_2)]'_{\theta_1=0} &= 0, \\
P^b_{a-}[D_{2b}\Phi(\theta_1, \theta_2)]'_{\theta_2=0} &= 0, \qquad (31)
\end{aligned}$$

$$\begin{aligned}
P^b_{a-}[D_{1b}\Phi(\theta_1, \theta_2)]'_{\theta_1=0} &= 0, \\
P^b_{a+}[D_{2b}\Phi(\theta_1, \theta_2)]'_{\theta_2=0} &= 0, \qquad (32)
\end{aligned}$$

$$\begin{aligned}
P^b_{a-}[D_{1b}\Phi(\theta_1, \theta_2)]'_{\theta_1=0} &= 0, \\
P^b_{a-}[D_{2b}\Phi(\theta_1, \theta_2)]'_{\theta_2=0} &= 0. \qquad (33)
\end{aligned}$$

It is important to note that these boundary conditions are invariant under half the generators of supersymmetry. This is needed for these boundary conditions to hold under supersymmetric transformations. Even though we can preserve half the on-shell supersymmetric of the original Lagrangian by imposing these boundary conditions, the advantage of using the present formalism is that it also preserves half of the off-shell supersymmetry.

3. Transformation of boundary fields

In this section, we will analyze the decomposition of the supercharges for a four dimensional theory with $\mathcal{N}=1$ supersymmetry. We can write the bulk supercharges as $\epsilon^{1a}Q_{1a}=\epsilon^{1+}Q_{1-}+\epsilon^{1-}Q_{1+}$, and $\epsilon^{2a}Q_{2a}=\epsilon^{2+}Q_{2-}+\epsilon^{2-}Q_{2+}$. Furthermore, the bulk supercharges $Q_{1\pm}$, $Q_{2\pm}$, are related to boundary supercharges $Q'_{1\pm}$, $Q'_{2\pm}$, as

$$Q_{1-}=Q'_{1-}+\theta_{1-}\partial_3, \qquad Q_{1+}=Q'_{1+}-\theta_{1+}\partial_3,$$
$$Q_{2-}=Q'_{2-}+\theta_{2-}\partial_3, \qquad Q_{2+}=Q'_{2+}-\theta_{2+}\partial_3. \tag{34}$$

The boundary supercharges given by

$$Q'_{1+}=\partial_{1+}-\gamma^s\theta_{1-}\partial_s, \qquad Q'_{1-}=\partial_{1-}-\gamma^s\theta_{1+}\partial_s,$$
$$Q'_{2+}=\partial_{2+}-\gamma^s\theta_{2-}\partial_s, \qquad Q'_{2-}=\partial_{2-}-\gamma^s\theta_{2+}\partial_s, \tag{35}$$

where s is the index for the coordinates along the boundary, i.e., the case $\mu=3$ has been excluded for a boundary fixed at x_3. Now by definition $Q_{1\pm}, Q_{2\pm}$, are the generators of the half supersymmetry of the bulk fields and $Q'_{1\pm}, Q'_{2\pm}$, are the standard supersymmetry of the boundary fields. We can now express the boundary superfields in terms of bulk superfields as follows,

$$Q'_{1-}=\exp(+\theta_{1+}\theta_{1-}\partial_3)\,Q_{1-}\exp(-\theta_{1+}\theta_{1-}\partial_3),$$
$$Q'_{1+}=\exp(-\theta_{1-}\theta_{1+}\partial_3)\,Q_{1+}\exp(+\theta_{1-}\theta_{1+}\partial_3),$$
$$Q'_{2-}=\exp(+\theta_{2+}\theta_{2-}\partial_3)\,Q_{2-}\exp(-\theta_{2+}\theta_{2-}\partial_3),$$
$$Q'_{2+}=\exp(-\theta_{2-}\theta_{2+}\partial_3)\,Q_{2+}\exp(+\theta_{2-}\theta_{2+}\partial_3). \tag{36}$$

The original superfield also gets decomposed as follows,

$$\Phi=\exp(+\theta_{2-}\theta_{2+}\partial_3)\exp(+\theta_{1-}\theta_{1+}\partial_3)\Phi'_{2+1+},$$
$$\Phi=\exp(+\theta_{2-}\theta_{2+}\partial_3)\exp(-\theta_{1+}\theta_{1-}\partial_3)\Phi'_{2+1-},$$
$$\Phi=\exp(-\theta_{2+}\theta_{2-}\partial_3)\exp(+\theta_{1-}\theta_{1+}\partial_3)\Phi'_{2-1+},$$
$$\Phi=\exp(-\theta_{2+}\theta_{2-}\partial_3)\exp(-\theta_{1+}\theta_{1-}\partial_3)\Phi'_{2-1-}, \tag{37}$$

where $\Phi'_{2+1+},\Phi'_{2+1-},\Phi'_{2-1+},\Phi'_{2-1-}$, decompose into boundary superfields,

$$\epsilon^{1-}Q_{1+}\Phi=\exp(+\theta_{2-}\theta_{2+}\partial_3)\exp(+\theta_{1-}\theta_{1+}\partial_3)\epsilon^{1-'}Q'_{1+}\Phi'_{2+1+},$$
$$\epsilon^{1+}Q_{1-}\Phi=\exp(+\theta_{2-}\theta_{2+}\partial_3)\exp(-\theta_{1+}\theta_{1-}\partial_3)\epsilon^{1+'}Q'_{1-}\Phi'_{2+1-},$$
$$\epsilon^{2-}Q_{2+}\Phi=\exp(+\theta_{2-}\theta_{2+}\partial_3)\exp(+\theta_{1-}\theta_{1+}\partial_3)\epsilon^{2-'}Q'_{2+}\Phi'_{2+1+},$$
$$\epsilon^{2+}Q_{2-}\Phi=\exp(-\theta_{2+}\theta_{2-}\partial_3)\exp(+\theta_{1-}\theta_{1+}\partial_3)\epsilon^{2+'}Q'_{2-}\Phi'_{2-1+},$$
$$\epsilon^{1-}Q_{1+}\Phi=\exp(-\theta_{2+}\theta_{2-}\partial_3)\exp(+\theta_{1-}\theta_{1+}\partial_3)\epsilon^{1-'}Q'_{1+}\Phi'_{2-1+},$$
$$\epsilon^{1+}Q_{1-}\Phi=\exp(-\theta_{2+}\theta_{2-}\partial_3)\exp(-\theta_{1+}\theta_{1-}\partial_3)\epsilon^{1+'}Q'_{1-}\Phi'_{2-1-},$$
$$\epsilon^{2-}Q_{2+}\Phi=\exp(+\theta_{2-}\theta_{2+}\partial_3)\exp(-\theta_{1+}\theta_{1-}\partial_3)\epsilon^{2-'}Q'_{2+}\Phi'_{2+1-},$$
$$\epsilon^{2+}Q_{2-}\Phi=\exp(-\theta_{2+}\theta_{2-}\partial_3)\exp(-\theta_{1+}\theta_{1-}\partial_3)\epsilon^{2+'}Q'_{2-}\Phi'_{2-1-}. \tag{38}$$

Now we will analyze the superalgebra for the four dimensional theory with $\mathcal{N}=1$ supersymmetric theory, in the presence of a boundary. The non-vanishing part of the superalgebra is given by

$$\{Q_{1+a},Q_{1+b}\}=2(\gamma^s_{ab}P_+)\partial_s, \qquad \{D_{1+a},D_{1+b}\}=-2(\gamma^s_{ab}P_+)\partial_s,$$
$$\{Q_{1-a},Q_{1-b}\}=2(\gamma^s_{ab}P_-)\partial_s, \qquad \{D_{1-a},D_{1-b}\}=-2(\gamma^s_{ab}P_-)\partial_s,$$
$$\{Q_{1+a},Q_{1-b}\}=-2(P_-)_{ab}\partial_3, \qquad \{D_{1+a},D_{1-b}\}=2(P_-)_{ab}\partial_3,$$
$$\{Q_{2+a},Q_{2+b}\}=2(\gamma^s_{ab}P_+)\partial_s, \qquad \{D_{2+a},D_{2+b}\}=-2(\gamma^s_{ab}P_+)\partial_s,$$
$$\{Q_{2-a},Q_{2-b}\}=2(\gamma^s_{ab}P_-)\partial_s, \qquad \{D_{2-a},D_{2-b}\}=-2(\gamma^s_{ab}P_-)\partial_s,$$
$$\{Q_{2+a},Q_{2-b}\}=-2(P_-)_{ab}\partial_3, \qquad \{D_{2+a},D_{2-b}\}=2(P_-)_{ab}\partial_3. \tag{39}$$

It may be noted that $\{Q_{1\pm},Q_{2\pm}\}=\{D_{1\pm},D_{2\pm}\}=0$, and $\{Q_{1\pm},D_{2\pm}\}=\{Q_{1\pm},D_{1\pm}\}=\{Q_{2\pm},D_{2\pm}\}=\{Q_{2\pm},D_{1\pm}\}=0$. So, we have

$$D_{1-a}D_{1+b}=(P_-)_{ab}(\partial_3-D_1^2),$$
$$D_{1+a}D_{1-b}=-(P_-)_{ab}(\partial_3+D_1^2),$$
$$D_{2-a}D_{2+b}=(P_-)_{ab}(\partial_3-D_2^2),$$
$$D_{2+a}D_{2-b}=-(P_-)_{ab}(\partial_3+D_2^2). \tag{40}$$

Now contracting these equations and using $(P_-)^a_a=1$, we obtain the following result,

$$D_1^2+\partial_3=D_{1+}D_{1-}, \qquad D_2^2+\partial_3=D_{2+}D_{2-}, \tag{41}$$
$$D_1^2-\partial_3=D_{1-}D_{1+}, \qquad D_2^2-\partial_3=D_{2-}D_{2+}. \tag{42}$$

Thus, we can see how the Lagrangian with the measure preserves the right amount of supersymmetry on the boundary, because we can write

$$\mathcal{L}^{1+2+}=D_{2+}D_{2-}D_{1+}D_{1-}[\Phi]_{\theta_1=\theta_2=0},$$
$$\mathcal{L}^{1-2-}=D_{2-}D_{2+}D_{1-}D_{1+}[\Phi]_{\theta_1=\theta_2=0},$$
$$\mathcal{L}^{1+2-}=D_{2+}D_{2-}D_{1-}D_{1+}[\Phi]_{\theta_1=\theta_2=0},$$
$$\mathcal{L}^{1-2+}=D_{2-}D_{2+}D_{1+}D_{1-}[\Phi]_{\theta_1=\theta_2=0}. \tag{43}$$

We can write it in terms of boundary superfields as

$$\mathcal{L}^{1+2+}=-D'_{2+}D'_{1+}[\Omega'_{1-2-}]_{\theta_{1-}=\theta_{2-}=0},$$
$$\mathcal{L}^{1-2-}=-D'_{2-}D'_{1-}[\Omega'_{1+2+}]_{\theta_{1+}=\theta_{2+}=0},$$
$$\mathcal{L}^{1+2-}=-D'_{2-}D'_{1+}[\Omega'_{1-2+}]_{\theta_{1-}=\theta_{2+}=0},$$
$$\mathcal{L}^{1-2+}=-D'_{2+}D'_{1-}[\Omega'_{1+2-}]_{\theta_{1+}=\theta_{2--}=0}, \tag{44}$$

where $'$ means the quantity is evaluated at the boundary and

$$\Omega'_{1-2-}=D'_{2-}D'_{1-}[\Phi'_{1-2-}]_{\theta_{1-}=\theta_{2-}=0},$$
$$\Omega'_{1+2+}=D'_{2+}D'_{1+}[\Phi'_{1+2+}]_{\theta_{1+}=\theta_{2+}=0},$$
$$\Omega'_{1-2+}=D'_{2+}D'_{1-}[\Phi'_{1-2+}]_{\theta_{1-}=\theta_{2+}=0},$$
$$\Omega'_{1+2-}=D'_{2-}D'_{1+}[\Phi'_{1+2-}]_{\theta_{1+}=\theta_{2-}=0}. \tag{45}$$

The boundary measure only contains $D'_{2\pm}D'_{1\pm}$. So, on the boundary $\epsilon^{1\pm'}Q'_{1\mp}$ and $\epsilon^{2\pm'}Q'_{2\mp}$ act as independent supercharges. Thus, we can add a boundary Lagrangian to the original theory, which will still preserve half the supersymmetry of the original theory,

$$\mathcal{L}_t=\mathcal{L}+\mathcal{L}_b, \tag{46}$$

where $\mathcal{L}_t$ is the total Lagrangian for the bulk and the boundary theory, $\mathcal{L}$ is the Lagrangian for the original theory and $\mathcal{L}_b$ is the Lagrangian for the boundary theory. Thus, we can add the following terms to Lagrangian

$$\mathcal{L}^{1+2+}=-D'_{2+}D'_{1+}[\Omega'_{1-2-}+\omega'_{1-2-}]_{\theta_{1-}=\theta_{2-}=0},$$
$$\mathcal{L}^{1-2-}=-D'_{2-}D'_{1-}[\Omega'_{1+2+}+\omega'_{1+2+}]_{\theta_{1+}=\theta_{2+}=0},$$
$$\mathcal{L}^{1+2-}=-D'_{2-}D'_{1+}[\Omega'_{1-2+}+\omega'_{1-2+}]_{\theta_{1-}=\theta_{2+}=0},$$
$$\mathcal{L}^{1-2+}=-D'_{2+}D'_{1-}[\Omega'_{1+2-}+\omega'_{1+2-}]_{\theta_{1+}=\theta_{2--}=0}. \tag{47}$$

Here $\omega'_{1\pm2\pm}$ are only defined on the boundary,

$$\omega'_{1-2-} = D'_{2-}D'_{1-}[\lambda'_{1-2-}]_{\theta_{1-}=\theta_{2-}=0},$$
$$\omega'_{1+2+} = D'_{2+}D'_{1+}[\lambda'_{1+2+}]_{\theta_{1+}=\theta_{2+}=0},$$
$$\omega'_{1-2+} = D'_{2+}D'_{1-}[\lambda'_{1-2+}]_{\theta_{1-}=\theta_{2+}=0},$$
$$\omega'_{1+2-} = D'_{2-}D'_{1+}[\lambda'_{1+2-}]_{\theta_{1+}=\theta_{2-}=0}, \quad (48)$$

where $\lambda'_{1\pm2\pm}$ can be purely boundary Lagrangian. We can take a suitable gauge invariant coupling between this purely boundary fields and the bulk fields. It may be noted that on the boundary only the supersymmetry generated by $\epsilon^{1\pm'}Q'_{1\mp}$ and $\epsilon^{2\pm'}Q'_{2\mp}$ is preserved.

4. Super-Yang–Mills theory

In this section, we will write the action for super-Yang–Mills theory as using a vector field $V^A T_A$, where T_A are the generators of the gauge symmetry, $[T_A, T_B] = if^C_{AB}T_C$. We can write the Lagrangian for the super-Yang–Mills theory as using a vector superfield $V = V^A T_A$,

$$\begin{aligned}\mathcal{L} &= D^2[W^a W_a]_{\theta=0} + \bar{D}^2[\bar{W}^a\bar{W}_a]_{\bar{\theta}=0} \\ &\quad + \bar{D}^2 D^2[\mathcal{V}(\Phi,\bar{\Phi}) + \bar{\Phi}e^V\Phi]_{\theta=\bar{\theta}=0} \\ &= D^2\bar{D}^2[\Box^{-1}D^2W^aW_a + \Box^{-1}\bar{D}^2\bar{W}^a\bar{W}_a \\ &\quad + \mathcal{V}(\Phi,\bar{\Phi}) + \bar{\Phi}e^V\Phi]_{\theta=\bar{\theta}=0}, \end{aligned} \quad (49)$$

where the superfield strengths are given by $W_a = -i\bar{D}^2 \times (e^{-V}D_a e^V)/4$ and $\bar{W}_a = -iD^2(e^{-V}\bar{D}_a e^V)/4$. Here the potential $\mathcal{V}(\Phi,\bar{\Phi})$ is a function of Φ and $\bar{\Phi}$. Even though this action looks like a non-local action, the component action in the bulk will be a local action. This is because it is another way of writing a local action. It may be noted, as we were only interested in analyzing the amount of supersymmetry preserved, we will did not need the explicit form of super-Yang–Mills action in real superfields. It may be noted that even though the expression for it would involve an complicated expression containing the non-local operator, the component action would be local. This is because it can be transformed back into the local action. However, it is not clear if the resultant boundary action is local or not, as it cannot be transformed into any local action. So, we will express this Lagrangian into an alternative formalism, and in that formalism we will be able to obtain a local action for the super-Yang–Mills theory even in presence of a boundary.

The gauge transformations of the superfield V transforms are given by $e^V \to e^{i\bar{\Lambda}}e^V e^{-i\Lambda}$, where Λ and $\bar{\Lambda}$ are chiral and anti-chiral gauge parameters. So, it is possible to write a covariant derivative which transforms under gauge transformation as $\nabla_a = e^{-V}D_a e^V \to e^{i\Lambda}\nabla_a e^{-i\Lambda}$ and $\bar{\nabla}_a = \bar{D}_a \to e^{i\Lambda}\nabla_a e^{-i\Lambda}$, and another covariant derivative which transforms under gauge transformation as $\tilde{\nabla}_a = D_a \to e^{i\bar{\Lambda}}\tilde{\nabla}_a e^{-i\bar{\Lambda}}$ and $\tilde{\bar{\nabla}}_a = e^V\bar{D}_a e^{-V} \to e^{i\bar{\Lambda}}\tilde{\bar{\nabla}}_a e^{-i\bar{\Lambda}}$. However, it is also possible to define another covariant derivative which transforms under a real gauge parameter u as $\nabla_a \to u\nabla_a u^{-1}$ and $\bar{\nabla}_a \to u\bar{\nabla}_a u^{-1}$ [26]. Now we can express this covariant derivative in terms of θ_{1a} and θ_{2a} rather than θ_{1a} and θ_{2a}. We can absorb the Jacobian using field redefinition, and then use the modified measure on the boundary. However, it would be more convenient to express the original covariant derivative in terms of the real spinor superfield and then work out the modification by the boundary theory. So, we define two gauge valued spinor superfields $\Gamma_{1a} = \Gamma^A_{1a}(\theta_1)T_A$ and $\Gamma_{2a} = \Gamma^A_{2a}(\theta_2)T_A$, and use them to construct gauge covariant derivatives for matter fields $\Phi(\theta_1,\theta_2)$ and $\bar{\Phi}(\theta_1,\theta_2)$,

$$\begin{aligned}\nabla_{1a}\Phi &= D_{1a}\Phi - i\Gamma_{1a}\Phi, & \nabla_{2a}\Phi &= D_{2a}\Phi - i\Gamma_{2a}\Phi, \\ \nabla_{1a}\bar{\Phi} &= D_{1a}\bar{\Phi} + i\bar{\Phi}\Gamma_{1a}, & \nabla_{2a}\bar{\Phi} &= D_{2a}\bar{\Phi} + i\bar{\Phi}\Gamma_{2a}. \end{aligned} \quad (50)$$

These fields transform under the gauge transformation as, $\Gamma_{1a} \to u\nabla_{1a}u^{-1}$, and $\Gamma_{2a} \to u\nabla_{2a}u^{-1}$, and so the covariant derivatives transform as $\nabla_{1a} \to u\nabla_{1a}u^{-1}$ and $u\nabla_{2a}u^{-1}$. It may be noted if we define ∇_a and $\bar{\nabla}_a$ as a linear combination of ∇_{1a} and u, then we will get to correct transformation for the original covariant derivatives. This is because $\nabla_a = x_{11}\nabla_{1a} + x_{12}\nabla_{2a} \to u[x_{11}\nabla_{1a} + x_{12}\nabla_{2a}]u^{-1} = u\nabla_a u^{-1}$ and $\bar{\nabla}_a = x_{22}\nabla_{2a} + x_{21}\nabla_{1a} \to u[x_{22}\nabla_{2a} + x_{21}\nabla_{1a}]u^{-1} = u\bar{\nabla}_a u^{-1}$, where x_{ij} are complex numbers. We can also construct the field strengths as follows,

$$\begin{aligned} W_{1a} &= \frac{1}{2}D^b_1 D_{1a}\Gamma_{1b} - \frac{i}{2}\{\Gamma^b_1, D_{1b}\Gamma_{1a}\} - \frac{1}{6}[\Gamma^b_1, \{\Gamma_{1b},\Gamma_{1a}\}], \\ W_{2a} &= \frac{1}{2}D^b_2 D_{2a}\Gamma_{2b} - \frac{i}{2}\{\Gamma^b_2, D_{2b}\Gamma_{2a}\} - \frac{1}{6}[\Gamma^2_1, \{\Gamma_{2b},\Gamma_{2a}\}]. \end{aligned} \quad (51)$$

Now these field strengths transform as $W_{1a} \to uW_{1a}u^{-1}$, and $W_{2a} \to uW_{2a}u^{-1}$. Now we can write the action for super-Yang–Mills theory as

$$\begin{aligned}\mathcal{L} &= D^2_1 D^2_2[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + D^2_1[W^a_1 W_{1a}]_{\theta_1=0} + D^2_2[W^a_2 W_{2a}]_{\theta_2=0}, \end{aligned} \quad (52)$$

where $\mathcal{V}[\Phi,\bar{\Phi}]$ is a potential term which is given by product of superfields Φ and $\bar{\Phi}$.

Now we can write the Lagrangian for super-Yang–Mills theory which preserves various supercharges as follows,

$$\begin{aligned}\mathcal{L}^{1-2-} &= (D^2_1 - \partial_3)(D^2_2 - \partial_3)[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + (D^2_1 - \partial_3)[W^a_1 W_{1a}]_{\theta_1=0} + (D^2_2 - \partial_3)[W^a_2 W_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1-2+} &= (D^2_1 - \partial_3)(D^2_2 + \partial_3)[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + (D^2_1 - \partial_3)[W^a_1 W_{1a}]_{\theta_1=0} + (D^2_2 + \partial_3)[W^a_2 W_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1+2-} &= (D^2_1 + \partial_3)(D^2_2 - \partial_3)[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + (D^2_1 + \partial_3)[W^a_1 W_{1a}]_{\theta_1=0} + (D^2_2 - \partial_3)[W^a_2 W_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1+2+} &= (D^2_1 + \partial_3)(D^2_2 + \partial_3)[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + (D^2_1 + \partial_3)[W^a_1 W_{1a}]_{\theta_1=0} + (D^2_2 + \partial_3)[W^a_2 W_{2a}]_{\theta_2=0}. \end{aligned} \quad (53)$$

This result can also be written as

$$\begin{aligned}\mathcal{L}^{1+2+} &= D_{2+}D_{2-}D_{1+}D_{1-}[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + D_{1+}D_{1-}[W^a_1 W_{1a}]_{\theta_1=0} + D_{2+}D_{2-}[W^a_2 W_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1-2-} &= D_{2-}D_{2+}D_{1-}D_{1+}[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + D_{1-}D_{1+}[W^a_1 W_{1a}]_{\theta_1=0} + D_{2-}D_{2+}[W^a_2 W_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1+2-} &= D_{2+}D_{2-}D_{1-}D_{1+}[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + D_{1-}D_{1+}[W^a_1 W_{1a}]_{\theta_1=0} + D_{2+}D_{2-}[W^a_2 W_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1-2+} &= D_{2-}D_{2+}D_{1+}D_{1-}[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + D_{1+}D_{1-}[W^a_1 W_{1a}]_{\theta_1=0} + D_{2-}D_{2+}[W^a_2 W_{2a}]_{\theta_2=0}. \end{aligned} \quad (54)$$

It is thus transparent that this modified Lagrangian only preserves half the original supersymmetry.

5. The Born–Infeld action

This action can be thought as a low energy action generated from the Born–Infeld action, which is the action for D3-branes. It is possible to write the full Born–Infeld Lagrangian in superspace [27–30]. The abelian Born–Infeld can be written as [31]

$$S = \frac{1}{(2\pi\alpha')^2}\int d^4x\sqrt{-\det(\eta_{\mu\nu} + (2\pi\alpha')F_{\mu\nu})}, \tag{55}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. It is also possible to express the abelian Born–Infeld action using complex bosonic variables,

$$\begin{aligned} &\omega = \alpha + i\beta, \qquad \bar\omega = \alpha - i\beta, \\ &\alpha = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \qquad \beta = \frac{1}{4}F^{\mu\nu}\tilde F_{\mu\nu}, \end{aligned} \tag{56}$$

where $\tilde F_{\mu\nu}$ is defined as $\tilde F_{\mu\nu} = \epsilon^{\mu\nu\tau\rho}F_{\mu\nu}/2$. So, the abelian Born–Infeld Lagrangian can be written as [27]

$$\int d^4x\mathcal{L} = \int d^4x - \frac{1}{2}(\omega + \bar\omega) + (2\pi\alpha')^2\omega\bar\omega\mathcal{B}(\omega,\bar\omega). \tag{57}$$

The function $\mathcal{B}(\omega,\bar\omega)$ can be expressed as

$$\begin{aligned} B(\omega,\bar\omega) = \Bigg[1 - \frac{(2\pi\alpha')^2}{2}\omega_+ \\ + \sqrt{1 + (2\pi\alpha')^2\omega_+ + \frac{(2\pi\alpha')^4}{4}\omega_-^2}\Bigg]^{-1}, \end{aligned} \tag{58}$$

where $\omega_+ = (\omega + \bar\omega)$ and $\omega_- = (\omega - \bar\omega)$.

It is possible to write a supersymmetric version of this action. This can be done by first defining $K = D^2[W^aW_a]$, and $\bar K = \bar D^2[\bar W_2^a\bar W_{2a}]$, and then written the supersymmetric abelian Born–Infeld Lagrangian as

$$\begin{aligned} \mathcal{L} = &D^2[W^aW_a]_{\theta=0} + \bar D^2[\bar W^a\bar W_a]_{\bar\theta=0} \\ &+ \bar D^2D^2[W^aW_a\bar W^b\bar W_b\mathcal{B}(K,\bar K)]_{\theta=\bar\theta=0}. \end{aligned} \tag{59}$$

The constraint $\mathcal{B}(K,K)$ can be written as [27]

$$\begin{aligned} B(K,\bar K) = \Bigg[1 - \frac{(2\pi\alpha')^2}{2}K_+ \\ + \sqrt{1 + (2\pi\alpha')^2K_+ + \frac{(2\pi\alpha')^4}{4}K_-^2}\Bigg]^{-1}, \end{aligned} \tag{60}$$

where $K_+ = (K + \bar K)$ and $K_- = (K - \bar K)$. The abelian Born–Infeld Lagrangian can be written as

$$\begin{aligned} \mathcal{L} &= D^2[W^aW_a]_{\theta=0} + \bar D^2[\bar W^a\bar W_a]_{\bar\theta=0} \\ &\quad + \bar D^2D^2[W^aW_a\bar W^b\bar W_b\mathcal{B}(K,\bar K)]_{\theta=\bar\theta=0} \\ &= D^2\bar D^2[\square^{-1}D^2W^aW_a + \square^{-1}\bar D^2\bar W^a\bar W_a \\ &\quad + W^aW_a\bar W^b\bar W_b\mathcal{B}(K,\bar K)]_{\theta=\bar\theta=0}. \end{aligned} \tag{61}$$

We can transform this Lagrangian to the one containing W_{1a} and W_{2a} as follows,

$$\begin{aligned} \mathcal{L} = &D_1^2D_2^2[W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2)]_{\theta_1=\theta_2=0} \\ &+ D_1^2[W_1^aW_{1a}]_{\theta_1=0} + D_2^2[W_2^aW_{2a}]_{\theta_2=0}, \end{aligned} \tag{62}$$

where $K_2 = D_1^2[W_1^aW_{1a}]$, and $K_2 = D_2^2[W_2^a\bar W_{2a}]$. So, we can write the abelian Born–Infeld Lagrangian in presence of a boundary as

$$\begin{aligned} \mathcal{L}^{1-2-} &= (D_1^2 - \partial_3)(D_2^2 - \partial_3)[W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2)]_{\theta_1=\theta_2=0} \\ &\quad + (D_1^2 - \partial_3)[W_1^aW_{1a}]_{\theta_1=0} + (D_2^2 - \partial_3)[W_2^aW_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1-2+} &= (D_1^2 - \partial_3)(D_2^2 + \partial_3)[W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2)]_{\theta_1=\theta_2=0} \\ &\quad + (D_1^2 - \partial_3)[W_1^aW_{1a}]_{\theta_1=0} + (D_2^2 + \partial_3)[W_2^aW_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1+2-} &= (D_1^2 + \partial_3)(D_2^2 - \partial_3)[W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2)]_{\theta_1=\theta_2=0} \\ &\quad + (D_1^2 + \partial_3)[W_1^aW_{1a}]_{\theta_1=0} + (D_2^2 - \partial_3)[W_2^aW_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1+2+} &= (D_1^2 + \partial_3)(D_2^2 + \partial_3)[W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2)]_{\theta_1=\theta_2=0} \\ &\quad + (D_1^2 + \partial_3)[W_1^aW_{1a}]_{\theta_1=0} + (D_2^2 + \partial_3)[W_2^aW_{2a}]_{\theta_2=0}. \end{aligned} \tag{63}$$

This result can also be written as

$$\begin{aligned} \mathcal{L}^{1+2+} &= D_{2+}D_{2-}D_{1+}D_{1-}[W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2)]_{\theta_1=\theta_2=0} \\ &\quad + D_{1+}D_{1-}[W_1^aW_{1a}]_{\theta_1=0} + D_{2+}D_{2-}[W_2^aW_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1-2-} &= D_{2-}D_{2+}D_{1-}D_{1+}[W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2)]_{\theta_1=\theta_2=0} \\ &\quad + D_{1-}D_{1+}[W_1^aW_{1a}]_{\theta_1=0} + D_{2-}D_{2+}[W_2^aW_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1+2-} &= D_{2+}D_{2-}D_{1-}D_{1+}[W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2)]_{\theta_1=\theta_2=0} \\ &\quad + D_{1-}D_{1+}[W_1^aW_{1a}]_{\theta_1=0} + D_{2+}D_{2-}[W_2^aW_{2a}]_{\theta_2=0}, \\ \mathcal{L}^{1-2+} &= D_{2-}D_{2+}D_{1+}D_{1-}[W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2)]_{\theta_1=\theta_2=0} \\ &\quad + D_{1+}D_{1-}[W_1^aW_{1a}]_{\theta_1=0} + D_{2-}D_{2+}[W_2^aW_{2a}]_{\theta_2=0}. \end{aligned} \tag{64}$$

The abelian Born–Infeld Lagrangian can couple to a background dilaton ϕ and an axion C. The supersymmetric version of this action will also require a dilatino field λ_a. To write the action for the system, we define a complex scalar $\rho = e^{-\phi} + iC$. We can write $A = \rho + \theta^a\lambda_a + \theta^2F$. and $\bar A = \bar\rho + \bar\theta^a\bar\lambda_a + \bar\theta^2\bar F$. Here F and $\bar F$ are auxiliary fields. We can also define $\mathcal{A} = A + \bar A$. This Lagrangian for this system can now be written as [27]

$$\begin{aligned} \mathcal{L} &= D^2[W^aW_a]_{\theta=0} + \bar D^2[\bar W^a\bar W_a]_{\bar\theta=0} \\ &\quad + \bar D^2D^2[\mathcal{A}^2W^aW_a\bar W^b\bar W_b\mathcal{B}(K,\bar K,\mathcal{A})]_{\theta=\bar\theta=0} \\ &= D^2\bar D^2[\square^{-1}D^2W^aW_a + \square^{-1}\bar D^2\bar W^a\bar W_a \\ &\quad + \mathcal{A}^2W^aW_a\bar W^b\bar W_b\mathcal{B}(K,\bar K,\mathcal{A})]_{\theta=\bar\theta=0}. \end{aligned} \tag{65}$$

The constraint $\mathcal{B}(K,\bar K,\mathcal{A})$ can be written as

$$\begin{aligned} \mathcal{B}(K,\bar K,\mathcal{A}) = \Bigg[1 - \frac{(2\pi\alpha')^2}{2}\mathcal{A}_+ \\ + \sqrt{(1 + (2\pi\alpha')^2\mathcal{A}_+ + \frac{(2\pi\alpha')^4}{4}\mathcal{A}_-^2}\Bigg]^{-1}, \end{aligned} \tag{66}$$

where $2\mathcal{A}_+ = (\mathcal{A}K + \mathcal{A}\bar K)$ and $2\mathcal{A}_- = (\mathcal{A}K - \mathcal{A}\bar K)$. We can again transform this Lagrangian to the one containing W_{1a} and W_{2a} as follows,

$$\begin{aligned} \mathcal{L} = &D_1^2D_2^2[\mathcal{A}^2W_1^aW_{1a}W_2^aW_{2a}\mathcal{B}(K_1,K_2,\mathcal{A})]_{\theta_1=\theta_2=0} \\ &+ D_1^2[W_1^aW_{1a}]_{\theta_1=0} + D_2^2[W_2^aW_{2a}]_{\theta_2=0}. \end{aligned} \tag{67}$$

Here the $\mathcal{A}$ has also been transformed to the superspace coordinates θ_1 and θ_2, and the Jacobian of the transformation has been absorbed in the field redefinition. Now in presence of a boundary, a Born–Infeld Lagrangian coupled to a dilaton and an axion is given by

$$
\begin{aligned}
\mathcal{L}^{1-2-} &= (D_1^2 - \partial_3)(D_2^2 - \partial_3)[\mathcal{A}^2 W_1^a W_{1a} W_2^a W_{2a} \\
&\quad \times \mathcal{B}(K_1, K_2, \mathcal{A})]_{\theta_1=\theta_2=0} \\
&\quad + (D_1^2 - \partial_3)[W_1^a W_{1a}]_{\theta_1=0} + (D_2^2 - \partial_3)[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1-2+} &= (D_1^2 - \partial_3)(D_2^2 + \partial_3)[\mathcal{A}^2 W_1^a W_{1a} W_2^a W_{2a} \\
&\quad \times \mathcal{B}(K_1, K_2, \mathcal{A})]_{\theta_1=\theta_2=0} \\
&\quad + (D_1^2 - \partial_3)[W_1^a W_{1a}]_{\theta_1=0} + (D_2^2 + \partial_3)[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1+2-} &= (D_1^2 + \partial_3)(D_2^2 - \partial_3)[\mathcal{A}^2 W_1^a W_{1a} W_2^a W_{2a} \\
&\quad \times \mathcal{B}(K_1, K_2, \mathcal{A})]_{\theta_1=\theta_2=0} \\
&\quad + (D_1^2 + \partial_3)[W_1^a W_{1a}]_{\theta_1=0} + (D_2^2 - \partial_3)[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1+2+} &= (D_1^2 + \partial_3)(D_2^2 + \partial_3)[\mathcal{A}^2 W_1^a W_{1a} W_2^a W_{2a} \\
&\quad \times \mathcal{B}(K_1, K_2, \mathcal{A})]_{\theta_1=\theta_2=0} \\
&\quad + (D_1^2 + \partial_3)[W_1^a W_{1a}]_{\theta_1=0} + (D_2^2 + \partial_3)[W_2^a W_{2a}]_{\theta_2=0}.
\end{aligned}
\tag{68}
$$

This result can also be written as

$$
\begin{aligned}
\mathcal{L}^{1+2+} &= D_{2+} D_{2-} D_{1+} D_{1-}[\mathcal{A}^2 W_1^a W_{1a} W_2^a W_{2a} \\
&\quad \times \mathcal{B}(K_1, K_2, \mathcal{A})]_{\theta_1=\theta_2=0} \\
&\quad + D_{1+} D_{1-}[W_1^a W_{1a}]_{\theta_1=0} + D_{2+} D_{2-}[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1-2-} &= D_{2-} D_{2+} D_{1-} D_{1+}[\mathcal{A}^2 W_1^a W_{1a} W_2^a W_{2a} \\
&\quad \times \mathcal{B}(K_1, K_2, \mathcal{A})]_{\theta_1=\theta_2=0} \\
&\quad + D_{1-} D_{1+}[W_1^a W_{1a}]_{\theta_1=0} + D_{2-} D_{2+}[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1+2-} &= D_{2+} D_{2-} D_{1-} D_{1+}[\mathcal{A}^2 W_1^a W_{1a} W_2^a W_{2a} \\
&\quad \times \mathcal{B}(K_1, K_2, \mathcal{A})]_{\theta_1=\theta_2=0} \\
&\quad + D_{1-} D_{1+}[W_1^a W_{1a}]_{\theta_1=0} + D_{2+} D_{2-}[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1-2+} &= D_{2-} D_{2+} D_{1+} D_{1-}[\mathcal{A}^2 W_1^a W_{1a} W_2^a W_{2a} \\
&\quad \times \mathcal{B}(K_1, K_2, \mathcal{A})]_{\theta_1=\theta_2=0} \\
&\quad + D_{1+} D_{1-}[W_1^a W_{1a}]_{\theta_1=0} + D_{2-} D_{2+}[W_2^a W_{2a}]_{\theta_2=0}.
\end{aligned}
\tag{69}
$$

The abelian Born–Infeld Lagrangian in absence of a dilaton and axion can also be written as a non-linear sigma model [27],

$$
\mathcal{L} = D^2[\chi]_{\theta=0} + \bar{D}^2[\bar{\chi}]_{\bar{\theta}=0}, \tag{70}
$$

where

$$
\begin{aligned}
\chi + \frac{(2\pi\alpha')^2}{2}\chi \bar{D}^2 \bar{\chi} &= \frac{1}{4} W^a W_a, \\
\bar{\chi} + \frac{(2\pi\alpha')^2}{2}\bar{\chi} D^2 \chi &= \frac{1}{4} \bar{W}^a \bar{W}_a.
\end{aligned}
\tag{71}
$$

It is possible to extend this formalism to non-abelian gauge theories. This can be done by defining [28]

$$
\begin{aligned}
\xi + \frac{(2\pi\alpha')^2}{2}\xi \bar{D}^2 (e^{2V} \bar{\xi} e^{2V}) &= \frac{1}{4} W^a W_a, \\
\bar{\xi} + \frac{(2\pi\alpha')^2}{2}\bar{\xi} D^2 (e^{-2V} \xi e^{2V}) &= \frac{1}{4} \bar{W}^a \bar{W}_a,
\end{aligned}
\tag{72}
$$

where W^a and $\bar{W}^a$ are field strengths for non-abelian gauge theories. Now the non-abelian Born–Infeld Lagrangian can be written as

$$
\begin{aligned}
\mathcal{L} &= D^2[\xi]_{\theta=0} + \bar{D}^2[\bar{\xi}]_{\bar{\theta}=0} \\
&= D^2 \bar{D}^2[\Box^{-1}\xi]_{\theta=0} + \bar{D}^2 D^2[\Box^{-1}\bar{\xi}]_{\bar{\theta}=0}.
\end{aligned}
\tag{73}
$$

Now we define $\tilde{\zeta}(\theta, \bar{\theta}) = \Box^{-1}\xi + \Box^{-1}\bar{\xi}$, and transform it to

$$
\zeta(\theta_1, \theta_2) = \mathcal{J}\tilde{\zeta}(\theta_1, \theta_2) \tag{74}
$$

where $\mathcal{J}$ is the Jacobian for transformation from $\theta, \bar{\theta}$ to θ_1, θ_2. So, we can write the non-abelian Born–Infeld Lagrangian as

$$
\mathcal{L} = D_1^2 D_2^2[\zeta(\theta_1, \theta_2)]_{\bar{\theta}=0}. \tag{75}
$$

It is possible to couple this action to matter fields and write the combined action as

$$
\mathcal{L} = D_1^2 D_2^2[\zeta(\theta_1, \theta_2) + \nabla^a \Phi \bar{\nabla}_a \bar{\Phi} + \mathcal{V}[\Phi, \bar{\Phi}]]_{\bar{\theta}=0}. \tag{76}
$$

So, we can write the action for the non-abelian Born–Infeld Lagrangian coupled to matter fields as

$$
\begin{aligned}
\mathcal{L}^{1-2-} &= (D_1^2 - \partial_3)(D_2^2 - \partial_3)[\zeta(\theta_1, \theta_2) + \nabla^a \Phi \bar{\nabla}_a \bar{\Phi} \\
&\quad + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0}, \\
\mathcal{L}^{1-2+} &= (D_1^2 - \partial_3)(D_2^2 + \partial_3)[\zeta(\theta_1, \theta_2) + \nabla^a \Phi \bar{\nabla}_a \bar{\Phi} \\
&\quad + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0}, \\
\mathcal{L}^{1+2-} &= (D_1^2 + \partial_3)(D_2^2 - \partial_3)[\zeta(\theta_1, \theta_2) + \nabla^a \Phi \bar{\nabla}_a \bar{\Phi} \\
&\quad + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0}, \\
\mathcal{L}^{1+2+} &= (D_1^2 + \partial_3)(D_2^2 + \partial_3)[\zeta(\theta_1, \theta_2) + \nabla^a \Phi \bar{\nabla}_a \bar{\Phi} \\
&\quad + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0}.
\end{aligned}
\tag{77}
$$

This result can also be written as

$$
\begin{aligned}
\mathcal{L}^{1+2+} &= D_{2+} D_{2-} D_{1+} D_{1-}[\zeta(\theta_1, \theta_2) + \nabla^a \Phi \bar{\nabla}_a \bar{\Phi} \\
&\quad + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0}, \\
\mathcal{L}^{1-2-} &= D_{2-} D_{2+} D_{1-} D_{1+}[\zeta(\theta_1, \theta_2) + \nabla^a \Phi \bar{\nabla}_a \bar{\Phi} \\
&\quad + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0}, \\
\mathcal{L}^{1+2-} &= D_{2+} D_{2-} D_{1-} D_{1+}[\zeta(\theta_1, \theta_2) + \nabla^a \Phi \bar{\nabla}_a \bar{\Phi} \\
&\quad + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0}, \\
\mathcal{L}^{1-2+} &= D_{2-} D_{2+} D_{1+} D_{1-}[\zeta(\theta_1, \theta_2) + \nabla^a \Phi \bar{\nabla}_a \bar{\Phi} \\
&\quad + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0}.
\end{aligned}
\tag{78}
$$

Thus, we have been able to analyze the non-abelian Born–Infeld Lagrangian coupled to matter fields, in presence of a boundary. This Lagrangian also preserves only half the supersymmetry of the original Lagrangian.

6. Conclusion

In this paper, we have analyzed the restoration of half the supersymmetry for a four dimensional theory in $\mathcal{N} = 1$ superspace formalism, on manifolds with a boundary. We first use the fact that a total derivative term is obtained from the supersymmetric variation of a Lagrangian for a four dimensional theory with $\mathcal{N} = 1$ supersymmetry. This total derivative term vanishes in absence of a boundary. However, in presence of a boundary, this total derivative term generates a boundary term, which breaks half the supersymmetry of the original theory. However, half of the original supersymmetry can be preserved by adding new boundary terms to the original Lagrangian. The supersymmetric variation of these new boundary terms exactly canceled the boundary terms generated by the supersymmetric transformation of the original bulk Lagrangian.

We explicitly constructed such boundary terms for the four dimensional theory with $\mathcal{N}=1$ supersymmetry. We also related the bulk supercharges to the boundary supercharges. The bulk supercharges behaved as two independent supercharges on the boundary. However, the inclusion of the new boundary terms only preserved the supersymmetry only with respect to one of these projections. Thus, it was demonstrated that only half of the supersymmetry of the original theory was preserved. This analysis was done using the real superfields, and the Jacobian of transformation was absorbed in field redefinitions. We finally applied our results to the super-Yang–Mills theory. We explicitly constructed the Lagrangian which preserves half the supersymmetry of the original theory. We also study the Born–Infeld Lagrangian in presence of a boundary. We study the coupling of the Born–Infeld Lagrangian to a dilaton and an axion field. We also study the non-abelian Born–Infeld action. We demonstrate that the Born–Infeld Lagrangian preserves half the supersymmetry of the original theory, in presence of a boundary.

It is possible to generalize this present analysis to theories which have higher amount of supersymmetry. In fact, the analysis of this present paper can be used for analyzing various aspects of the *AdS*/*CFT* correspondence [32–35]. This is because according to the *AdS*/*CFT* correspondence type IIB string theory on $AdS_5 \times S^5$ is dual to the $\mathcal{N}=4$ super-Yang–Mills theory on its conformal boundary. Thus, the theory which describes the low energy limit of the action for a stack of D3-branes on $AdS_5 \times S^5$ is the $\mathcal{N}=4$ super-Yang–Mills action with the gauge group $U(N)$. The four world-volume coordinates of the D3-branes become the Minkowski coordinates, and six transverse coordinates to the D3-branes give rise to the six gauge valued scalar fields of $\mathcal{N}=4$ super-Yang–Mills theory. This theory also contains eight gauge valued fermions, and a gauge field. It would be interesting to analyze $\mathcal{N}=4$ super-Yang–Mills theory in presence of a boundary. It is again expected that the $\mathcal{N}=4$ super-Yang–Mills theory in presence of a boundary will preserve only half the supersymmetry of the original theory.

It has been demonstrated that using the Horava–Witten theory, one of the low energy limits of the heterotic string theory can be obtained from the eleven dimensional supergravity in presence of a boundary [36–39]. In this construction, it has been possible to obtain a unification of gauge and gravitational couplings. It would be interesting to analyze the connection between the Horava–Witten theory and the boundary supersymmetry discussed in this paper. In order to do that, it might be interesting to first generalize the results of this paper to five dimensions. This is because, motivated by Horava–Witten theory, a five dimensional globally supersymmetric Yang–Mills theory coupled to a four dimensional hypermultiplet on the boundary has been constructed [40].

It may be noted that in Randall–Sundrum models our four dimensional universe is thought to be located on a three-brane in a five dimensional spacetime with negative cosmological constant [41,42]. These models provide a geometrical solution to the electroweak hierarchy problem. A supersymmetric generalization of such models have also been analyzed in [43,44]. In fact, it has been argued that for the supersymmetric Randall–Sundrum models to be consistent, the issue of supersymmetric boundary conditions has to be analyzed [45]. We are hoping to generalize such a procedure to help constructing supersymmetric Randall–Sundrum models.

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