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Effect of Rayleigh thermal number in Double diffusive Non-Darcy mixed convective flow in vertical pipe filled with porous medium

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Abstract

In the present manuscript effect of different Rayleigh number of non-darcy fully developed mixed convection in a vertical pipe filled with porous media is numerical investigation. The motion in the pipe caused by external pressure gradient and byoucey force. Non-Darcy Brinkmen-Forchheimer extended model has been introduces in momentum equation. The basic state of the flow model using fundamental assumption is formed in the form of coupled differential equation which is solved using Chebyshev Spectral collocation technique. The study is best based on double diffusive mixed convection which is governed in mathematical formulation of the problem , in which the velocity profile posses point of inflection beyond the threshold value of Ra_T (Positive Rayilegh thermal number). In case of Negative Rayliegh thermal number , the velocity profile may contain point of inflection in the centre zone and point of separation at the vicinity of the wall. Point of separation is create a back flow near the wall.

Keywords: Double-diffusive, Non-Darcy, Porous media, Spectral collocation method, Rayleigh Number

1. Introduction

The study of mixed convective heat and mass transfer in a saturated porous medium has received a noticeable attention from the researchers during the last few decades . These include geothermal engineering, thermal insulation systems, packed bed chemical reactors, porous heat exchangers, oil separation from sand by steam, underground disposal of nuclear waste materials, food storage, electronic device cooling, to name a few applications [1]. In the case of double-diffusive convection in vertical porous enclosure, most of the works [2–3] have considered the flow as smooth as well as stable, and obtained numerical and analytical solutions. Alavyoon [2] found multiple steady state solutions and flow oscillation in isotropic porous enclosure, when flow is induced by opposing fluxes of heat and mass on the vertical walls. Latter on, it was extended to anisotropic porous media by Bera and Khalili [3], who investigated the correlation between the existence of multiple steady-state solutions and oscillation as a function of buoyancy ratio, aspect ratio, and anisotropic parameters the detailed review can be obtained from the [4] in which they investigated instability of pressure gradient driven double-diffusive mixed convection in a vertical channel filled with a fluid-saturated porous medium. But the flow through the vertical system is not given much attention in the recent past,Yao[5], has taken a challenge of the problem of vertical pipe in which he assumed that flow is fully developed and non-isothermal which is a substantial idea taken from the Weissberg [6], in this study laminar flow in vertical pipe filled with a porous medium is considered after that Yao and Rogers [7], considered a

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similar kind of geometry called annulus is taken and illustrate the linear stability analysis along with basic flow in viscous media then Murlidhar [8] did this in porous media along with mixed convection. Not only this good number of papers [9-12] are available in vertical channel filled with porous medium, I which, observed that the velocity profile having flow of separation and point of inflexion. Hence Su and chen [12] extended the above idea in vertical pipe but in viscous media he reported that velocity profile having point of separation and in another situation it crates a back flow near the wall of different Rayleigh number . Recently Ashok and Bera [14] investigated the flow in vertical pipe filled with porous media in which they found that, In case of buoyancy-opposed flow, the velocity profile may contain point of inflection in the center zone and point of separation at the vicinity of the wall. The appearance of point of separation causes back flow at the vicinity of the wall (i.e. velocity becomes negative at the vicinity of the wall). Based on the values of other controlling parameters, there exist a minimum value of Ra, beyond it, the temperature profile possess point of inflection. A kind of distortion on the velocity as well as on the temperature is found on the further enhancement of Ra. In opposed flow, enhancement of absolute value of Ra increases the maximum magnitude of temperature as well as velocity which appear at the center of the pipe. Whereas, in case of assisted flow magnitude of both velocity as well as temperature decreases on increasing Ra at center of the pipe, and maximum magnitude of the velocity increases, the goal of this study is to investigate the influence of Rayleigh thermal number on fully developed mixed convection flow in a vertical pipe filled with porous medium

Nomenclature

c_F	= Form drag constant
C_1	=Axial temperature gradient
Da	=Darcy Number = $\frac{K}{R_0^2}$
F	= Forchheimer number = $\frac{R_0 c_F}{K^{\frac{1}{2}}}$
F'	= Modified Forchheimer number = $Re\epsilon^2 F \frac{dp}{dz}$
g	= gravitational acceleration
p	= dimensional pressure
P	= non-dimensional pressure
Ra_T	= Thermal Rayleigh number =$\frac{g\beta_T C_1 R_0^4}{\nu\alpha}$
Ra_S	= Solutal Rayleigh number =$\frac{g\beta_s C_2 R_0^4}{\beta\alpha}$
Re	= Reynolds number = $\frac{W_c r_0}{\upsilon}$
N	= Buoyancy ratio = $\frac{Ra_S}{Ra_T}$
ε	= Porosity
β_T	= Thermal expansion coefficient
β_s	= Solutal expansion coefficient
ρ	= Fluid density

2. Mathematical Formulation

The geometry under consideration is depicted in fig1. which is fully developed double-diffusive mixed convective in a vertical pipe filled with a porous media. The flow is induced by external pressure gradient and buoyancy forces (due to temperature and concentration differences).

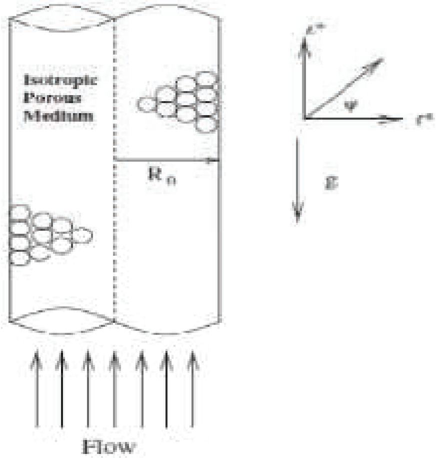


Figure 1 Schematic of vertical pipe flow and the coordinate system

The wall temperature and concentration are linearly varying with vertical component of coordinate, z^*, as $T_w = T_0 + A_T R_C Z$ and $C_w = C_0 + A_C R_0 Z$ respectively. Where A_T, A_c are a constant. The upstream reference wall temperature and concentration are $T_0 \, and \, C_0$ respectively, and, $R_0$is radius of the pipe. The gravitational force is aligned in the negative z^*-direction. Here all thermal physical properties assumed to be constant except of density, the effective viscosity and fluid velocity are equal. The Boussinesq approximation is valid. There is no internal radiation in the pipe. The objective of the paper is to understand the fluid flow, heat and transfer mechanism of the steady, unidirectional fully developed flow (basic flow). Therefore, it is assumed that flow is in vertical direction only i.e. the velocity vector is (0,0,w*). From the continuity equation, it is clear that w* is function of r* only. As a consequence of this, the governing differential equations for momentum and energy, in cylindrical coordinate, of the basic flow can be written as

3. Basic state

On the basis of assumption that flow is fully developed, unidirectional , laminar ,we get the following set of equation

$$\rho_f \left| \frac{c_F}{K^{1/2}} \right| w \; \left| w \; - \frac{dp}{dz} \right| + \tilde{\mu} \left| \frac{d^2 w}{dr^{*2}} + \frac{1}{r} \frac{dw}{dr} \right| - \frac{\mu_f}{K} w \; = \; \rho_f g(\beta_T (T \; - T_w) + \beta_C (C \; - C_w) \quad (5)$$

$$w \; \frac{\partial T}{\partial z} = \; \alpha_e \left| \frac{\partial^2 T}{\partial r^{*2}} + \frac{1}{r} \frac{\partial T}{\partial r} \right| \qquad (6)$$

$$w \; \frac{\partial CT}{\partial z} = \; \beta_e \left| \frac{\partial^2 C}{\partial r^{*2}} + \frac{1}{r} \frac{\partial C}{\partial r} \right| \qquad (7)$$

Using non – dimensional quantities:

$$r = \frac{r}{R_0}, \quad U_0 = \frac{w}{W_c} \; , z = \frac{z}{R_0},$$

$$p = \frac{p}{\rho_f W_c^{\,2}}, \Theta_0 = \frac{T_w - T}{A_T R_0 RePr},$$

$$, \Phi_0 = \frac{C_{w-}C}{A_C R_0 ReSc}$$

The above governing equations may be written as

$$\frac{d^2 U_0}{dr^2} + \frac{1}{r}\frac{dU_0}{dr} - \frac{}{Da} U_0 - (Ra_T \Theta_0 + Ra_c \Phi_0) - Re\frac{dp}{dz} - \; ReF|U_0|U_0 = 0 \qquad (8)$$

$$\frac{d^2\Theta_0}{dr^2} + \frac{1}{r}\frac{d\Theta_0}{dr} = -U_0 \quad (9)$$

$$\frac{d^2\Phi_0}{dr^2} + \frac{1}{r}\frac{d\Phi_0}{dr} = -U_0 \quad (10)$$

The corresponding boundary conditions are given by

$$\frac{dU_0}{dr} = \frac{d\Theta_0}{dr} = \frac{d\Phi_0}{dr} = 0 \; at \; r = 0 \quad \text{and} \quad \Theta_0 = \Phi_0 = U_0 = 0 \; at \; r = 1 \quad (11)$$

4. Numerical solutions of basic flow

The spectral collocation method has been adopted for the solution following coupled nonlinear differential equation (8)-(10) with the boundary condition (11). The governing equations (8- 10) and the corresponding boundary conditions in terms of Chebyshev variable ξ are

$$-F'|U|U - 1 + \left(4\frac{d^2U}{d\xi^2} - \frac{2}{1-\xi}\frac{dU}{d\xi}\right) - \frac{1}{Da}U + (Ra_T\Theta + Ra_C\Phi) = 0 \quad (12)$$

$$4\frac{d^2\Theta}{d\xi^2} - \frac{2}{1-\xi}\frac{d\Theta}{d\xi} = -U \quad (13)$$

$$4\frac{d^2\Phi}{d\xi^2} - \frac{2}{1-\xi}\frac{d\Phi}{d\xi} = -U \quad (14)$$

In Which $U = \frac{U_0}{\left(Re\frac{dE}{dz}\right)}, \Theta = \frac{\Theta_0}{\left(Re\frac{dE}{dz}\right)}, \Phi = \frac{\Phi_0}{\left(Re\frac{dE}{dz}\right)}$

With boundary conditions

$$U = \Phi = \Theta = 0 \text{ at } \xi = -1, \quad \text{and} \quad \frac{dU}{d\xi} = \frac{d\Theta}{d\xi} = \frac{d\Phi}{d\xi} = 0 \; at \; \xi = 1 \quad (15)$$

$$Nu/Sh = \frac{h_1 R_0}{k_1/D_1} \quad (16)$$

5. Result and discussion

The present study is basically is the extension of Ashok.et.al (2011) while including the solutel effect in the form of mass transfer equation, here our objective is to understand the impact of Rayleigh Thermal number (Ra_T) in entire flow dynamics, the entire result section is split into four different section in the first section we shows the correct implementation of spectral collocation technique using grid impendency test in next two two section we illustrate the effect of positive and negative Ra_T, and in the last section we discussed the heat and mass transfer phenomena in the form of nusslet and Sherwood number

Code validation:

In order to validate the numerical result , Table 1. Shows the grid indecency test , using different collocation point

Numbr of term (N)	Velocity (W)	Temperature (Θ)
5	0.50536067	0.01357833
25	0.51233826	0.01372147
45	0.50987219	0.01311157
65	0.51234211	0.01372154
75	0.51234221	0.01372156

From our rigors numerical experiment we found that , the N=60 is appropriate for the solution of the above problem.

6.1 Effect of positive Ra_T

In order to understand the effect of media anisotropy on the basic flow in a vertical pipe filled with anisotropic porous media . Here , the velocity , temperature and concentration are plotted as a function of different physical parameter s. The effect of Rayleigh thermal number on velocity , temperature concentration is plotted in fig 2(a)-2(c) at $Da=10^{-4}$, Ras = 100, F=10. It can be observed from the fig 2(a) that the basic velocity profile contains a point of inflexion for $Ra_T > 10^5$ for lover value of Ra_T ($<10^5$) the basic velocity profile is nearly flat in the most of the domain, and it will be continue as Rat is increased further. The point of inflexion the velocity profile moves from centre to the wall of the pipe.

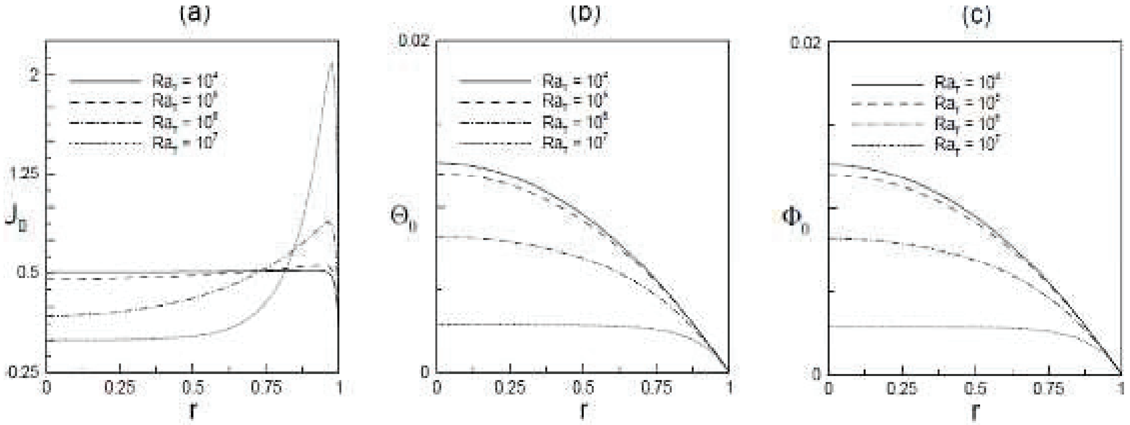


Figure 2 Effect of Rayleigh thermal number on velocity , temperature and concentration, Da=10^{-2}, Ra$_s$=10^2,F=10

The appearance of the point of inflexion is a sufficient condition for the instability of the basic flow . therefore it is expected that for Da=10^{-4}, Ra_S = 100, F=10, , there exists a minimum pressure gradient such that the basic flow will remain no longer as an one dimensional flow for $Ra_T > 10^5$. Another imp finding is that higher value of Rat indicate higher magnitude of the velocity near the wall of the pipe and its become negligible (almost zero) in the centre zone of the pipe . It is also noted that the magnitude of the velocity is decreased at the centre of the pipe and it is increases at the vicinity of the wall of the pipe. The corresponding temperature as well as concentration profile shows that the maximum magnitude of the temperature decreases on increasing of Rat and the nature of their profiles is parabolic . The concentration and temperature are exactly same, it can be seen from the differential equation of the basic flow.

6.2 Effect of negative Ra_T

In order to understand the flow dynamics in a vertical pipe, when buoyancy force is in the against of the force flow, Similar study is also presented here. The effect of Raleigh thermal number on velocity, temperature as well as concentration is plotted in Fig3(a)-(b),when Da=10^{-2}, F=10, Ra_S=100. As can be observed from Fig 3(a)-(b), that for Ra_T 10^6 the velocity profile contains point of separation at the vicinity of the wall along with point of inflection at the centre zone of the pipe , this point of separation indicates the reverse flow and causes the Rayleigh Taylor instability of the flow (e.g. in pipe [6] in channel [10]) . At the same time, the drastic change is observed in the corresponding temperature as concentration profiles in comparison to the buoyancy assisted case. The profiles of temperature as well as concentration are parabolic up to Ra_T -10^5, but at Ra_T -10^6 the concavity of their profiles changes and as a result the point of inflection appeared in the temperature as well as on concentration profile. The temperature as well as concentration profiles are parabolic for Rayleigh thermal number Ra_T -10^5. An interesting finding is that velocity, temperature as well as concentration contains an unnatural deviation that is a kind of distortion for Ra_T -10^6.

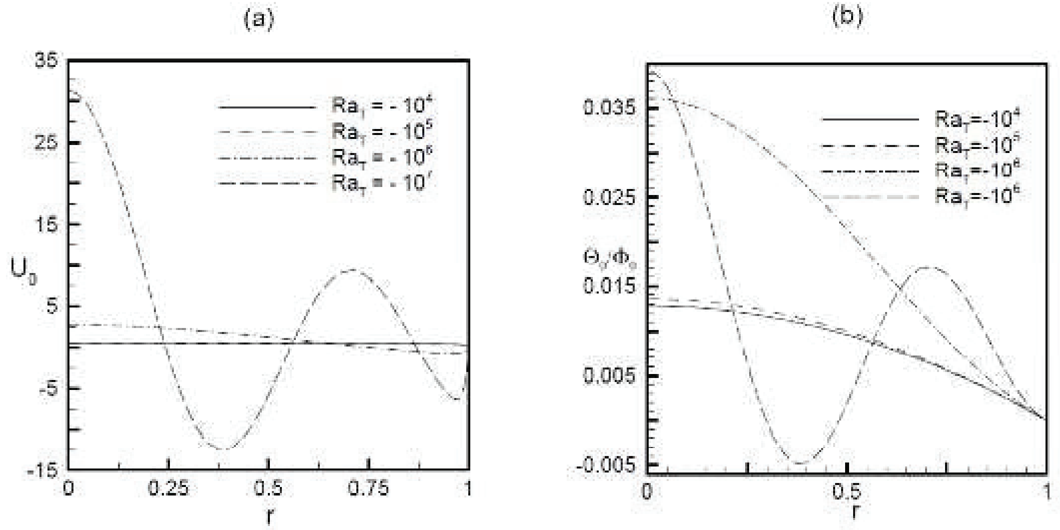
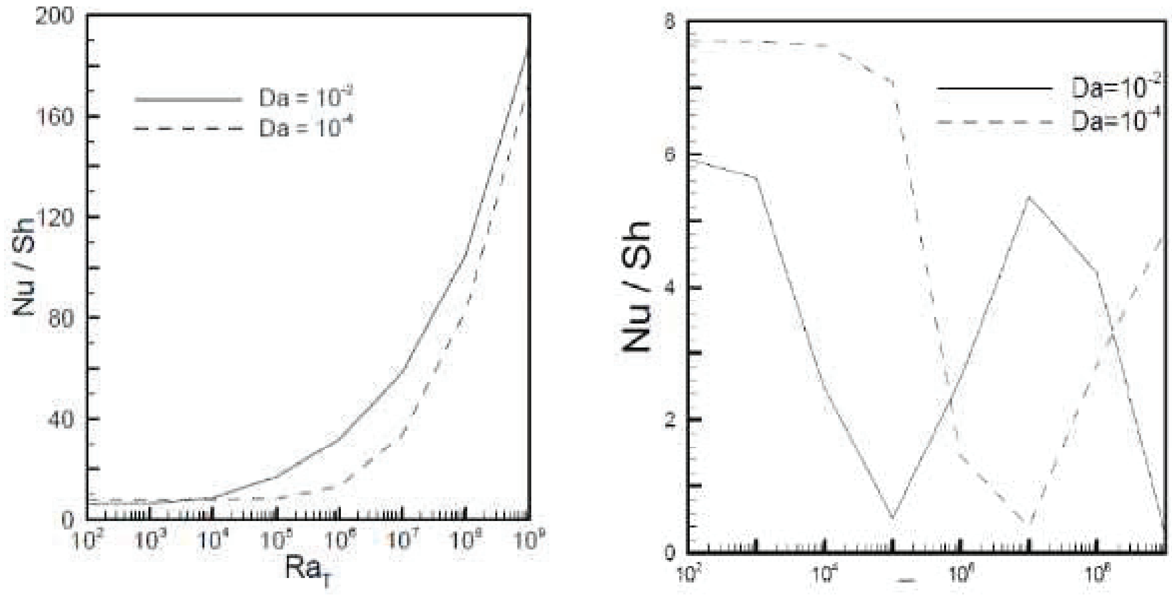


Figure 3 Effect of Rayleigh thermal number on velocity, temperature and concentration Da=10^{-2}, Ra_s=10^2,F=10

(See fig 3(a)-(b)).As can be observed from these figs that increasing of Rat increasing number of zeros on the velocity, temperature as well as concentration profiles. And number of zeros on their profiles tends to inf as Ra_T . The similar type of unnatural deviation (Distortion) on basic flow in horizontal pipe has also observed by other researcher, while studying the Poiseulle flow.

6.3 Nusslet /Sherwood Number

When the buoyancy force is in the favor of forced flow that is buoyancy assisted flow , the variation of the Nusselt number, Nu, as a function of the Rayleigh thermal number for different values different values of Da= 10^{-2}, 10^{-4} at, F=10, Ra_s=100., is depicted in fig 7(a). from the fig 7(a), it can be pointed out that the rate of heat transfer as well as mass transfer increases on increasing Rayleigh thermal number



. It is also observed that the change in heat transfer rate is negligible upto a threshold value, of Ra_T and beyond it, Nu/Sh increasing significantly when $Ra_T \times Da$ 100. The influence of Rayleigh thermal number on Nu/Sh for the buoyancy opposed to each other case , is shown in Fig5 and 6 respectively. For different value of Da=10^{-2}, 10^{-4}. It can be seen from the Fig6 that Nu /Sh various abruptly as a function of Rayleigh thermal number , which is a consequence of sudden variation in the temperature profile. In order to characterize the media permeability on Nu /Sh is plotted in fig 3The influence of Rayleigh thermal number on Nu/Sh for the buoyancy opposed to each other case , is shown in Fig 3 respectively. For different value of Da=10^{-2}, 10^{-4}. It can be seen from the Fig6 that Nu /Sh various abruptly as a function of Rayleigh thermal number , which is a consequence of sudden variation in the temperature profile. In order to characterize the media permeability on Nu /Sh is plotted in fig 3

6. Conclusion

An attempted has been taken to understand the flow of Double diffusive, Non –Darcy mixed convective flow in vertical pipe and its dependency on anisotropic porous medium parameters, where flow is induced by external pressure gradient and buoyancy force. The wall temperature and concentration of the pipe varies linearly with vertical coordinate. To this end, we have adopted the non-Darcy Brinkmane Forchheimere Wooding extended model. The fully developed, one dimensional, nonlinear coupled equations are solved numerically by Chebyshev spectral-collocation method. The above study state that there is some important finding is the distortion and point of infection , separation is die out in the system as the Thermal Rayleigh number is effected . the values of the parameters the velocity profile posses point of inflection beyond the threshold value of Ra_T (Rayleigh thermal number . In case of Negative Rayleigh thermal number, the velocity profile may contain point of inflection in the centre zone and point of separation at the vicinity of the wall. Point of separation is create a back flow near the wall. The spectral element is also successfully implemented in the system of coupled differential equation.

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