MHD non-Newtonian fluid flow over a slendering stretching sheet in the presence of cross-diffusion effects

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KEYWORDS
MHD; Soret; Dufour; Williamson fluid; Velocity slip parameter; Slendering sheet

Abstract In this study, we inquired the cross-diffusion effects on the magnetohydrodynamic Williamson fluid flow across a variable thickness stretching sheet by viewing velocity slip. With the aid of Runge-Kutta based shooting process, we resolved the transformed differential equations numerically. The effects of different dimensionless parameters on three usual profiles (velocity, temperature, concentration) along with skin friction coefficient, heat transfer rate and mass transfer rate are examined with the support of plots and tables. Dual solutions are exhibited for two cases i.e., Newtonian fluid and non-Newtonian fluid. Results reveal that the Soret and Dufour numbers have drift to control the thermal and concentration boundary layers. We also found a good agreement of the present results by comparing with the published results.

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1. Introduction

Several investigators have allured by the concept of heat and mass transfer as they have immense applications in diverse disciplines such as classification of moisture and temperature among orchard of fruit trees and farming fields. We know that when heat and mass transfer occur simultaneously, complex behavior was observed in the connections between the fluxes and the guiding potentials. And also, it is noticed that the energy flux can be furnished by both concentration and temperature gradients. The energy flux instigated by concentration gradient is called diffusion thermo or Dufour effect and the energy flux instigated by temperature gradient is called thermal diffusion or Soret or thermodiffusion effect. These effects may be disregarded as they are of less order in magnitude when compared with the effects instigated by Fourier’s and Fick’s laws. But they have their own moment in the fields such as hydrology and geosciences. The Soret effect is employed in the detachment of isotopes and in mix among the gases with light molecular weight (Hydrogen or Helium) and medium molecular weight (Hydrogen or Air).

Pseudoplastic fluids are non-Newtonian fluids with lessen viscosity when considered shear strain. For sample, recent paints are pseudoplastic materials. Since the Navier Stokes equations are inadequate to illustrate the physical properties of pseudoplastic fluids, some physical models were insinuated to fill this gap such as Carreau model, power law model, Ellis model and Cross-model. The flow across a stretching surface

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allured numerous authors due to its vast applications in several areas such as polymer extrusion, plastic films, metal spinning, and metallurgical processes. Good example for the boundary layer flow across a stretching sheet is the process of melting where the extrudate is stretched into a sheet or filament when it is dragged from the die. At the end, this sheet becomes solid when it passes by the controlled cooling system.

Initially Eckert et al. [1] recognized the significance of Soret and Dufour effects and discussed the consequence of these effects on heat transfer across a cylinder by viewing helium injection. They observed that the velocity gradient and pressure distribution are unaffected by the injection process till the laminar separation point. Later some advanced work contributed for Soret and Dufour effects by considering different channels [2–6]. Further by viewing magnetic field and chemical reaction along with these effects, Postelnicu [7,8] discussed many cases to examine the qualities of heat and mass transfer in the convective fluid flow where the fluid is plunged in a porous medium. He noticed that raising magnetic field lessen the Soret and Dufour values. In the same way, Alam and Rahman [9] have done the problem by considering magneto hydrodynamic flow and Cheng [10,11] discussed the problem by assuming concentration, wall temperature, wall mass and heat fluxes as immutable. One of their important findings is the growth in Soret parameter causes to lessen the temperature and heighten the concentration field. Later Srinivasacharya and Swamy Reddy [12] examined the same with power-law fluid. They observe that improving power-law index parameter, rises the velocity, temperature and concentration. Hayat et al. [13,14] have written a mathematical formulation for the MHD flow of a Casson fluid and explained the peristaltic transport of nanofluid by considering the slip, Joule heating and applied magnetic field effects. They found that the enhancement in slip parameter reduces the velocity. In addition to these effects, by viewing radiation effect, Raju et al. [15] inquired the MHD nanofluid flow across a vertical moving plate plunged in porous medium and noticed that the buoyancy parameter and Soret number assist in raising the rate of heat transfer.

In 1929, Williamson [16] designed a model to examine the pseudoplastic fluid flow. Unlike the other models, this model counts the minimal and maximal viscosities of the fluid which are wanted for pseudoplastic fluids. Later Williamson fluid flow was investigated through different channels (inclined channel, stretching and non-stretching surfaces) [17–20]. Some of their findings are that the enhancement in Williamson fluid parameter lessens the velocity and density of the momentum boundary layer. Also the skin friction coefficient diminishes with the rise in Williamson fluid parameter. In 1961, Sakiadis [21] pioneered the work on the Blasius type flow past continuous solid surfaces. This was extended by Crane [22] which is helpful in the pulling of plastic films. Later many authors studied the MHD viscoelastic flow across a stretching sheet [23–27]. Some of their observations are choosing viscoelastic liquids with insignificant viscous dissipation as a cooling liquid, all over the flow region, enhancement in elastic deformation effect reduces the temperature and thermal conductivity but rises the skin friction coefficient. Hayat et al. [28] explained the MHD flow across a non-linear stretching sheet.
Sandeep et al. [29] designed a numerical model for MHD nanofluid flow plunged in a porous medium by viewing thermophoresis and heat source/sink. They noticed that the thermophoresis parameter rises the concentration and temperature but depreciates the Nusselt number. Furthermore, Raju et al. [30,31] made contribution to this domain by considering different parameters and different fluids. In 1967, Lee [32] introduced the idea of variable thickness sheet by tenuous needles. Later, work on variable thickness sheet was continued by Fang et al. [33] and Anjali Devi and Prakash [34–36]. Recently, the researchers [37,38] studied the peristaltic transport of non-Newtonian fluids in the presence of transverse magnetic field by considering various physical effects. Peristaltic transport of Johnson–Segalman and Williamson fluids without slip and with slip conditions is respectively studied by the researchers [39,40]. Mulvandi et al. [41] discussed the nanofluid entropy generation of over a flat plate, and he presented an analytical solution for this study. Magnetohydrodynamic and slip effects on peristaltic motion of nanofluid were numerically reported by Hayat et al. [42]. Kothandapani and Prakash [43] discussed the thermal radiation effect on peristaltic transport of the nanofluid through porous medium. The researchers [44,45] analyzed the peristaltic transport of Newtonian and non-Newtonian flows by considering thermal radiation, Soret and Dufour effects.

All the above mentioned studies limited their investigation for analyzing the heat and mass transfer characteristics of the MHD flows over a uniform thickness stretching sheet. But due to numerous applications in polymer extrusion, plastic films, metal spinning, metallurgical processes, etc. In this study we analyzed the heat and mass transfer characteristics of Williamson fluid flow across a stretching sheet with variable thickness bearing Soret and Dufour effects. To the writer’s utmost cognizance, so far no writing has reported these types of investigation. The governing equations transmuted as ordinary differential equations using appropriate similarity transmutations and resolved numerically. The influence of dissimilar dimensionless parameters on three usual profiles (velocity, temperature and concentration) along with the skin friction coefficient, local Nusselt number (heat transfer rate) and Sherwood number (mass transfer rate) is examined with the support of plots and tables.

2. Mathematical formulation

We consider the non-linear steady two-dimensional laminar MHD flow of an incompressible Williamson fluid past a slendering stretching sheet (variable thickness sheet). The x-axis is taken as the way of the sheet motion and the y-axis is normal to it as displayed in Fig. 1. Assume that \( y = A(x + b)^{m-1} \), \( u_u(x) = U_u(x + b)^m \) and \( v_u = 0, m \neq 1 \) and external electric field as negligible.

With these assumptions, the governing equations for continuity, velocity, temperature and concentration, in a steady two-dimensional flow of a Williamson fluid are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{u}{\partial x} + \frac{v}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B(x)u}{\rho} - \sqrt{2\nu^2} \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2}, \tag{2}
\]

\[
\frac{\partial T}{\partial x} + \frac{1}{\rho C_p} \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_r}{C_{m}} \frac{\partial^2 C}{\partial y^2}, \tag{3}
\]

\[
\frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = \frac{D_m}{C_{m}} \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_r}{C_{m}} \frac{\partial^2 T}{\partial y^2}, \tag{4}
\]

The corresponding boundary conditions are

\[
\begin{aligned}
&u(x, y) = U_u(x) + h_1 \frac{\partial u}{\partial y}, \quad v(x, y) = 0, \\
&T(x, y) = T_u(x) + h_2 \frac{\partial T}{\partial y}, \quad C(x, y) = C_u(x) + h_3 \frac{\partial C}{\partial y},
\end{aligned}
\]

\[
\begin{aligned}
&u = 0, \quad T = T_\infty, \quad C = C_\infty \text{ at } y = \infty
\end{aligned}
\]

where

\[
\begin{aligned}
h_1 &= \frac{2 - f_1}{f_1} \bar{z}_1(x + b)^{m-1}, \quad \bar{z}_2 = \left( \frac{2}{y + 1} \right)^{m-1} \frac{\bar{z}_1}{Pr}, \\
h_2 &= \frac{2 - a}{a} \bar{z}_2(x + b)^{m-1}, \quad \bar{z}_3 = \left( \frac{2}{y + 1} \right)^{m-1} \frac{\bar{z}_2}{Pr}, \\
h_3 &= \frac{2 - d}{d} \bar{z}_3(x + b)^{m-1}, \quad B(x) = B_0(x + b)^{m-1},
\end{aligned}
\]

\[
T_u(x) = T_\infty + \bar{B}(x) \text{ and } C_u(x) = C_\infty + \bar{B}(x)\tag{5}
\]

To convert the governing equations into a set of nonlinear ordinary differential equations, we introduce the following similarity transformations:

\[
\phi(x, y) = f(\eta) \left( \frac{2}{m + 1} \right)^{m-1} u(x + b)^m \tag{6}
\]

\[
\eta = \sqrt{\frac{m + 1}{2}} \frac{U_u(x + b)^{m-1}}{u}, \tag{7}
\]

\[
\theta = \frac{T - T_\infty}{T_u(x) - T_\infty} \text{ and } \phi = \frac{C - C_\infty}{C_u(x) - C_\infty} \tag{8}
\]

If stream function \( \psi \) is defined as \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \) then \( \psi \) and \( v \) satisfy the equation of continuity and become as

\[
u = -\sqrt{\frac{m + 1}{2}} v U_u(x + b)^{m-1} \left[ \psi'(\eta) \eta \left( \frac{m - 1}{m + 1} \right) + \psi(\eta) \right]. \tag{9}
\]
With the help of (12), (13) and (14), Eqs. (2)–(4) converted as the below nonlinear partial differential equations:

\[(1 + Af^n)f'' - \frac{2m}{m+1} f'^2 + f^n - Mf' = 0\]  \hspace{1cm} (14)

\[\theta'' - Pr \left( \frac{1-m}{m+1} f' \theta - \beta \theta' - Du \phi'' \right) = 0\]  \hspace{1cm} (15)

\[\phi'' - Sc \left( \frac{1-m}{m+1} f' \phi - f \phi' - Sr \theta'' \right) = 0\]  \hspace{1cm} (16)

And the corresponding boundary conditions are

\[f(0) = \lambda \left( \frac{1}{m} \right) [1 + h_1 f^n(0)], \quad f'(0) = [1 + h_1 f^n(0)], \quad \theta(0) = [1 + h_2 \theta'(0)], \quad \phi(0) = [1 + h_3 \phi'(0)], \quad f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0\]  \hspace{1cm} (17)

where \(A, M, Pr, Du, Sc, Sr\) are defined as

\[A = F\sqrt{(m+1)\ell_0^{b(x+b)h_c^{-1}}}, \quad M = \frac{2\mu_b}{\rho x (m+1)}, \quad Pr = \frac{\mu_b^2}{\rho x}, \quad Du = \frac{D_b(C_0-C_c)}{C_0 \beta x^2 (T_x-T_\infty)}, \quad Sc = \frac{\mu_b}{Du}, \quad Sr = \frac{D_b(k(T_x-T_\infty))}{\rho x^2 (C_0-C_c)}\]  \hspace{1cm} (18)

The skin-friction coefficient \(C_f\), local Nusselt number \(Nu_x\) and local Sherwood number \(Sh_x\) are defined as

\[C_f = \frac{\mu_b}{2 \rho U_x^2}, \quad Nu_x = \frac{(x+b) \frac{d \theta}{dy}}{T_x(x) - T_\infty}, \quad Sh_x = \frac{(x+d) \frac{d \phi}{dy}}{C_u(x) - C_\infty}\]  \hspace{1cm} (19)

By using (5), (19) becomes

\[C_f = 2 \sqrt{\frac{m+1}{2}} (Re_x)^{0.5} f^n(0) \cdot Nu_x = - \sqrt{\frac{m+1}{2}} (Re_x)^{0.5} \theta'(0) \quad \text{and} \quad X = (x+b)\]  \hspace{1cm} (20)

where \(Re_x = \frac{\rho U_x}{\mu_b}\) and \(X = (x+b)\).

3. Results and discussion

The group of nonlinear ordinary differential Eqs. (14)–(16) subject to the restrictions (17) is solved numerically by employing Runge-Kutta based shooting process. We reveal the results to keep up the influence of several non-dimensional parameters such as Soret, Dufour and other parameters on the three usual profiles (velocity, temperature and concentration). Also we examined the same parameters on skin friction coefficient, heat transfer rate and mass transfer rate with the aid of table. In this paper, we have chosen the non-dimensional parameter values as \(Sc = 0.2, Pr = 2, m = 0.75, M = 4, Du = 0.3, Sr = 5, A = 0.1, h_1 = 0.3, h_2 = 0.3, \lambda = 1\). These values are maintained as invariable in this study unless the varied parameters as depicted in the figures.

It is observed from Figs. 2–4 that the rise in velocity slip parameter lessens the velocity but boosts up the concentration and temperature profiles. Velocity of the flow is important to dispel the temperature of the sheet. The velocity of the flow is lessened by the slip parameter near the sheet. This in turn heightens the temperature. Generally, rising the slip causes to enhance the wall friction and leads to produce the additional
heat to the flow. It is evident from Figs. 5–7 that the velocity and concentration are fall but temperature rises with the improvement in $M$. If the ratio of the electrical conductivity of the fluid to the density of the fluid is less, then the magnetic interaction parameter increases which reduces the momentum boundary layer. Physically, increasing values of $M$ develops the force opposite to the flow, which is called drag force. This may be the reason for depreciation in momentum boundary layer. It is apparent from Figs. 8 and 9 that there is a reduction in the temperature profile but reverse effect on concentration profile with the hike in Soret number. Soret number is the ratio of thermodiffusion coefficient to the diffusion coefficient. So, the temperature difference rises with the gain in the Soret number.

It is noticed from Figs. 10 and 11 that as Dufour number increases, temperature decreases and concentration increases. The Dufour number is the ratio of the energy flux owing to the concentration gradient. So improving the Dufour number, downturns the temperature and rises the concentration.

Figure 5  Velocity profile for different values of magnetic interaction parameter $M$.

Figure 6  Temperature profile for different values of magnetic interaction parameter $M$.

Figure 7  Concentration profile for different values of magnetic interaction parameter $M$.

Figure 8  Temperature profile for different values of Soret number $Sr$.

Figure 9  Concentration profile for different values of Soret number $Sr$. 
Figure 10  Temperature profile for different values of Dufour number $Du$.

Figure 11  Concentration profile for different values of Dufour number $Du$.

Figure 12  Velocity profile for different values of thermal slip parameter $h_2$.

Figure 13  Temperature profile for different values of thermal slip parameter $h_2$.

Figure 14  Temperature profile for different values of diffusion slip parameter $h_3$.

Figure 15  Concentration profile for different values of diffusion slip parameter $h_3$. 

Green : non-Newtonian fluid
Red  : Newtonian fluid

$Du=0.1,0.2,0.3$

$h_2=0.1,0.3,0.5$

$h_3=0.1,0.6,1.2$
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Table 1 Values of the skin friction coefficient, local Nusselt and Sherwood number for different physical parameters in the both Newtonian and non-Newtonian fluid cases.

| $M$ | $Sr$ | $Du$ | $h_1$ | $h_2$ | $h_3$ | $C_f$ | $-Nu_x$ | $-Sh_x$
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     | $A = 0$ | $A = 0.2$ | $A = 0$ | $A = 0.2$
| 1   |     |     |     |     |     | -2.180932 | -2.232527 | 0.491866 | 0.487824 | 0.621523 | 0.612501
| 1.5 |     |     |     |     |     | -2.459006 | -2.522645 | 0.455531 | 0.450817 | 0.540283 | 0.529592
| 2   | 0.1 |     |     |     |     | -2.747013 | -2.822965 | 0.421898 | 0.416492 | 0.452003 | 0.435928
|     | 0.2 |     |     |     |     | -2.747014 | -2.822971 | 0.392954 | 0.388011 | 0.382698 | 0.381267
|     | 0.3 |     |     |     |     | -2.747014 | -2.822971 | 0.395267 | 0.390203 | 0.372428 | 0.371623
| 0.1 |     | 0.1 |     |     |     | -2.747014 | -2.822971 | 0.397443 | 0.392227 | 0.364924 | 0.361849
| 0.3 |     | 0.2 |     |     |     | -2.747014 | -2.822971 | 0.351619 | 0.345907 | 0.307034 | 0.301797
| 0.5 | 0.1 |     |     |     |     | -2.747014 | -2.822971 | 0.401337 | 0.395672 | 0.436474 | 0.419239
|     | 0.3 |     |     |     |     | -2.127096 | -2.162213 | 0.369257 | 0.364358 | 0.335768 | 0.321009
|     | 0.5 | 0.1 |     |     |     | -2.747014 | -2.822971 | 0.423148 | 0.416551 | 0.412101 | 0.395545
|     | 0.6 | 0.2 |     |     |     | -2.747014 | -2.822971 | 0.410337 | 0.397678 | 0.364747 | 0.343923
| 1.2 | 0.1 | 0.3 |     |     |     | -2.747014 | -2.822971 | 0.415999 | 0.409827 | 0.310434 | 0.487900
|     | 0.6 | 0.4 |     |     |     | -2.747014 | -2.822971 | 0.385889 | 0.380608 | 0.358546 | 0.346166
|     | 1.2 | 0.5 |     |     |     | -2.747014 | -2.822971 | 0.367188 | 0.362161 | 0.264204 | 0.256686

Table 2 Comparison of the values of $f'(0)$ when $R = M = Du = Sr = Sc = 0$, $m = 0.5$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$h_1$</th>
<th>Khader and Megahed [36]</th>
<th>Present study</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0</td>
<td>$-0.924828$</td>
<td>$-0.924829$</td>
</tr>
<tr>
<td>0.25</td>
<td>0.2</td>
<td>$-0.733395$</td>
<td>$-0.733396$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>$-0.759570$</td>
<td>$-0.759570$</td>
</tr>
</tbody>
</table>

Figs. 12 and 13 show that the temperature and concentration fields are reduced with the rise in thermal slip parameter. However, the increase in thermal jump parameter improves the thermal accommodation coefficient. This can lead to reduce the thermal diffusion toward the flow. Due to this, temperature boundary layer also gets thinner. It is clear to mention that the non-Newtonian fluid occupies higher temperature field compared with Newtonian fluid. It is remarked from Figs. 14 and 15 that the temperature is an increasing function of concentration slip parameter $h_3$ and concentration is a decreasing function of $h_1$. Physically, higher the variation in concentration slip causes to release the heat energy to the flow. This may be the reason for rise in temperature field. It is interesting to mention that the non-Newtonian fluid is highly influenced in temperature and concentration fields compared with Newtonian fluid.

Table 1 shows the effect of non-dimensional parameters on three parameters (skin friction coefficient, local Nusselt number, Sherwood number) in two cases (Newtonian fluid and non-Newtonian fluid). It is apparent that rise in the magnetic field parameter lessens the friction coefficient along with the heat transfer rate and mass transfer rate. But Soret number and thermal slip parameter show mixed behavior on heat and mass transfer rate. An increase in the Dufour number rises the heat and mass transfer rate. Rise in the velocity slip parameter depreciates the heat transfer rate and mass transfer rate but improves the friction factor. The diffusion slip parameter lessens the Sherwood number as well as the local Nusselt number. Table 2 displays the validation of present results with the existed. We found a good agreement of the present results with the published work under some limited cases.

4. Conclusions

This paper deals with the Soret and Dufour effects on the Williamson fluid flow across a variable thickness stretching sheet by viewing slip parameters. The influence of different dimensionless parameters on the three usual profiles (velocity, temperature, concentration) together with skin friction coefficient, local Nusselt number (heat transfer rate) and Sherwood number (mass transfer rate) are discussed through plots and tables. From the resultants of the present numerical study, conclusions can be referred as follows:

- The magnetic interaction parameter $M$ improves the temperature profiles of the flow.
- Rise in Dufour number enhances the heat transfer rate.
- Rise in Soret number enhances the heat transfer rate and reduces the mass transfer rate.
- Velocity slip parameter rises the skin friction coefficient, Sherwood number but depreciates the Nusselt number.
- The thermal slip parameter has tendency to boost up the temperature field which can lead to reduce the heat transfer rate.
- The rate of heat transfer performance is very high in Newtonian fluid compared with non-Newtonian fluid.

References


