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Mechanics of accommodation of the human eye

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Abstract

The classical Helmholtz theory of accommodation has, over the years, not gone unchallenged and most recently has been opposed by Schachar at al. (1993) (Annals of Ophthalmology, 25 (1) 5–9) who suggest that increasing the zonular tension increases rather than decreases the power of the lens. This view is supported by a numerical analysis of the lens based on a linearised form of the governing equations. We propose in this paper an alternative numerical model in which the geometric non-linear behaviour of the lens is explicitly included. Our results differ from those of Schachar et al. (1993) and are consistent with the classical Helmholtz mechanism. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Helmholtz theory; Zonular tension; Numerical analysis

1. Introduction

According to the classical Helmholtz theory (Helmholtz 1909; Fincham 1937; Fisher, 1977) in the unaccommodated state the lens of the human eye is flattened by passive tension in the zonular fibres that run radially from its equator to the ciliary body on the wall of the globe. Accommodation is achieved by contraction of the annular ciliary muscle. This reduces the tension in the zonular fibres and causes the curvature of the lens surfaces to increase with a consequential increase in optical power. This view has recently been challenged by Schachar, Huang and Huang (1993) and Schachar, Cudmore, Torti, Black and Huang (1994), who suggest that a contraction of the ciliary muscle increases zonular tension and tends to increase (rather than decrease) the lens power. This proposal is supported by a simple physical model consisting of a gelatine-filled balloon (Schachar et al., 1994) and also by numerical studies of the accommodation process (Schachar et al., 1993). It is not immediately clear why this numerical model gives results that are so profoundly counter-intuitive; this is the question that we seek to address in this paper.

Schachar et al. (1993) modelled the lens as an axisymmetric elastic membrane (the capsule) enclosing an incompressible fluid (the matrix). The shape of the capsule was derived from measured data (Brown, 1973) from the unaccommodated lens of a 29-year old. An energy method was used to compute the capsule displacements when a zonular tension was applied, subject to the constraint that the enclosed volume was conserved. This analysis is subject to several important assumptions. The lens matrix was assumed to be incompressible, which is the usual assumption although published measurements of Poisson's ratio do not appear to be available. The potential role of the vitreous in influencing the mechanical behaviour of the lens was ignored. The analysis was based on linear theory and therefore neglected the important geometric non-linear terms that govern the behaviour of membranes as displacements become large. This third feature of the Schachar et al. (1993) analysis is discussed below.

1.1. Geometric non-linear behaviour of membranes

Geometric non-linear behaviour of membranes is

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well-understood by, for example, engineers working on the design of fabric roof structures (e.g. Barnes, 1994) and is demonstrated by the following simple example. Consider a spherical elastic membrane of initial radius R_o which is inflated to radius R by internal pressure p as shown in Fig. 1a. The membrane has thickness t, a Young's modulus E and (for simplicity) zero Poisson's ratio. Hooke's Law combined with the requirements of radial equilibrium gives:

$$\frac{dp}{dR}=\frac{2t(E-\sigma)}{R^2} \tag{1}$$

where the hoop and meridional stresses are both equal to σ.

In conventional small displacement structural analysis, Eq. (1) would be linearised to give:

$$\frac{dp}{du}=\frac{2Et}{R_o^2} \tag{2}$$

where u is radial displacement of the membrane. Eq. (2) is adequate provided that the radial displacement remains small compared with R_o. If changes of radius become large, however, then Eq. (2) becomes unsatisfactory. In this case, the response of the membrane is obtained by integration of Eq. (1) to give:

$$p=\frac{2Et}{R}\ln\left(\frac{R}{R_o}\right) \tag{3}$$

Eqs. (2) and (3) are plotted in Fig. 1b. The solutions differ significantly for large values of R/R_o. In particular, Eq. (3) indicates that pressure reduces with radius after the membrane reaches a certain size and this explains why a balloon becomes easier to inflate as it becomes bigger.

Membranes support an applied pressure by the combined action of geometric curvature and in-plane stresses. In conventional linear theory, the geometry of the membrane is not modified by the displacements. This assumption becomes increasingly unacceptable when displacements become large, as illustrated by the above example. There is no general rule about how large is too large; this will depend on the system being modelled.

2. Numerical model of accommodation

We have carried out a numerical study of the lens capsule using the finite element package ABAQUS (ABAQUS–Hibbitt, Karlsson and Sorensen, UK, Ltd.) which is widely used in civil and mechanical engineering. These calculations were based on the assumption (also adopted by Schachar et al., 1993) that the lens matrix is incompressible. Two sets of analyses were carried out with the same initial lens geometry that was used by Schachar et al. (1993). One set of analyses was based on a linearised form of the equations that governs the mechanical behaviour of the lens capsule. The other set was based on an analysis procedure in which geometric non-linear terms were included in the governing equations and the mesh geometry was updated during the calculation process. The capsule was modelled with a single layer of 55 8-node elastic isoparametric quadrilateral elements as shown in Fig. 2. The elements were assigned a geometric thickness, t, of 15 μm; values of Young's modulus, E, were adopted such that the product Et matched closely the variation assumed by Schachar et al. (1993). Poisson's ratio for each element was set to 0.47, which was the value adopted by Schachar et al. (1993). Computations were carried out by applying a zonular tension, F, to the lens equator and using a numerical scheme to adjust the pressure within the capsule to ensure that the enclosed volume was conserved, as it must be for an incompressible matrix.

The computed displacements from the linear analysis agree well, both quantitatively and qualitatively, with those given by Schachar et al. (1993). A deformed mesh (with unfactored displacements) for an equatorial strain of 6.25%, is shown in Fig. 3a. Equatorial strain, for a linear analysis, is defined:

$$\varepsilon=\frac{u}{R_o} \tag{4}$$

where R_o is the initial equatorial radius of the lens and u is the radial displacement at the lens equator. In this analysis $R_o = 4.3$ mm and the displacements shown in Fig. 3a correspond to a radial displacement, u, of 0.27 mm. In a linearised analysis such as this, the geometry of the lens capsule cannot change and equilibrium may only be achieved by the development of appropriate membrane stresses. These stresses are developed as a

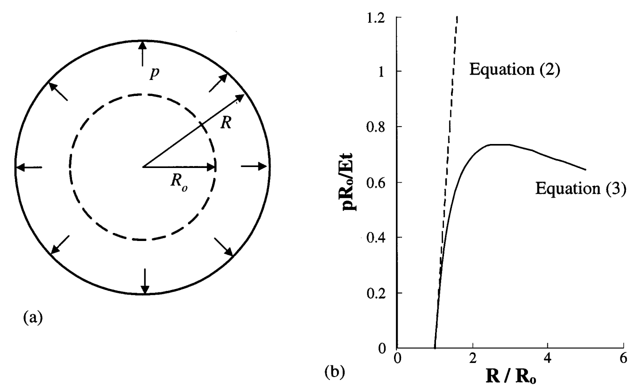

Fig. 1. (a) Expansion of elastic spherical membrane of initial radius R_o, subjected to internal pressure p resulting in an increased radius R. (b) Non-dimensional plot of pR_o/Et, (where Et is the product of Young's modulus and membrane thickness) as a function of fractional radial expansion, R/R_o. Eq. (2) refers to a linear analysis; Eq. (3) refers to an analysis in which geometric non-linearities are included. Note the significant difference between the two results for R/R_o greater than about 1.5.

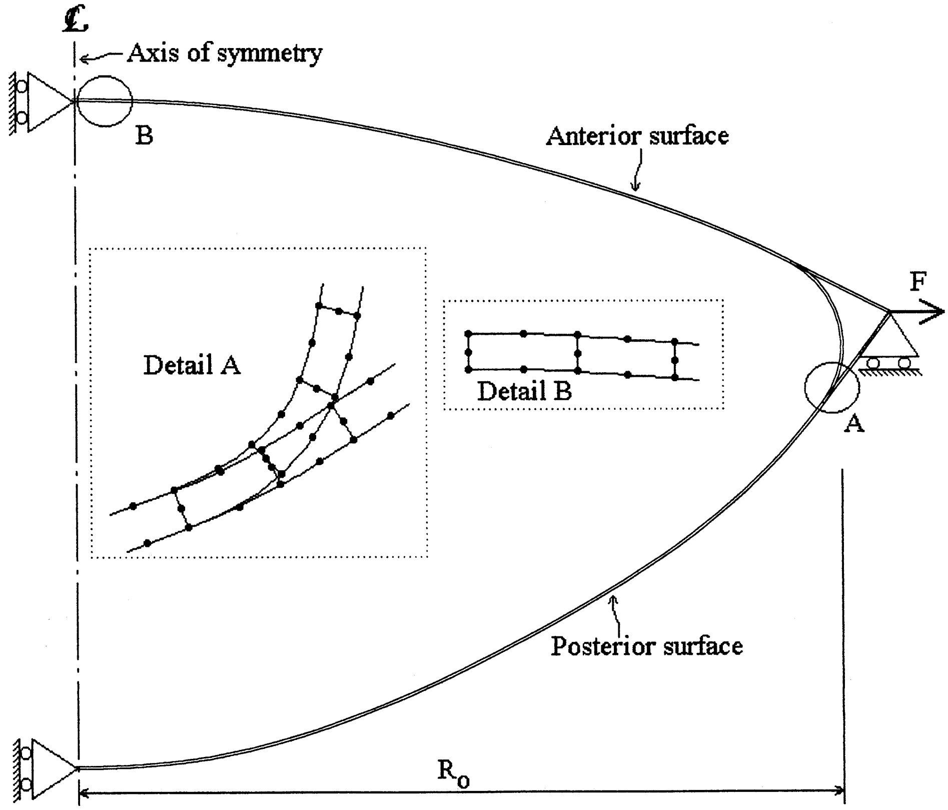

Fig. 2. Axisymmetric finite element mesh of eight-noded quadrilateral elements used to model the lens capsule. Detail A shows the connection between the zonules and the capsule; detail B shows the detail on the lens axis. Young's modulus, E, was varied with position to ensure that the product Et (where t is the membrane thickness) followed closely that adopted by Schachar et al. (1993). R_o is the initial equatorial radius (equal to 4.3 mm) and F is the zonular force that was applied during the finite element analysis. On the lens axis, the capsule is constrained to move only in an axial direction; the zonular force is applied at right-angles to the lens axis.

result of strains within the membrane and result in the prediction of displacements that follow the curious undulating pattern shown in Fig. 3a. The computed curvature of the lens surfaces is clearly subject to a large variability. On the lens axis the curvature of both the posterior and anterior surface are both seen to increase. This confirms the main result of Schachar's analysis, and leads to the suggestion that the optical power of the lens is increased by the application of zonular tension.

The analysis was repeated using the geometric non-linear formulation available in ABAQUS. A difficulty exists, however, because in a non-linear analysis the lens behaviour will depend on the stresses in the capsule at the start of the analysis (i.e. when the zonular tension is zero). For simplicity, however, we assumed that the initial stresses were all zero. A deformed mesh, for an equatorial strain of 6.25%, is shown in Fig. 3b. Note that for a geometric non-linear analysis the equatorial strain is defined as:

$$\varepsilon = \frac{R - R_o}{R_o} \qquad (5)$$

where R is the current equatorial radius. Fig. 3b shows a mode of behaviour that differs fundamentally from that obtained from the linear analysis. For a non-linear analysis, the application of a zonular tension is seen to decrease the anterior curvature on the lens axis leading to a reduction in optical power. Moreover, changes in the position and curvature of the posterior surface are negligible; this is consistent with the classical view of the accommodation process.

The lens geometry adopted by Schachar et al. (1993) (and used as the initial geometry in the calculations described above) is based on measured curvatures of an *unaccommodated* eye whereas the accommodated geometry is thought to be more appropriate initial state. To improve the model, further non-linear calculations were carried out in which the initial geometry was based on measured lens curvatures (Brown, 1973) for a 29-year old subjected to an accommodation demand of 10 Dioptres; this was thought to represent a fully accommodated state. These new analyses were based on a mesh consisting of 245 8-node isoparametric quadrilateral elements. The results of these analyses were used to obtain the variation of lens power with equatorial strain shown in Fig. 4. To calculate these lens power data from the

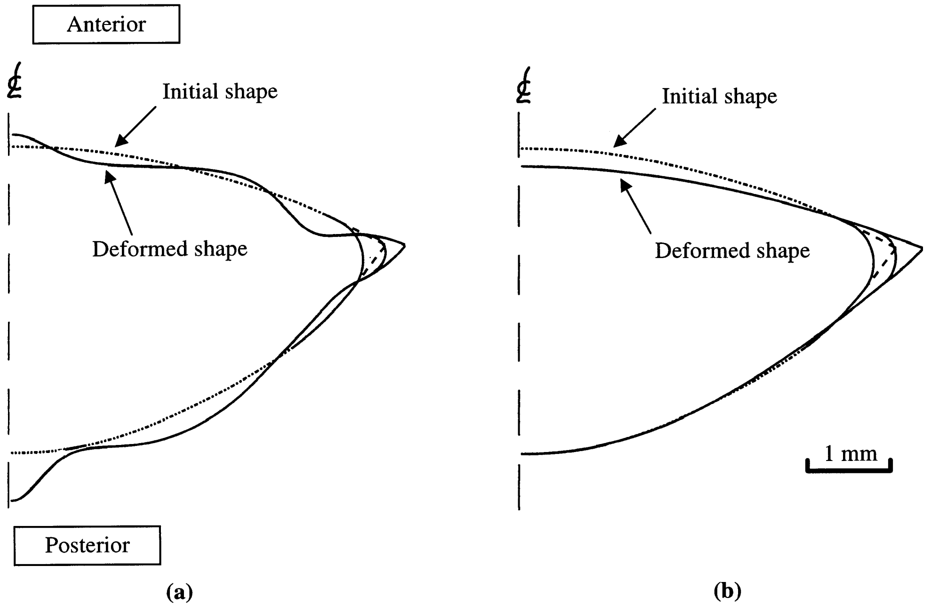

Fig. 3. (a) Deformed mesh obtained from a linear analysis for an equatorial strain, u/R_o, of 6.25%, where u is radial displacement of the lens equator and R_o is the initial lens equatorial radius. Note the undulating shape of the deformed capsule and the increased curvature on the lens axis. (b) Deformed mesh obtained from a geometric non-linear analysis for an equatorial strain, $(R-R_o)/R_o$, of 6.25% where R is radius of the lens equator after application of the zonular force. Note that the zonular force causes the curvature of the anterior surface to *decrease* on the lens axis. The scale bar shown applies to both horizontal and vertical axes

finite element results, a least-squares procedure was used to fit spherical surfaces to the portion of the computed capsule geometry within a radial distance of 0.8 mm of the lens axis for both the anterior and posterior surfaces. The curvatures of these spheres, and the thickness of the lens, were then used to determine the optical power of the lens on its axis using the conventional thick lens formula, with refractive indices of 1.336 for the aqueous and vitreous, and 1.42 for the matrix (these are the same as the values adopted by Schachar et al., 1993). Fig. 4 indicates conventional Helmholtz behaviour with optical power reducing with increasing equatorial strain. The data show that a strain increase of 10% from the initial state causes a power reduction of about 10 Dioptres, giving an average power reduction rate of approximately 1 Dioptre per percent strain. Moreover, for all the strain values examined, the power reduced with strain, which is contrary to the Schachar result.

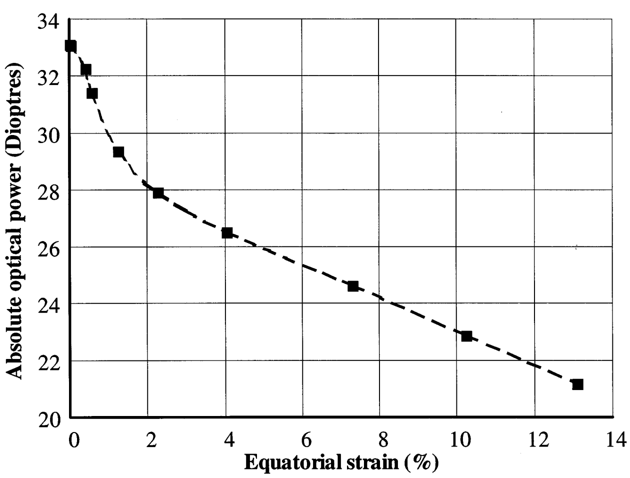

Fig. 4. Variation of lens power with equatorial strain for a non-linear analysis taking the accommodated geometry (Brown, 1973) as the initial state. Equatorial strain is defined as $(R-R_o)/R_o$, where R_o is the equatorial radius for zero zonular force (and zero capsular tension). This plot is consistent with the Helmholtz mechanism (power reducing with equatorial strain). For strains less than 2%, the power falls more rapidly than for larger strains.

2.1. Comparison with physiological measurements

In vitro measurements (Glasser & Campbell, 1998; Glasser, 1998) of lens equatorial diameter and lens power show that in young (11–20 year old) eyes the optical power of the lens reduced at a rate of between 0.69 and 1.25 Dioptres per percent equatorial strain. In vivo measurements (Storey & Rabie, 1985) on young subjects indicated a rate of optical power reduction of 1.2 Dioptres per percent equatorial strain. These results compare favourably with our numerical data.

3. Conclusion

It is not claimed that these numerical results capture all of the complex physiological aspects of the accommodation process. It is suggested, however, that a correct mathematical treatment of the lens capsule is needed to carry out meaningful analyses of this problem. Our results show that for eyes that are relatively young (i.e. not presbyopic) the Helmholtzian mechanism of accommodation remains the most likely. It must of course be remembered that our model (and Schachars) neglects age-related changes in the mechanical and optical properties of the lens matrix. It is generally accepted that these processes will modify the behaviour of the lens and more complex analyses will be needed to investigate this.

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