Hydromagnetic natural convection flow with heat and mass transfer of a chemically reacting and heat absorbing fluid past an accelerated moving vertical plate with ramped temperature and ramped surface concentration through a porous medium

G.S. Seth a,*, S.M. Hussain b, S. Sarkar a

a Department of Applied Mathematics, Indian School of Mines, Dhanbad 826004, India
b Department of Mathematics, O. P. Jindal Institute of Technology, Raigarh 496109, India

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Abstract
Unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and heat absorbing fluid past an accelerated moving vertical plate with ramped temperature and ramped surface concentration through a porous medium in the presence of thermal and mass diffusions is studied. The exact solutions of momentum, energy and concentration equations, under the Boussinesq approximation, are obtained in closed form by Laplace Transform technique. The expressions for skin friction, Nusselt number and Sherwood number are also derived. The variations in fluid velocity, fluid temperature and species concentration are displayed graphically whereas numerical values of skin friction, Nusselt number and Sherwood number are presented in tabular form for various values of pertinent flow parameters. Natural convection flow near a ramped temperature plate with ramped surface concentration is also compared with the flow near an isothermal plate with uniform surface concentration.

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1. Introduction

The problems of hydromagnetic convective flow in a porous medium have drawn considerable attentions of several researchers owing to its importance in various scientific and technological applications viz. problems of boundary layer...
flow control, plasma studies, geothermal energy extraction, metallurgy, chemical, mineral and petroleum engineering, etc. and on the performance of so many engineering devices using electrically conducting fluids, namely, MHD generators, MHD pumps, MHD accelerators, MHD flow-meters, nuclear reactors, plasma jet engines, etc. Raptis and Kafousias [1] investigated steady hydromagnetic free convection flow through a porous medium bounded by an infinite vertical plate with constant suction velocity. Raptis [2] discussed unsteady two-dimensional natural convection flow of an electrically conducting, viscous and incompressible fluid along an infinite vertical plate embedded in a porous medium. Chamkha [3] studied unsteady MHD free convection flow through a porous medium supported by a surface. Chamkha [4] also studied MHD natural convection flow near an isothermal inclined surface adjacent to a thermally stratified porous medium. Aldoss et al. [5] investigated combined free and forced convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Kim [6] analyzed unsteady MHD free convection flow past a moving semi-infinite vertical porous plate embedded in a porous medium with variable suction. Theoretical/experimental investigations of convective boundary layer flow with heat and mass transfer induced due to a moving surface with a uniform or non-uniform velocity play an important role in several manufacturing processes in industry which include the boundary layer flow along material handling conveyers, extrusion of plastic sheets, cooling of an infinite metallic plate in cooling bath, glass blowing, continuous casting and levitation, design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of trees, damage of crops due to freezing, common industrial sight especially in power plants, etc. In keeping view the importance of such study, Jha [7] considered hydromagnetic free convection and mass transfer flow past a uniformly accelerated moving vertical plate through a porous medium. Ibrahim et al. [8] investigated unsteady hydromagnetic free convection flow of micro-polar fluid and heat transfer past a vertical porous plate through a porous medium in the presence of thermal and mass diffusions with a constant heat source. Makinde and Sibanda [9] studied MHD mixed convective flow with heat and mass transfer past a vertical plate embedded in a porous medium with constant wall suction. Makinde [10] analyzed hydromagnetic mixed convection flow and mass transfer over a vertical porous plate with constant heat flux embedded in a porous medium. Makinde [11] also investigated MHD boundary layer flow with heat and mass transfer over a moving vertical plate with a convective surface boundary condition.

It is noticed that there may be an appreciable temperature difference between the surface of the solid body and ambient fluid in so many fluid flow problems of practical interests. This prompted many researchers to consider temperature dependent heat sources and/or sinks, which may have strong influence on heat transfer characteristics [12]. The research studies related to heat generating and/or heat absorbing fluid flow are of considerable importance in several physical problems viz. fluids undergoing exothermic and/or endothermic chemical reaction [12], its applications in the field of nuclear energy [13], convection in Earth’s mantle [14], post accident heat removal [15], fire and combustion modeling [16], development of metal waste from spent nuclear fuel [17], etc. Exact mathematical modeling of internal heat generation/absorption is very much complicated. It is found that some simple mathematical models yet idealized may present their average behavior for most of the physical situations. Taking into consideration of this fact Sparrow and Cess [18] discussed the effects of temperature dependent heat absorption in their research study on steady stagnation point flow and heat transfer. Moalem [19] investigated steady heat transfer in a porous medium with temperature dependent heat source. Chamkha and Khaled [20] considered hydromagnetic combined heat and mass transfer by natural convection from a permeable vertical plate embedded in a fluid saturated porous medium in the presence of heat generation or absorption. Kamel [21] investigated unsteady hydromagnetic convection flow due to heat and mass transfer through a porous medium bounded by an infinite vertical porous plate with temperature dependent heat

<table>
<thead>
<tr>
<th>Nomenclature</th>
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<tr>
<td>$B_0$ uniform magnetic field</td>
<td>$Q_0$ heat absorption coefficient</td>
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<tr>
<td>$C'$ species concentration</td>
<td>$S_c$ Schmidt number</td>
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<tr>
<td>$D$ chemical molecular diffusivity</td>
<td>$T'$ fluid temperature</td>
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<tr>
<td>$G_r$ solutal Grashof number</td>
<td>$u'$ fluid velocity in $\eta$ direction</td>
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<tr>
<td>$K_{1}$ permeability parameter</td>
<td>$\zeta'$ thermal diffusivity</td>
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<tr>
<td>$K_{1}'$ permeability of porous medium</td>
<td>$\beta'$ volumetric coefficient of thermal expansion for species concentration</td>
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<tr>
<td>$P_r$ Prandtl number</td>
<td>$\nu$ kinematic coefficient of viscosity</td>
</tr>
<tr>
<td>$R$ dimensionless constant</td>
<td>$\phi$ dimensionless heat absorption coefficient</td>
</tr>
<tr>
<td>$T$ dimensionless fluid temperature</td>
<td>$\rho$ fluid density</td>
</tr>
<tr>
<td>$u$ dimensionless fluid velocity in $\eta$ direction</td>
<td>$\beta''$ volumetric coefficient of expansion</td>
</tr>
<tr>
<td>$C$ dimensionless species concentration</td>
<td>$\sigma$ electrical conductivity</td>
</tr>
<tr>
<td>$C_p$ specific heat at constant pressure</td>
<td>$\sigma$ electrical conductivity</td>
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sources and sinks. Chamkha [22] studied unsteady hydromagnetic two dimensional convective laminar boundary layer flow with heat and mass transfer of a viscous, incompressible, electrically conducting and temperature dependent heat absorbing fluid along a semi infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field. Makinde [23] investigated heat and mass transfer by MHD mixed convection stagnation point flow toward a vertical plate embedded in a highly porous medium with radiation and internal heat generation. In all these investigations, numerical/analytical solution is obtained by assuming conditions for the velocity and temperature at the plate as continuous and well defined. However, there are several problems of practical interests which may require non-uniform or arbitrary conditions at the plates. Keeping in view this fact, several researchers, namely, Hayday et al. [24], Kelleher [25], Kao [26], Lee and Yovanovich [27] and Chandran et al. [28] studied natural convection flow from a vertical plate with step discontinuities in the surface temperature considering different aspects of the problem. Patra et al. [29] investigated the effects of radiation on natural convection flow of a viscous and incompressible fluid near a vertical flat plate with ramped temperature. They compared the effects of radiative heat transfer on natural convection flow near a ramped temperature plate with the flow near an isothermal plate. Seth and Ansari [30] investigated unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and temperature dependent heat absorbing fluid past an impulsively moving vertical plate with ramped temperature in a porous medium taking into account the effects of thermal diffusion. Subsequently, Seth et al. [31] extended the problem studied by Seth and Ansari [30] to consider the effects of rotation on flow-field.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid. Chemical reactions can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. These processes take place in numerous industrial applications viz. polymer production, manufacturing of ceramics or glassware, food processing, etc. Afify [32] studied the effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field. Muthucumaraswamy and Chandrakala [33] investigated radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. Ibrahim et al. [34] analyzed the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source. Bakr [35] discussed the effects of chemical reaction on MHD free convection and mass transfer flow of a micro-polar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. Chamkha [36] studied MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. Chamkha et al. [37] discussed the effects of Joule heating, chemical reaction and thermal radiation on unsteady hydromagnetic natural convection boundary layer flow with heat and mass transfer of a micro-polar fluid from a semi-infinite heated vertical porous plate in the presence of a uniform transverse magnetic field. Bhattcharya and Layek [38] obtained similarity solution of MHD boundary layer flow with mass diffusion and chemical reaction over a porous flat plate with suction/blowing. Recently, Mohamed et al. [39] investigated unsteady MHD free convection heat and mass transfer boundary layer flow of viscous, incompressible, optically thick and electrically conducting fluid through a porous medium along an impulsively moving hot vertical plate in the presence of homogeneous chemical reaction of first order and temperature dependent heat sink. They obtained analytical solution of the governing equations in closed form by Laplace transform technique.

Objective of present investigation is to study unsteady hydromagnetic natural convection flow with heat and mass transfer of a chemically reacting and heat absorbing fluid past an accelerated moving vertical plate with ramped temperature and ramped surface concentration through a porous medium. Such study may find application in solar collection systems, fire dynamics in insulations, geothermal energy systems, catalytic reactors, nuclear waste repositories, recovery of petroleum products and gases (e.g. CBM: Coal Bed Methane and UCG: Underground Coal Gasification), etc.

2. Formulation of the problem and its solution

Consider unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and temperature dependent heat absorbing fluid past an accelerated moving infinite vertical plate embedded in a porous medium in the presence of thermal and mass diffusions. Choose the co-ordinate system in such a way that \( x \)-axis is along the plate in upward direction, \( y \)-axis normal to the plane of the plate and \( z \)-axis perpendicular to \( x'y' \)-plane. The fluid is permeated by uniform transverse magnetic field \( B_0 \) applied parallel to \( y' \)-axis. Initially, i.e. at time \( t \leq 0 \), both the fluid and plate are at rest and maintained at uniform temperature \( T_{x0} \) and uniform surface concentration \( C_{w0} \). At time \( t > 0 \), plate starts moving in \( x' \)-direction against the gravitational field with time dependent velocity \( U(t) \). Temperature of the plate is raised or lowered to \( T_{x0} + (T_{x1} - T_{x0})t/t_0 \) and the level of concentration at the surface of the plate is raised or lowered to \( C_{w0} + (C_{w1} - C_{w0})t/t_0 \) when \( 0 < t < t_0 \). Thereafter, i.e. at \( t > t_0 \), plate is maintained at the uniform temperature \( T_{x1} \) and the level of concentration at the surface of the plate is preserved at uniform concentration \( C_{w1} \). Here \( t_0 \) is characteristic time. It is assumed that there exists a homogeneous chemical reaction of first order with constant rate \( K_2 \) between the diffusing species and the fluid. The schematic diagram of the physical problem is shown in Fig. 1. Since the plate is of infinite extent along \( x' \) and \( z' \)-directions and is electrically non-conducting, all physical quantities except pressure depend on \( y' \) and \( t' \) only. The induced magnetic field produced by fluid motion is neglected in comparison with applied one. This statement is justified because magnetic Reynolds number is very small for liquid metals and partially ionized fluid [40]. Also no external electric field is applied so the effect of polarization of fluid is negligible. This corresponds to the case where no energy is added or extracted from the fluid by electrical means [40].

Taking into consideration the assumptions made above, the governing equations for natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and temperature dependent heat absorbing fluid through a porous medium in the presence of
thermal and mass diffusions, under Boussinesq approximation, are given by
\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= \frac{\partial \rho'}{\partial t} - \frac{\sigma B_e^2}{\rho} \frac{\partial T'}{\partial t} - \frac{v}{K_i'} u' + g \beta'(T' - T_w) + g \beta'(C - C_w'), \quad (2.1) \\
\frac{\partial T'}{\partial t} &= \frac{\partial T'}{\partial t} + \frac{Q_g}{\rho C_p} (T' - T_w), \quad (2.2) \\
\frac{\partial C'}{\partial t} &= D \frac{\partial C'}{\partial t} - K_e (C - C_w'), \quad (2.3)
\end{align*}
\]

Appropriate initial and boundary conditions for fluid velocity, fluid temperature and species concentration are given by
\[
\begin{align*}
\rho' &= 0, \quad T' = T_w' \quad C' = C_w' \quad \text{for} \quad y' \geq 0 \quad \text{and} \quad t' < 0, \quad (2.4a) \\
\rho' &= U(t') \quad \text{at} \quad y' = 0 \quad \text{for} \quad t' > 0, \quad (2.4b) \\
T' &= T_w' + (T_w' - T_w)(t'/t_0), \quad C' = C_w' + (C_w' - C_w')(t'/t_0) \quad \text{at} \quad y' = 0 \quad \text{for} \quad 0 < t' \leq t_0, \quad (2.4c) \\
T' &= T_w', \quad C' = C_w' \quad \text{at} \quad y' = 0 \quad \text{for} \quad t' > t_0, \quad (2.4d) \\
\rho' &\to 0, \quad T' \to T_w', \quad C \to C_w' \quad \text{as} \quad y' \to \infty \quad \text{for} \quad t' > 0. \quad (2.4e)
\end{align*}
\]

In order to represent Eqs. (2.1)-(2.3) along with initial and boundary conditions Eq. (2.4) in dimensionless form, following dimensionless quantities and parameters are introduced.
\[
\begin{align*}
\frac{y'}{U_0 t_0}, \quad \frac{u'}{U_0}, \quad \frac{t'}{t_0}, \quad \frac{T'}{T_w}, \quad \frac{T_w'}{T_w} \quad \text{for} \quad y' \geq 0 \quad \text{and} \quad t' \leq 0, \quad (2.5) \\
C = (C - C_w')/(C_w' - C_w'), \quad G_e = \frac{v g \beta'}{T_w U_0^3}, \quad G_w = \frac{v g \beta'}{T_w U_0^3}, \quad G_t = \frac{v g \beta'}{T_w U_0^3}, \quad G_f = \frac{v g \beta'}{T_w U_0^3}, \quad G_L = \frac{v g \beta'}{T_w U_0^3}, \quad G_p = v / \beta, \quad G_s = v / \beta, \quad G_L = \frac{v g \beta'}{T_w U_0^3}, \quad G_s = \frac{v g \beta'}{T_w U_0^3}, \quad (2.6)
\end{align*}
\]

Using dimensionless quantities and parameters defined in (2.5), Eqs. (2.1)-(2.3), in dimensionless form, become
\[
\begin{align*}
\frac{\partial u'}{\partial t} - \frac{\partial^2 u'}{\partial y^2} = M u' - \frac{u}{K_i'} + G_T + G_C, \quad (2.7)
\end{align*}
\]

According to above non-dimensionlization process, characteristic time \(t_0\) may be defined as
\[
t_0 = \frac{v}{U_0^2}, \quad (2.9)
\]

Appropriate initial and boundary conditions (2.4), in dimensionless form, assume the following form
\[
\begin{align*}
\rho &= 0, \quad T = 0, \quad C = 0 \quad \text{for} \quad y > 0 \quad \text{and} \quad t < 0, \quad (2.10a) \\
\rho &= U(t) \quad \text{at} \quad y = 0 \quad \text{for} \quad t > 0, \quad (2.10b) \\
T &= t, \quad C = t \quad \text{at} \quad y = 0 \quad \text{for} \quad 0 < t \leq 1, \quad (2.10c) \\
T = 1, \quad C = 1 \quad \text{at} \quad y = 0 \quad \text{for} \quad t > 1, \quad (2.10d) \\
\rho \to 0, \quad T \to T_w, \quad C \to C_w \quad \text{as} \quad y \to \infty \quad \text{for} \quad t > 0, \quad (2.10e)
\end{align*}
\]

where \(U(t) = U(t)/U_0\).

The fluid flow described by the Eqs. (2.6)-(2.8) subject to initial and boundary conditions (2.10) is quite general. In order to analyze the flow features of the fluid flow we now consider a particular case of interest, namely, uniformly accelerated movement of the plate, i.e. \(F(t) = R t\) where \(R\) is dimensionless constant.

Eqs. (2.6)-(2.8) are solved analytically with the help of Laplace transform technique subject to the initial and boundary conditions (2.10) and the exact solutions for fluid velocity \(F(y, t), \) fluid temperature \(T(y, t)\) and species concentration \(C(y, t)\) are obtained and are presented after simplification in the following form
\[
\begin{align*}
\rho(y, t) &= \frac{R}{2} \left( \left( \frac{t + \sqrt{2}y}{2} \right) d_1 + \left( \frac{t - \sqrt{2}y}{2} \right) d_2 \right) \\
- \frac{d_1}{d_1} \left( F_1(y, t) - H(t-1)F_1(y, t-1) \right) \\
- \frac{d_1}{d_1} \left( F_2(y, t) - H(t-1)F_2(y, t-1) \right), \quad (2.11)
\end{align*}
\]
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\[ T(y, t) = P(y, t) - H(t - 1)P(y, t - 1), \]
\[ C(y, t) = Q(y, t) - H(t - 1)Q(y, t - 1), \]
where
\[ F_1(y, t) = \int \frac{\phi}{\theta} \left( e^{-\frac{1}{2} \left( e^{-\frac{1}{2} \left( e^{-\frac{1}{2}} \right)} \right)} e^{\left( \sqrt{\langle \lambda + \delta_q \rangle} t + \frac{y}{\sqrt{t}} \right)} + e^{-\frac{1}{2} \left( e^{-\frac{1}{2} \left( e^{-\frac{1}{2}} \right)} \right)} e^{\left( \sqrt{\langle \lambda + \delta_q \rangle} t + \frac{y}{\sqrt{t}} \right)} \right) + e^{\frac{1}{2} \left( e^{-\frac{1}{2} \left( e^{-\frac{1}{2}} \right)} \right)} e^{\left( \sqrt{\langle \lambda + \delta_q \rangle} t + \frac{y}{\sqrt{t}} \right)} \right) d\lambda \]
\[ F_2(y, t) = \int \frac{\phi}{\theta} \left( e^{-\frac{1}{2} \left( e^{-\frac{1}{2} \left( e^{-\frac{1}{2}} \right)} \right)} e^{\left( \sqrt{\langle \lambda + \delta_q \rangle} t + \frac{y}{\sqrt{t}} \right)} + e^{-\frac{1}{2} \left( e^{-\frac{1}{2} \left( e^{-\frac{1}{2}} \right)} \right)} e^{\left( \sqrt{\langle \lambda + \delta_q \rangle} t + \frac{y}{\sqrt{t}} \right)} \right) + e^{\frac{1}{2} \left( e^{-\frac{1}{2} \left( e^{-\frac{1}{2}} \right)} \right)} e^{\left( \sqrt{\langle \lambda + \delta_q \rangle} t + \frac{y}{\sqrt{t}} \right)} \right) d\lambda \]
\[ P(y, t) = \frac{1}{2} \left( t + \frac{y}{\sqrt{t}} \right) d\lambda \]
\[ C(y, t) = \frac{1}{2} \left( t + \frac{y}{\sqrt{t}} \right) d\lambda \]
where
\[ \lambda = \left( M + \frac{1}{K_2} \right), \]  
\[ \delta_1 = G_v/(1 - P_v), \]  
\[ \delta_2 = G_v/(1 - S_v), \]  
\[ \delta_3 = (P_v \phi - \lambda)/(1 - P_v), \]  
\[ \delta_4 = (S_v K_2 - \lambda)/(1 - S_v), \]
and
\[ d_1 = e^{-\sqrt{\lambda} \theta} \left( \sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right), \]
\[ d_2 = e^{-\sqrt{\lambda} \theta} \left( -\sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right), \]
\[ d_3 = e^{\sqrt{\lambda} \theta} \left( \sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right), \]
\[ d_4 = e^{\sqrt{\lambda} \theta} \left( -\sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right), \]
\[ d_5 = e^{\sqrt{\lambda} \theta} \left( \sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right), \]
\[ d_6 = e^{\sqrt{\lambda} \theta} \left( -\sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right). \]

3. Solution in the case of isothermal plate with uniform surface concentration

In order to highlight the effects of ramped temperature and ramped surface concentration on fluid flow, it may be worthwhile to compare such flow with the one near an accelerated moving vertical plate with uniform temperature and uniform surface concentration. Keeping in view the assumptions made in this paper, the solution for fluid velocity, fluid temperature and species concentration for natural convection flow past an accelerated moving vertical isothermal plate with uniform surface concentration is obtained and is expressed in the following form

\[ u(y, t) = \frac{1}{2} \left( \begin{array}{c}
R(t - \frac{y}{\sqrt{t}}) + \frac{\delta_1}{\delta_2} \phi \left( \begin{array}{c}
\delta_2 \left( \sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right)
\end{array} \right) + \frac{\delta_1}{\delta_2} \phi \left( \begin{array}{c}
\sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right)
\end{array} \right) + \frac{\delta_1}{\delta_2} \phi \left( \begin{array}{c}
\sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right)
\end{array} \right) + \frac{\delta_1}{\delta_2} \phi \left( \begin{array}{c}
\sqrt{\lambda} + \frac{y}{\sqrt{\lambda}} \right)
\end{array} \right)
\]

4. Skin friction, Nusselt number and Sherwood number

The expressions for the skin friction \( \tau \), Nusselt number \( Nu \), and Sherwood number \( Sh \), which are measures of the shear stress, rate of heat transfer and rate of mass transfer at the plate respectively, are presented in the following form for the plate with ramped temperature and ramped surface concentration and isothermal plate with uniform surface concentration:
For the plate with ramped temperature and ramped surface concentration
\[
\tau = R \left[ \frac{1}{2\sqrt{\lambda}} + t\sqrt{\lambda} \right] \{ \text{erfc}(\sqrt{\lambda}t) - 1 \} - \sqrt{\frac{t}{\pi}} e^{-\lambda t} \]
\[
- \frac{\delta}{\delta t} \left[ F_0(0, t) - H(t-1)F_0(0, t-1) \right]
\]
\[
- \frac{\delta}{\delta t} \left[ F_0(0, t) - H(t-1)F_0(0, t-1) \right],
\]
where
\[
N_a = \sqrt{P_e \phi \{ \text{erfc}(\sqrt{\lambda}t) - 1 \} - \sqrt{\frac{P_e}{\pi t}} e^{-\phi t}},
\]
\[
S_b = \sqrt{S_K \{ \text{erfc}(\sqrt{K_s t}) - 1 \} - \sqrt{\frac{S_K}{\pi t}} e^{-K_s t}}.
\]
plate with ramped surface concentration and isothermal plate with uniform surface concentration, fluid velocity increases on increasing either $\text{Gr}$ or $G_c$. This implies that fluid velocity is getting accelerated due to enrichment in either thermal buoyancy force or species buoyancy force.

Fig. 5 demonstrates the effects of heat absorption on fluid velocity. It is noticed that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, fluid velocity decreases on increasing $\phi$. This implies that, both for ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, heat absorption tends to retard fluid velocity throughout the boundary layer region. This may be attributed to the fact that the tendency of heat absorption (thermal sink) is to reduce the fluid temperature which causes the strength of thermal buoyancy force to decrease resulting in a net reduction in the fluid velocity.

Fig. 6 displays the effect of chemical reaction on fluid velocity. It is observed that fluid velocity decreases on increasing $\mathcal{K}_2$. This implies that chemical reaction has a retarding influence on fluid flow for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration. This implies that chemical reaction has a retarding influence on fluid flow for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration. Fig. 7 depicts the effects of time on fluid velocity. It is observed that fluid velocity increases on increasing time $t$. This implies that there is an enhancement in fluid velocity for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration with the progress of time. It is noted from Figs. 2–7 that fluid velocity is faster in the case of isothermal plate with uniform surface concentration than that in the case of ramped temperature plate with ramped surface concentration.
The numerical solutions for fluid temperature and species concentration, computed from analytical solutions reported in Sections 2 and 3, are depicted graphically in Figs. 8–11 for different values of heat absorption coefficient $\phi$, chemical reaction parameter $K_2$ and time $t$. It is observed from Figs. 8–11 that, fluid temperature and species concentration are maximum at the surface of the plate and decrease properly on increasing boundary layer coordinate $y$ to approach free stream value. Figs. 8 and 9 illustrate the effects of heat absorption coefficient $\phi$ and time $t$ on fluid temperature. It is evident that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, fluid temperature decreases on increasing $\phi$ whereas it increases on increasing time $t$. This implies that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, heat absorption has a tendency to reduce the fluid temperature and there is a rise in fluid temperature with the progress of time in the boundary layer region. Figs. 10 and 11 demonstrate the effects of chemical reaction parameter $K_2$ and time $t$ on species concentration. It is noticed that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, species concentration decreases on increasing $K_2$ whereas it increases on increasing time $t$. This implies that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, chemical reaction tends to reduce species concentration and there is enrichment in species concentration with the progress of time. It may be noted from Figs. 8–11 that the fluid temperature and species concentration are lower in the case of ramped temperature plate with ramped surface concentration than that in the case of isothermal plate with uniform surface concentration.

The numerical values of skin friction $C_0$, computed from the analytical expressions reported in Section 4, are presented in tabular form in Tables 1–3 for various values of $M$, $G_r$, $G_s$, $K_2$, $\phi$ and $t$ taking $R = 1$, $K_1 = 0.5$, $S_e = 0.22$ and $P_r = 0.71$ whereas those of Nusselt number $N_u$ and Sherwood number $S_h$ calculated from the analytical expressions reported in Section 4, are exhibited in tabular form in Tables 4 and 5 for different values of $\phi$, $K_2$ and $t$.

It is found from Table 1 that, for ramped temperature plate with ramped surface concentration, skin friction $C_0$ increases on increasing time $t$ whereas, for isothermal plate with uniform surface concentration, it decreases in magnitude on increasing $t$ when $M \geq 4$. This implies that, for ramped temperature plate with ramped surface concentration, skin friction is getting enhanced with the progress of time whereas, for isothermal plate with uniform surface concentration, skin friction is getting reduced with the progress of time when $M \geq 4$. It is worthy to note from Table 1 that there exists flow separation at the isothermal plate on increasing time $t$ when $M = 6$ and on increasing $M$ when $t = 0.7$. It is observed from Tables 2
and 3 that, for ramped temperature plate with ramped surface concentration, skin friction $-\tau$ increases on increasing either $\phi$ or $K_2$ and it decreases on increasing either $G_e$ or $G_c$. For isothermal plate with uniform surface concentration, $-\tau$ decreases in magnitude, attains a minimum and then increases on increasing $K_2$ when $G_e = 3$ and it decreases in magnitude on increasing $K_2$ when $G_e = 5$ and $7$. $-\tau$ increases in magnitude on increasing either $G_e$ or $G_c$ for isothermal plate with uniform surface concentration. This implies that, for ramped temperature plate with ramped surface concentration, heat absorption and chemical reaction have tendency to enhance skin friction whereas thermal and species buoyancy forces have reverse effect on it. For isothermal plate with uniform surface concentration, heat absorption has a tendency to reduce skin friction whereas chemical reaction tends to reduce skin friction when $G_c \geq 5$ and thermal and species buoyancy forces have tendency to enhance skin friction. It is evident from Table 3 that there exists flow separation at the isothermal plate on increasing $K_2$ when $G_e = 3$ and on increasing $G_c$ when $K_2 = 5$. It is perceived from Table 4 that, for ramped temperature plate with ramped surface concentration, Nusselt number $-Nu$ increases on increasing $\phi$ or $t$. For isothermal plate with uniform surface concentration, $-Nu$ increases on increasing $\phi$ and decreases on increasing $t$. This implies that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, heat

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Skin friction $-\tau$ when $K_1 = 0.5$, $G_e = 4$, $G_c = 5$, $K_2 = 0.2$ and $\phi = 1$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ramp</td>
<td>Isothermal</td>
</tr>
<tr>
<td>$M_{fl}$</td>
<td>$M_{fl}$</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
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<tr>
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</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Skin friction $-\tau$ when $M = 4$, $G_c = 5$, $K_1 = 0.5$, $K_2 = 0.2$ and $t = 0.5$.</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>$G_e$, $\phi$</td>
<td>$G_e$, $\phi$</td>
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<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
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</table>

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Skin friction $-\tau$ when $M = 4$, $G_c = 4$, $K_1 = 0.5$, $\phi = 1$ and $t = 0.5$.</th>
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<tbody>
<tr>
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</tr>
<tr>
<td>$G_e$, $K_2$</td>
<td>$G_e$, $K_2$</td>
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<table>
<thead>
<tr>
<th>Table 4</th>
<th>Nusselt number $-N_u$ when $Pr = 0.71$.</th>
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<tbody>
<tr>
<td>Ramp</td>
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</tr>
<tr>
<td>$\phi$, $t$</td>
<td>$\phi$, $t$</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Sherwood number $-S_h$ when $Sc = 0.22$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ramp</td>
<td>Isothermal</td>
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<td>$K_2$, $t$</td>
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</table>
absorption tends to enhance rate of heat transfer at the plate. Rate of heat transfer at the plate is getting enhanced for ramped temperature plate with ramped surface concentration whereas rate of heat transfer at the plate is getting reduced for isothermal plate with uniform surface concentration with the progress of time.

It is observed from Table 5 that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, Sherwood number $-S_h$ increases on increasing $K_2$. On increasing time $t$, $-S_h$ increases for ramped temperature plate with ramped surface concentration whereas it decreases for isothermal plate with uniform surface concentration. This implies that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, chemical reaction tends to enhance rate of mass transfer at the plate. For ramped temperature plate with ramped surface concentration, rate of mass transfer at the plate is getting enhanced whereas for isothermal plate with uniform surface concentration rate of mass transfer at the plate is getting reduced with the progress of time.

6. Conclusions

The present study brings out the following significant findings:

(i) For both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration:

- Magnetic field, heat absorption and chemical reaction have retarding influence on fluid flow. Fluid velocity is getting accelerated with the progress of time. Heat absorption has tendency to reduce fluid temperature and there is a rise in fluid temperature with the progress of time. Chemical reaction tends to reduce species concentration and there is enrichment in species concentration with the progress of time.

(ii) For ramped temperature plate with ramped surface concentration: skin friction is getting enhanced with the progress of time when $M \geq 4$ whereas heat absorption and chemical reaction have an accelerating influence on fluid flow whereas thermal and species buoyancy forces have reverse effect on it.

(iii) For isothermal plate with uniform surface concentration: Skin friction is getting reduced with the progress of time when $M \geq 4$. Heat absorption has a tendency to reduce skin friction whereas chemical reaction tends to reduce skin friction when $G_c \geq 5$ and thermal and species buoyancy forces have tendency to enhance skin friction. There exists flow separation at the plate on increasing time $t$ when $M = 6$ and on increasing $M$ when $t = 0.7$. Also, there exists flow separation at the plate on increasing $K_3$ when $G_c = 3$ and on increasing $G_c$ when $K_3 = 5$.

(iv) For both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, heat absorption tends to enhance rate of heat transfer at the plate. Rate of heat transfer at the plate is getting enhanced for ramped temperature plate with ramped surface concentration whereas rate of heat transfer at the plate is getting reduced for isothermal plate with uniform surface concentration with the progress of time.

(v) For both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, chemical reaction tends to enhance rate of mass transfer at the plate. For ramped temperature plate with ramped surface concentration, rate of mass transfer at the plate is getting enhanced whereas for isothermal plate with uniform surface concentration rate of mass transfer at the plate is getting reduced with the progress of time.

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