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Chiral SUSY theories with a suppressed SUSY charge

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ABSTRACT

We discuss the SUSY Ward Identities in terms of the BRST Master Equation. This requires that the theory be coupled to Supergravity. In this paper we will mostly truncate the Supergravity part and consider the simpler Chiral and Gauge Realizations of SUSY.

The well-known Chiral and Gauge SUSY Actions realize the SUSY charge in terms of transformations among the Fields. In the context of the Master Equation, there is a simple kind of 'Exchange Transformation' in SUSY theories which allows us to transform any Chiral Action so that its Scalar Fields are replaced by Sources, while preserving the form of the Master Equation. This generates a new action which does not conserve the Supercharge, but which is still constrained by the new Master Equation. There is a close relationship between this theory with Suppressed SUSY Charge and the original theory with Conserved SUSY Charge. We examine the new theories to see what they can do for the problems of mass splitting in SUSY theories.

1. Introduction

In spite of a large amount of work, there is still much that is unknown about SUSY [1–6]. Even its representation theory is filled with unanswered questions [7]. Much recent work has concerned itself with geometric and duality issues in supersymmetric theories. Some of this is associated with Branes, M theory and AdS CFT [8–11]. A large body of SUSY work has, understandably, focussed on phenomenology and experimental signals, while leaving the problems of the origin of the spectrum of mass splitting for future work [12–14]. The progress reported in this paper is the direct result of a study of the algebraic problem of the local BRST cohomology of the Chiral Multiplet [15–17], which examined the solution to the tachyon problem. Previous work, troubled by the tachyon problem, was in [18–23] using some of the methods in [24,25]. All of that work was based on the Master Equation formulation of symmetries [26–31].

From the beginning of research into SUSY, it has been noticed by many authors that SUSY seems to hint at solutions to various problems. But these hints then turn to disappointment, because the effort to remove the mass degeneracy of the supermultiplets, using spontaneous SUSY breaking, tends to spoil the nice properties of the theory [32][39]. This has led some authors to wonder whether the mass degeneracy of SUSY can be removed, even when SUSY itself is not really spontaneously (or explicitly) broken at all [33].

The theory presented here shows how the mass degeneracy can be removed for Chiral Multiplets, without spontaneous or explicit breaking of SUSY. The result is a sort of compromise between the usual SUSY theories that have a conserved SUSY charge, and theories which have no SUSY at all. We use an Exchange Transformation[1] to change the original normal SUSY theory to a theory with a non-conserved, but still very relevant, SUSY charge. The new theory satisfies a Master Equation that is very similar to the Master Equation of the parent SUSY theory. The method here does not apply to SUSY Gauge theory so as to remove its SUSY charge. But the presence of SUSY Gauge theory, or even Supergravity, is not a problem for the method.

2. Half-Chiral Multiplets and Un-Chiral Multiplets

For each Chiral Multiplet in an Action, there is a choice to be made. One can simply leave it as a Chiral Multiplet, or one can use an Exchange Transformation to transform it to a new kind of Multiplet, with a new Action and new physics. This Exchange Trans-

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[1] This is like a canonical transformation – it ensures that the new Master Equation yields zero with the new Action. See Section 14 below.

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formation transforms some or all of the Scalar Field A to a Zinn Source[2] J and the Zinn Source Γ to an Antighost Field η. The Zinn Source Γ is the Source for variations of the Scalar Field A (in the old Action). The Zinn Source J is the Source for variations of the Antighost Field η (in the new Action).

The Exchange Transformations of the Master Equation that we will introduce in this paper will be said to give rise to a **Half-Chiral Multiplet**, when one Scalar remains, and to an **Un-Chiral Multiplet**, when no Scalars remain. This decision can be made for each Chiral Multiplet in the theory, separately. Since the two Scalars in a Chiral Multiplet are not equivalent to each other (they differ in parity for example), there are really three different Exchange Transformations possible for each Chiral Multiplet. The Exchange Transformation is, in terms of its construction, a sort of Canonical Transformation, except that it takes one theory to a different theory for the Master Equation case. This is explained in Section 14 below.

The Half-Chiral Multiplets are useful for the spontaneous breaking of Gauge symmetry. However, some of the mass degeneracy survives for the Gauge/Higgs sector, as will be shown in [34]. The Un-Chiral Multiplets are useful to make an Action with no Scalars, and we will use them for the SSM in [34], to eliminate all the Squarks and Sleptons, leaving no mass degeneracy in that Matter sector. Clearly this is a form of 'SUSY Charge Suppression', which eliminates the SUSY charge in a particular sector, while preserving the Master Equation, which has a strong influence from SUSY.

3. BRST cohomology of the Chiral Multiplet

The BRST cohomology of the Chiral Multiplet is immense, but much of it has unsaturated Lorentz Spinor indices [18]. By and large, these uncontracted Lorentz indices have been taken to mean that '*this cohomology cannot appear in a Lorentz invariant Action and so it is of no interest*'. **But this pessimistic view is not valid**. The cohomology is very important, because it can only be relevant if one couples the unsaturated indices to something new, so that the cohomology can appear in an Action. *The point is that this opens up a new view on SUSY.* The simplest such new object is evidently a Chiral Dotted Spinor Superfield [15]. But then the question is whether such an object makes sense by itself, and the answer has been discouraging for a long time. The most obvious Action for the Chiral Dotted Spinor Superfield has higher derivatives and also tachyons. But recently some progress was made on this problem [15–17]. A tachyon free Action for the Irreducible Chiral Dotted Spinor Superfield was found and used in a free theory.[3] These Exchange Transformations were found while trying to make that theory an interacting one. In fact they go beyond the results from the cohomology. We will return to the Majorana version of the Irreducible Chiral Dotted Spinor Superfield in Section 8.

4. The Chiral Scalar Superfield

We start with the well-known Chiral Multiplet theory. It has the total Action:

$$\mathcal{A}_{\text{Chiral Total}} = \mathcal{A}_{\text{Chiral Kinetic}} + \mathcal{A}_{\text{Chiral Mass and Interaction}} + \mathcal{A}_{\text{Chiral Zinn}} \tag{1}$$

where the free massless kinetic Action is:

$$\mathcal{A}_{\text{Chiral Kinetic}} = \int d^4x \left\{ F\overline{F} - \psi_\alpha \partial^{\alpha\dot\beta} \overline{\psi}_{\dot\beta} + \frac{1}{2}\partial_{\alpha\dot\beta} A \partial^{\alpha\dot\beta}\overline{A} \right\} \tag{2}$$

and the mass and interaction terms look like[4]:

$$\mathcal{A}_{\text{Chiral Mass and Interaction}} = \int d^4x \left\{ m_1 AF - \frac{1}{2} m_1 \psi^\alpha \psi_\alpha + g_1 A^2 F - g_1 A \psi^\alpha \psi_\alpha + * \right\} \tag{3}$$

and the Zinn Action is:

$$\mathcal{A}_{\text{Chiral Zinn}} = \int d^4x \Big\{ \Gamma \left(C^\alpha \psi_\alpha + \xi \cdot \partial A \right) + Y^\alpha \left(\overline{C}^{\dot\delta} \partial_{\alpha\dot\delta} A + C_\alpha F + \xi \cdot \partial \psi_\alpha \right) \tag{4}$$

$$+ \Lambda \left(\overline{C}^{\dot\beta} \partial_{\alpha\dot\beta} \psi^\alpha + \xi \cdot \partial F \right) + * \Big\} + \mathcal{A}_{\text{SUSY}} \tag{5}$$

where we define the following rigid term (it is not integrated over spacetime):

$$\mathcal{A}_{\text{SUSY}} = -C^\alpha \overline{C}^{\dot\alpha} h_{\alpha\dot\alpha} \tag{6}$$

Here A is a complex Scalar Field, ψ_α is a two-component Weyl Spinor Field, and F is a complex auxiliary Scalar Field. The Zinn Action has the form $\int d^4x \, \{\Gamma\delta A + Y\delta\psi + \Lambda\delta F + *\}$, which is a sum of Zinn Sources coupled to the SUSY variations of the Fields, augmented with translations for reasons explained below.

In the above C_α is a Grassmann even, space–time constant Weyl Spinor ghost Field, and $\overline{C}_{\dot\alpha}$ is its complex conjugate. The object $\xi^{\alpha\dot\alpha}$ is a Grassmann odd, space–time constant, vector ghost Field corresponding to the translation. We define

$$\xi \cdot \partial \equiv \xi^{\alpha\dot\alpha} \partial_{\alpha\dot\alpha} \tag{7}$$

The term $h_{\alpha\dot\alpha}$ in Equation (6) is a Grassmann even, spacetime constant, vector field which is a Zinn source for the variation of $\xi_{\alpha\dot\alpha}$. Note that the term in Equation (6) is not integrated over spacetime, because this is a rigid theory. If this theory included Supergravity, the related term would be integrated over spacetime. These terms take account of the fact that the SUSY algebra closes onto a translation. The BRST ghost of that translation for rigid SUSY is the constant anticommuting vector $\xi^{\alpha\dot\alpha}$.

The SUSY invariance can be summarized by the fact that the above Action $\mathcal{A}_{\text{Chiral Total}}$ as defined by Equation (1) yields zero when we take the lowest order term in the 1PI Generating Functional, $\mathcal{G} \to \mathcal{A}_{\text{Chiral Total}}$, in the Master Equation:

$$\mathcal{P}_{\text{Chiral}}[\mathcal{G}] = \int d^4x \left\{ \frac{\delta\mathcal{G}}{\delta A}\frac{\delta\mathcal{G}}{\delta\Gamma} + \frac{\delta\mathcal{G}}{\delta\psi_\alpha}\frac{\delta\mathcal{G}}{\delta Y^\alpha} + \frac{\delta\mathcal{G}}{\delta F}\frac{\delta\mathcal{G}}{\delta\Lambda} + * \right\} + \frac{\partial\mathcal{G}}{\partial h_{\alpha\dot\alpha}}\frac{\partial\mathcal{G}}{\partial\xi^{\alpha\dot\alpha}} \tag{8}$$

[2] The original formulation of the Master Equation as an 'antibracket' was given by Zinn-Justin [26–31]. The Sources in the Master Equation are sometimes called 'Antifields', following [35]. The author thinks that this term is misleading and confusing. The so-called 'Antifields' are very definitely not Fields – they are Sources. The term 'anti' is also confusing, because the term 'Antifield' would naturally means the 'Field that creates Antiparticles'. So here these Sources are called Zinn Sources. The Zinn Sources do not get integrated in the Feynman path integral, whereas Fields do get integrated, and so, of course, do their Complex Conjugates, the Antifields. We use the term 'Zinn Action' to denote the part of the Action that is at least linear in Zinn Sources. It is unphysical, but useful to keep track of the symmetry.

[3] That progress was not very exciting however, because it dealt only with free theories. Moreover, efforts to make those theories interacting have been fraught with obstructions of the kind mentioned in [16]. However this changes if one integrates the auxiliary W, as will be seen below.

[4] Here $+*$ means 'add the Complex Conjugate of the previous terms'.

For the case when we are using $\mathcal{G} \to \mathcal{A}_{\text{Chiral Total}}$, the Master Equation term $\frac{\partial \mathcal{G}}{\partial \xi^{\alpha\dot\alpha}}$ at the end of equation (8) consists of terms of the following form:

$$\frac{\partial \mathcal{G}}{\partial \xi^{\alpha\dot\alpha}} \equiv \frac{\partial \mathcal{A}_{\text{Chiral Zinn}}}{\partial \xi^{\alpha\dot\alpha}} = \int d^4x \left\{ -\Gamma\left(\partial_{\alpha\dot\alpha} A\right) + Y^{\delta}\left(\partial_{\alpha\dot\beta}\psi_{\delta}\right) - \Lambda\left(\partial_{\alpha\dot\beta} F\right) + * \right\} \tag{9}$$

For the case where $\mathcal{G} \to \mathcal{A}_{\text{Chiral Total}}$ in the Master Equation, the terms $\frac{\partial \mathcal{G}}{\partial h_{\alpha\dot\alpha}} \frac{\partial \mathcal{G}}{\partial \xi^{\alpha\dot\alpha}}$ generate

$$\int d^4x \left\{ \Gamma\left(C^{\alpha}\overline{C}^{\dot\alpha}\partial_{\alpha\dot\alpha} A\right) + \cdots \right\} \tag{10}$$

which exactly cancel similar terms that arise from the variations in the first terms of Equation (8). For the case where $\mathcal{G}$ in Equation (8) is the full 1PI Functional $\mathcal{G}$, this is trickier, and we address that issue below in section 5.

5. Some remarks about closing the algebra and Supergravity

The following terms must always be added to every Master Equation of rigid SUSY theories:

$$\mathcal{P}_{\text{SUSY}}[\mathcal{G}] = \frac{\partial \mathcal{G}}{\partial h_{\alpha\dot\alpha}} \frac{\partial \mathcal{G}}{\partial \xi^{\alpha\dot\alpha}} = -C^{\alpha}\overline{C}^{\dot\alpha} \frac{\partial \mathcal{G}}{\partial \xi^{\alpha\dot\alpha}} \equiv -C^{\alpha}\overline{C}^{\dot\alpha} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot\alpha}} \tag{11}$$

Equation (11) summarizes the fact that two SUSY transformations with the SUSY parameter C_α act like a spacetime translation. One cannot implement the BRST method, and the Master Equation, unless one closes the algebra. We have noted in Equation (11) the fact that $\mathcal{G}$ is not going to get any contributions from loop level in this theory, and this is obvious from a look at the Action above.

There is a genuine issue here. Although there are identities which arise from this Master Equation that are true in perturbation theory for this rigid SUSY theory, those that require terms that come from the quantum nature of ξ are not true, because ξ is not integrated in the Feynman path integral, although it is more like a Field than a Zinn Source. This problem can be resolved by embedding the rigid theory in Supergravity, but of course that is quite complicated, and for present purposes we claim that it is unnecessary, because we are remaining at the tree level here (mostly). However when one is checking this theory at loop level, care will be needed for this issue.

6. The Chiral Scalar theory after integration of the auxiliary *F*

After integrating the F auxiliary Field in Section 4, and dropping the Source Λ for its variation, one gets.[5]

$$\begin{aligned}
\mathcal{A}_{\text{Chiral F Int}} &= \int d^4x \left\{ -\psi_\alpha \partial^{\alpha\dot\beta}\overline{\psi}_{\dot\beta} + \frac{1}{2}\partial_{\alpha\dot\beta} A \partial^{\alpha\dot\beta}\overline{A} \right. \\
&+ \Gamma\left(C^{\alpha}\psi_\alpha + \xi\cdot\partial A\right) \qquad (12)\\
&+ Y^{\alpha}\left(\overline{C}^{\dot\alpha}\partial_{\alpha\dot\alpha} A + \xi\cdot\partial\psi_\alpha\right) + \overline{\Gamma}\left(\overline{C}^{\dot\alpha}\overline{\psi}_{\dot\alpha} + \xi\cdot\partial\overline{A}\right) \\
&+ \overline{Y}^{\dot\alpha}\left(C^{\alpha}\partial_{\alpha\dot\alpha}\overline{A} + \xi\cdot\partial\overline{\psi}_{\dot\alpha}\right) \qquad (13)\\
&- \frac{1}{2}m_1\psi^{\alpha}\psi_\alpha - g_1 A\psi^{\alpha}\psi_\alpha - \frac{1}{2}\overline{m}_1\overline{\psi}^{\dot\alpha}\overline{\psi}_{\dot\alpha} - \overline{g}_1\overline{A}\overline{\psi}^{\dot\alpha}\overline{\psi}_{\dot\alpha} \qquad (14)\\
&\left. - \left(\overline{m}_1\overline{A} + \overline{g}_1\overline{A}^2 + \overline{Y}^{\dot\alpha}\overline{C}_{\dot\alpha}\right)\left(m_1 A + g_1 A^2 + Y^{\alpha}C_\alpha\right) \right\} \qquad (15)
\end{aligned}$$

which yields zero for the smaller Master Equation:

$$\mathcal{P}_{\text{Chiral F Int}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta\mathcal{A}}{\delta\overline{A}}\frac{\delta\mathcal{A}}{\delta\overline{\Gamma}} + \frac{\delta\mathcal{A}}{\delta\overline{\psi}_{\dot\alpha}}\frac{\delta\mathcal{A}}{\delta\overline{Y}^{\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta A}\frac{\delta\mathcal{A}}{\delta\Gamma} + \frac{\delta\mathcal{A}}{\delta\psi_\alpha}\frac{\delta\mathcal{A}}{\delta Y^{\alpha}} \right\} + \frac{\partial\mathcal{A}}{\partial h_{\alpha\dot\alpha}}\frac{\partial\mathcal{A}}{\partial\xi^{\alpha\dot\alpha}} \tag{16}$$

This is the Master Equation for a Chiral Multiplet where the auxiliary F has been integrated out. We will see this form again below. The Zinn Sources Y appear quadratically, and they keep the invariance intact. Now we shall put this derivation in a theorem that we will often need.

7. Theorem about auxiliary fields and Master Equations

The technique in the above example is frequently used in this paper, and it is worth making it into a theorem. We will use this theorem repeatedly in this paper and the paper [34].

Theorem. *Given an Action $\mathcal{A}$ that satisfies a given Master Equation $\mathcal{P}$, then:*

1. *Suppose that there is a Field F in that Action*[6] *which has an algebraically invertible quadratic term, and a linear term*[7] *in F, so that the total Action has the form:*

$$\mathcal{A} = \int d^4x \left\{ m_{ij}F^iF^j + F^iG_i + \Lambda_i\delta F^i + etc. \right\} + \mathcal{A}_{Other\ Terms} \tag{17}$$

and that the Master Equation has the form

$$\mathcal{P}[\mathcal{A}] = \int d^4x \left(\frac{\delta\mathcal{A}}{\delta\Lambda_i}\frac{\delta\mathcal{A}}{\delta F^i} \right) + \mathcal{P}_{Other\ Terms}[\mathcal{A}] \tag{18}$$

2. *Then we can integrate out the Field F and get a new Action and a new Master Equation as follows:*
   (a) *remove the Zinn Source term $\int d^4x\, \Lambda_i\delta F^i$ from the Action (17), and*
   (b) *remove the related term $\frac{\delta\mathcal{A}}{\delta\Lambda_i}\frac{\delta\mathcal{A}}{\delta F^i}$ from the Master Equation (18),*
   (c) *the new Action is*

$$\mathcal{A}_{New} = \int d^4x \frac{-1}{4}\left\{ (m^{-1})^{ij}G_iG_j \right\} + \mathcal{A}_{Other\ Terms} \tag{19}$$

   (d) *the new Action yields zero for the new Master Equation, which reduces to*

$$\mathcal{P}_{New}[\mathcal{A}] = \mathcal{P}_{Other\ Terms}[\mathcal{A}] \tag{20}$$

The proof is simple. Write the relevant terms in the Action (17) in the form

$$\mathcal{A} = \int d^4x \left\{ m_{ij}\left(F^i + \left(\frac{m^{-1}}{2}G\right)^i\right)\left(F^j + \left(\frac{m^{-1}}{2}G\right)^j\right) - m_{ij}\left(\frac{m^{-1}}{2}G\right)^i\left(\frac{m^{-1}}{2}G\right)^j \right\} \tag{21}$$

[5] Insert the Action into a Feynman path integral with Sources for the Fields and derive the Master Equation in the usual way. Then complete the quadratic in F and $\overline{F}$ and perform the same exercise, after dropping the Source Λ. Then shift the F and integrate it, which just leaves a number. This leaves the terms and Zinn Action shown.

[6] Here we assume that F is real. In Section 6, F was complex.

[7] Auxiliary Fields generally satisfy this condition.

Then shift and integrate the Field F^i, after placing it in a Feynman path integral. This yields an irrelevant constant plus the second term. It is important that the G^i does not contain F^i. But it can contain anything else including Zinn Sources.

The usual demonstration that the Master Equation yields zero goes through when this has been done, and no Zinn Source for the variation of F^i is needed because F^i is gone from the theory. This theorem is important because our Exchange Transformations here map Actions which have had their auxiliaries integrated, as we shall see.

8. The Majorana Irreducible Chiral Dotted Spinor Supermultiplet

Next we consider the simplest example of a Majorana Irreducible Chiral Dotted Spinor Supermultiplet. It is the Majorana version of the Dirac type theory.[8] We will use the notation of [15]. The Action is quite simple:

$$\mathcal{A}_{\mathrm{MI}} = \mathcal{A}_{\mathrm{MI\ Kinetic}} + \mathcal{A}_{\mathrm{MI\ Zinn}} \tag{22}$$

where

$$\mathcal{A}_{\mathrm{MI\ Kinetic}} = \int d^4x \left\{ -\phi^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}^{\alpha} - \frac{1}{2}W_{\alpha\dot\alpha}W^{\alpha\dot\alpha} \right.$$
$$\left. + \frac{1}{2}G\Box G - \frac{1}{\sqrt{2}}\eta\left(\phi^{\dot\delta}\overline{C}_{\dot\delta} + \overline{\phi}^{\delta}C_{\delta}\right)\right\} \tag{23}$$

In the above, G is a real Scalar Field, $\phi^{\dot\alpha}$ is a two component complex Weyl Spinor, W_μ is a real vector Field,[9] and it turns out to be auxiliary (no kinetic term), η is a real Grassmann odd Antighost Scalar Field and C_α is again the Grassmann even space–time constant Weyl Spinor ghost Field. The Zinn Action that we need is:

$$\mathcal{A}_{\mathrm{MI\ Zinn}} = \int d^4x Z^{\dot\alpha}\left(-i\frac{1}{\sqrt{2}}\partial_{\alpha\dot\alpha}GC^{\alpha} - W_{\alpha\dot\alpha}C^{\alpha} + \xi\cdot\partial\,\phi_{\dot\alpha}\right)$$
$$+ \overline{Z}^{\alpha}\left(i\frac{1}{\sqrt{2}}\partial_{\alpha\dot\alpha}G\overline{C}^{\dot\alpha} - W_{\alpha\dot\alpha}\overline{C}^{\dot\alpha} + \xi\cdot\partial\,\overline{\phi}_{\alpha}\right)$$
$$+ \Sigma^{\alpha\dot\alpha}\left(\sqrt{2}\eta\overline{C}_{\dot\alpha}C_{\alpha} - \frac{1}{2}\partial^{\dot\gamma}_{\alpha}\phi_{\dot\gamma}\overline{C}_{\dot\alpha} - \frac{1}{2}\partial^{\dot\gamma}_{\alpha}\phi_{\dot\alpha}\overline{C}_{\dot\gamma}\right.$$
$$\left. - \frac{1}{2}\partial^{\gamma}_{\dot\alpha}\overline{\phi}_{\gamma}C_{\alpha} - \frac{1}{2}\partial^{\gamma}_{\dot\alpha}\overline{\phi}_{\alpha}C_{\gamma} + \xi\cdot\partial\,W_{\alpha\dot\alpha}\right)$$
$$+ \Upsilon\left(-\frac{i}{\sqrt{2}}\overline{\phi}_{\beta}C^{\beta} + \frac{i}{\sqrt{2}}\phi_{\dot\beta}\overline{C}^{\dot\beta} + \xi\cdot\partial G\right)$$
$$+ J\left(\frac{1}{\sqrt{2}}\partial_{\gamma\dot\delta}W^{\gamma\dot\delta} + \xi\cdot\partial\eta\right) + \mathcal{A}_{\mathrm{SUSY}} \tag{24}$$

Any of the three Actions in (22) yields zero when inserted into the following Master Equation for SUSY:

$$\mathcal{P}_{\mathrm{MI}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta\mathcal{A}}{\delta Z^{\dot\alpha}}\frac{\delta\mathcal{A}}{\delta\phi_{\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta\overline{Z}^{\alpha}}\frac{\delta\mathcal{A}}{\delta\overline{\phi}_{\alpha}} + \frac{\delta\mathcal{A}}{\delta\Sigma^{\alpha\dot\alpha}}\frac{\delta\mathcal{A}}{\delta W_{\alpha\dot\alpha}}\right.$$
$$\left. + \frac{\delta\mathcal{A}}{\delta\Upsilon}\frac{\delta\mathcal{A}}{\delta G} + \frac{\delta\mathcal{A}}{\delta J}\frac{\delta\mathcal{A}}{\delta\eta}\right\} + \frac{\partial\mathcal{A}}{\partial h_{\alpha\dot\alpha}}\frac{\partial\mathcal{A}}{\partial\xi^{\alpha\dot\alpha}} \tag{25}$$

[8] This could be derived in exactly the same way as in [15–17], using BRST recycling, starting with the U(1) Gauge theory in this Majorana case. We start with this simplest case but the other (Dirac) case is also needed and it can be found in [40].

[9] $W_{\alpha\dot\alpha} = W_\mu\sigma^{\mu}_{\alpha\dot\alpha}$.

9. Integrate the auxiliary out of the Irreducible Chiral Dotted Spinor Supermultiplet

After [15–17] were written, the main problem for the new version of the Irreducible Chiral Dotted Spinor Superfield was whether it could be put into an interacting Action. That seemed very difficult at first. However, it turns out that interactions can be generated easily if we first integrate the auxiliary vector Field $W_{\alpha\dot\beta}$ out of the Action for the Majorana Irreducible Chiral Dotted Spinor Superfield in Section 8. We will give the result of that a new name: the Half-Chiral Multiplet Action. We use that name for the Half-Chiral Multiplet because it has half the Scalar degrees of freedom that a Chiral Multiplet has.

Once the integration of W is done, it is fairly easy to recognize that the resulting Half-Chiral Multiplet Action is really the result of an Exchange Transformation acting on a Chiral Multiplet that has had its auxiliary Field F integrated. This is the Exchange Transformation that we will be using. Because it will turn out that this theory can be obtained from an Exchange Transformation acting on a Chiral Multiplet, we can couple the Chiral Multiplet using known methods, and then using the inverse Exchange Transformation, we can deduce the interactions of the Half-Chiral Multiplet.

10. The Majorana Half-Chiral Multiplet

Drop the Σ terms in the action in Equation (22) in Section 8, and integrate W out of the Action.[10] This yields a closely related Action, which we will dignify by a new name, the Majorana Half-Chiral Multiplet:

$$\mathcal{A}_{\mathrm{MHC}} = \int d^4x \left\{-\phi^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}^{\alpha} + \frac{1}{2}G\Box G - \frac{1}{\sqrt{2}}\eta\left(\phi^{\dot\delta}\overline{C}_{\dot\delta} + \overline{\phi}^{\delta}C_{\delta}\right)\right\}$$
$$+ \int d^4x \left\{ Z^{\dot\alpha}\left(-i\frac{1}{\sqrt{2}}\partial_{\alpha\dot\alpha}GC^{\alpha} + \xi\cdot\partial\,\phi_{\dot\alpha}\right)\right.$$
$$+ \overline{Z}^{\alpha}\left(i\frac{1}{\sqrt{2}}\partial_{\alpha\dot\alpha}G\overline{C}^{\dot\alpha} + \xi\cdot\partial\,\overline{\phi}_{\alpha}\right)$$
$$\left. + \Upsilon\left(-\frac{i}{\sqrt{2}}\overline{\phi}_{\beta}C^{\beta} + \frac{i}{\sqrt{2}}\phi_{\dot\beta}\overline{C}^{\dot\beta} + \xi\cdot\partial G\right) + J\xi\cdot\partial\eta\right\}$$
$$+ \frac{1}{2}\int d^4x \left\{Z^{\dot\alpha}C^{\alpha} + \overline{Z}^{\alpha}\overline{C}^{\dot\alpha} + \frac{1}{\sqrt{2}}\partial^{\alpha\dot\alpha}J\right\}$$
$$\times \left\{Z_{\dot\alpha}C_{\alpha} + \overline{Z}_{\alpha}\overline{C}_{\dot\alpha} + \frac{1}{\sqrt{2}}\partial_{\alpha\dot\alpha}J\right\} + \mathcal{A}_{\mathrm{SUSY}} \tag{26}$$

Now this yields zero for:

$$\mathcal{P}_{\mathrm{MHC}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta\mathcal{A}}{\delta Z^{\dot\alpha}}\frac{\delta\mathcal{A}}{\delta\phi_{\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta\overline{Z}^{\alpha}}\frac{\delta\mathcal{A}}{\delta\overline{\phi}_{\alpha}} + \frac{\delta\mathcal{A}}{\delta\Upsilon}\frac{\delta\mathcal{A}}{\delta G} + \frac{\delta\mathcal{A}}{\delta J}\frac{\delta\mathcal{A}}{\delta\eta}\right\}$$
$$+ \frac{\partial\mathcal{A}}{\partial h_{\alpha\dot\alpha}}\frac{\partial\mathcal{A}}{\partial\xi^{\alpha\dot\alpha}} \tag{27}$$

11. Remarkable symmetries of the Action $\mathcal{A}_{\mathrm{MHC}}$

The above Action $\mathcal{A}_{\mathrm{MHC}}$ in Equation (26) has a remarkable symmetry which was not obvious before we integrated the auxiliary W. The Field η and the Source Υ appear in similar ways.

[10] Insert the Action into a Feynman path integral with Sources for the Fields, and derive the Master Equation in the usual way. Then complete the quadratic in W and perform the same exercise, while leaving its Source out, and shifting the W to integrate it. This leaves the terms and Zinn Action shown. This is an application of the theorem in Section 7.

The term $\eta\left(\frac{1}{\sqrt{2}}\overline{\phi}_\delta C^\delta+\frac{1}{\sqrt{2}}\phi_{\dot\delta}\overline{C}^{\dot\delta}+\xi\cdot\partial J\right)$ looks like a Zinn Source coupled to a variation, except that η is a Field. The Field G and the Source J also appear in similar ways. The term $\frac{1}{2}J\Box J$ looks like a kinetic term for J, except that J is a Source.

12. New variables for the Majorana Half-Chiral Multiplet

This symmetry can be exploited with a Generating Functional for an Exchange Transformation of the Action and Master Equation. The new Action will yield zero for the new Master Equation. Our new Action will have 'a new complex Field' $(S,\overline{S})$ and 'a new complex Zinn Source' $(\Gamma,\overline{\Gamma})$. These will replace the 'old real Fields' (η,G) and the 'old real Zinn Sources' (J,Υ). We will choose a Generating Functional of the new Zinn Sources $(\Gamma,\overline{\Gamma})$ and the old Field G and the old Zinn Source J.

$$\mathcal{G}_{\rm MHC}=\int d^4x\left\{\frac{1}{\sqrt{2}}\Gamma(J-iG)+\frac{1}{\sqrt{2}}\overline{\Gamma}(J+iG)\right\}\tag{28}$$

and the Exchange Transformations are:

$$S=\frac{\delta\mathcal{G}}{\delta\Gamma}=\frac{1}{\sqrt{2}}(J-iG);\ \overline{S}=\frac{\delta\mathcal{G}}{\delta\overline{\Gamma}}=\frac{1}{\sqrt{2}}(J+iG);\tag{29}$$

$$\eta=\frac{\delta\mathcal{G}}{\delta J}=\frac{1}{\sqrt{2}}\left(\Gamma+\overline{\Gamma}\right);\ \Upsilon=\frac{\delta\mathcal{G}}{\delta G}=\frac{1}{\sqrt{2}}(-i\Gamma+i\overline{\Gamma})\tag{30}$$

These have the following inverses:

$$G=\frac{1}{\sqrt{2}}(iS-i\overline{S});\ J=\frac{1}{\sqrt{2}}(S+\overline{S});$$
$$\Gamma=\frac{1}{\sqrt{2}}(\eta+i\Upsilon);\ \overline{\Gamma}=\frac{1}{\sqrt{2}}(\eta-i\Upsilon)\tag{31}$$

13. New Action after Exchange Transformation: it looks like the Chiral Action

The new Action expressed in terms of the new variables is:

$$\begin{aligned}\mathcal{A}_{\rm CM}=\int d^4x\,\Big\{&-\phi^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}^{\alpha}+S\Box\overline{S}+\overline{\Gamma}\left(\overline{\phi}_\delta C^\delta+\xi\cdot\partial\overline{S}\right)\\&+\Gamma\left(\phi_{\dot\delta}\overline{C}^{\dot\delta}+\xi\cdot\partial S\right)+Z^{\dot\alpha}\left(\partial_{\alpha\dot\alpha}SC^\alpha+\xi\cdot\partial\phi_{\dot\alpha}\right)\\&+\overline{Z}^{\alpha}\left(\partial_{\alpha\dot\alpha}\overline{SC}^{\dot\alpha}+\xi\cdot\partial\overline{\phi}_\alpha\right)-Z_{\dot\alpha}C^\alpha\overline{Z}_\alpha\overline{C}^{\dot\alpha}\Big\}\end{aligned}\tag{32}$$

The new Master Equation is:

$$\mathcal{P}_{\rm CM}[\mathcal{A}]=\int d^4x\left\{\frac{\delta\mathcal{A}}{\delta Z^{\dot\alpha}}\frac{\delta\mathcal{A}}{\delta\phi_{\dot\alpha}}+\frac{\delta\mathcal{A}}{\delta\Gamma}\frac{\delta\mathcal{A}}{\delta S}+*\right\}+\frac{\partial\mathcal{A}}{\partial h_{\alpha\dot\alpha}}\frac{\partial\mathcal{A}}{\partial\xi^{\alpha\dot\alpha}}\tag{33}$$

The invariance of the new Action $\mathcal{A}_{\rm CM}$ is expressed by:

$$\mathcal{P}_{\rm CM}[\mathcal{A}_{\rm CM}]=0\tag{34}$$

Note that the expressions in this Section are identical to the results for the Chiral Multiplet in Section 6 above, if one changes the names of the Fields and Zinn Sources, and sets $m_1=g_1=0$ in Section 6. Here is the mapping from Section 6 to this Section 13.

$$A\to\overline{S};\ \Gamma\to\overline{\Gamma};\ Y^\alpha\to\overline{Z}^\alpha;\ \psi_\alpha\to\overline{\phi}_\alpha\tag{35}$$

and their Complex Conjugates.

14. Poisson Brackets, Canonical Transformations, Exchange Transformations, and the Master Equation

The Master Equation [26–31] has the same form as a Poisson Bracket[11] in classical mechanics [37,38]. There is no essential distinction between coordinates and momentum for classical mechanics, but there is one for the Master Equation. The reason is that the Fields are quantized and the Zinn Sources are not. Nevertheless, Canonical Transformations play a role for both kinds of Poisson Brackets, because they leave the Poisson Bracket invariant [37,38]. For the Master Equation case we are calling these 'Exchange Transformations', because they can map one action to another, which a Canonical Transformation would never do in Classical Mechanics. But it is important to remember that these Exchange Transformations must always yield an Action which yields zero for the resulting Master Equation, and that is because they are Canonical Transformations in their mathematical form.

15. Finding mass and interaction terms for the Half-Chiral Multiplet by starting with the known mass and interaction terms for the Chiral Multiplet

So now we see that the Half-Chiral Multiplet arises from the Chiral Multiplet through the Exchange Transformation above. This is useful because we know how to make masses and interactions for the Chiral Multiplet, and we did this in Section 6. Can we put those into a Chiral Multiplet and then use the Exchange Transformation to deduce what they look like for the Half-Chiral Multiplet from that? The answer is yes! Let us see how this works in detail, by adding a mass term and a cubic interaction term to (32). This is a little tricky, because the two theories are related by an Exchange Transformation only when the auxiliaries in both theories have been integrated out of the theories. Also, the auxiliaries of the two theories are different – the Chiral Multiplet has a Scalar F and the Irreducible Chiral Dotted Spinor Superfield has a vector auxiliary $W_{\alpha\dot\alpha}$. When these auxiliaries are integrated then there is an Exchange Transformation that relates the two, and we call the Irreducible Chiral Dotted Spinor Superfield with its W auxiliary integrated, by the shorter and more descriptive name Half-Chiral Multiplet. The Chiral Multiplet with a mass term and an interaction term has the following Action, once the auxiliary has been integrated:

$$\begin{aligned}&\mathcal{A}_{\text{CM with Mass \& Interaction}}\\&=\int d^4x\,\Big\{-\phi^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}^{\alpha}+S\Box\overline{S}+\overline{\Gamma}\left(\overline{\phi}_\delta C^\delta+\xi\cdot\partial\overline{S}\right)\\&\quad+\Gamma\left(\phi_{\dot\delta}\overline{C}^{\dot\delta}+\xi\cdot\partial S\right)+Z^{\dot\alpha}\left(\partial_{\alpha\dot\alpha}SC^\alpha+\xi\cdot\partial\phi_{\dot\alpha}\right)\\&\quad+\overline{Z}^{\alpha}\left(\partial_{\alpha\dot\alpha}\overline{SC}^{\dot\alpha}+\xi\cdot\partial\overline{\phi}_\alpha\right)\\&\quad-(\frac{1}{2}m_1+g_1S)\phi^{\dot\alpha}\phi_{\dot\alpha}-(\frac{1}{2}\overline{m}_1+\overline{g}_1\overline{S})\overline{\phi}^{\alpha}\overline{\phi}_\alpha\\&\quad-\left(m_1S+g_1S^2+Z_{\dot\alpha}\overline{C}^{\dot\alpha}\right)\left(m_1\overline{S}+\overline{g}_1\overline{S}^2+\overline{Z}_\alpha C^\alpha\right)\Big\}\end{aligned}\tag{36}$$

We are using the notation in Section 13, rather than the notation in Sections 4 and 6. This is done to agree with the notation in [34].

[11] The Master Equation is a Poisson Bracket, except that it uses Grassmann anticommuting quantities (these actually simplify things somewhat).

16. The Half-Chiral Multiplet Action with mass and interaction

It is elementary to use the Exchange Transformation from Section 12 on the expression in Section 15. The result is the Action for the massive interacting Majorana Half-Chiral Multiplet:

$$\begin{aligned}
&\mathcal{A}_{\text{MHC with Mass \& Interaction}}\\
&= \int d^4x \left\{ -\phi^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}^{\alpha} + \frac{1}{\sqrt{2}}(J-iG)\Box\frac{1}{\sqrt{2}}(J+iG)\right.\\
&\quad + \frac{1}{\sqrt{2}}(\eta - i\Upsilon)\left(\overline{\phi}_\delta C^\delta + \xi\cdot\partial\frac{1}{\sqrt{2}}(J+iG)\right)\\
&\quad + \frac{1}{\sqrt{2}}(\eta + i\Upsilon)\left(\phi_{\dot\delta}\overline{C}^{\dot\delta} + \xi\cdot\partial\frac{1}{\sqrt{2}}(J-iG)\right)\\
&\quad + Z_{\dot\alpha}\left(\partial^{\alpha\dot\alpha}\frac{1}{\sqrt{2}}(J-iG)C_\alpha + \xi\cdot\partial\phi^{\dot\alpha}\right)\\
&\quad + \overline{Z}_{\alpha}\left(\partial^{\alpha\dot\alpha}\frac{1}{\sqrt{2}}(J+iG)\overline{C}_{\dot\alpha} + \xi\cdot\partial\overline{\phi}^{\alpha}\right)\\
&\quad - (\frac{1}{2}m_1 + g_1\frac{1}{\sqrt{2}}(J-iG))\phi^{\dot\alpha}\phi_{\dot\alpha}\\
&\quad - (\frac{1}{2}\overline{m}_1 + \overline{g}_1\frac{1}{\sqrt{2}}(J+iG))\overline{\phi}^{\alpha}\overline{\phi}_{\alpha} \qquad (37)\\
&\quad - \left(m_1\frac{1}{\sqrt{2}}(J-iG) + g_1\frac{1}{2}(J-iG)^2 + Z_{\dot\alpha}\overline{C}^{\dot\alpha}\right)\\
&\quad \times \left.\left(m_1\frac{1}{\sqrt{2}}(J+iG) + \overline{g}_1\frac{1}{2}(J+iG)^2 + \overline{Z}_\alpha C^\alpha\right)\right\} \qquad (38)
\end{aligned}$$

17. Details for the Majorana Half-Chiral Multiplet with mass, and the loss of the SUSY charge

Let us set $g_1 \to 0$ in Section 16. Then the expression in Section 16 is the Half-Chiral Multiplet with just a Majorana mass term. The only difference from the massless case is:

$$\begin{aligned}
&\mathcal{A}_{\text{MHC with Mass}} = \int d^4x\, \{\cdots \qquad (39)\\
&\quad -\frac{1}{2}m_1\phi^{\dot\alpha}\phi_{\dot\alpha} - \frac{1}{2}m_1\overline{\phi}^{\alpha}\overline{\phi}_{\alpha} - \left(m_1\frac{1}{\sqrt{2}}(J-iG) + Z_{\dot\alpha}C^\alpha\right)\\
&\quad \times\left(m_1\frac{1}{\sqrt{2}}(J+iG) + \overline{Z}_\alpha\overline{C}^{\dot\alpha}\right)\} \qquad (40)
\end{aligned}$$

We see that indeed there is a mass term here for the Scalar, namely

$$\mathcal{A}_{\text{MHC with Mass}} = \int d^4x \left\{\cdots - \frac{m_1^2}{2}(J^2+G^2)\right\} \qquad (41)$$

But note that there is also a 'mass' term for the Zinn Source J in Equation (41), and then there are extra terms that are all in the Zinn Action

$$\begin{aligned}
&\mathcal{A}_{\text{MHC with Mass}} = \int d^4x \left\{\cdots - \left(Z_{\dot\alpha}C^\alpha\right)\left(\overline{Z}_\alpha\overline{C}^{\dot\alpha}\right)\right. \qquad (42)\\
&\quad \left.- \left(\frac{m_1}{\sqrt{2}}(J-iG)\right)\left(\overline{Z}_\alpha\overline{C}^{\dot\alpha}\right) - \left(Z_{\dot\alpha}C^\alpha\right)\left(\frac{m_1}{\sqrt{2}}(J+iG)\right)\right\} \qquad (43)
\end{aligned}$$

So here is what we have discovered: The Half-Chiral Multiplet Action has one Scalar G and a Majorana Spinor ϕ and also the Source J. When we generate the Half-Chiral Multiplet mass term from the massive Chiral Multiplet plus the Exchange Transformation, we get a massive Spinor and a massive Scalar G. We also get an object that looks like a mass term for J, but J is a Zinn Source.

Because this is all a result of the Exchange Transformation, we are guaranteed that it will satisfy the Half-Chiral Master Equation in Equation (27).

But just looking at it we can see that it describes a Multiplet of SUSY that has one Scalar and a Spinor – but this is clearly not a proper mass multiplet that forms a representation of the SUSY algebra – that needs two Scalars and there is only one here. And yet there is some mass degeneracy here, as though the SUSY algebra is 'half-present'.

And that is the essential point of this entire paper! The Exchange Transformation has enabled us to build a SUSY theory that does not have a nice conserved Noether Charge – the physical theory here is not the proper one we would expect from a theory with a conserved Noether current – it has this Source J where the Scalar should be. And because of the Exchange Transformation, it has the right set of Zinn Source terms to satisfy the Half-Chiral Master Equation.

18. Details for the Half-Chiral Multiplet with mass and interactions

Now consider the case where $g_1 \neq 0$ in Section 16. The Action there has both mass and interactions. Note the complicated way that the Zinn Source is intertwined with the Scalar Field. We could do the same with a Chiral Action that also includes any other kind of interactions, with Gauge, other Chiral Multiplets, even Supergravity. We would end up with a lot of Zinn Source terms and an Action that only goes half-way towards a representation of the SUSY algebra.

The Action in Section 16 is probably the simplest possible Half-Chiral massive interacting theory, and it would be worth while to examine its nilpotent BRST operator δ_{BRST} (this is the 'square root' of the Master Equation), and its one loop diagrams to get a feel for how this Half-Chiral Multiplet works at one loop.

These Half-Chiral Multiplets will be used in [34] for the Higgs Multiplets. We will use a Dirac Half-Chiral Multiplet and a Majorana Half-Chiral Multiplet there.

19. Un-Chiral Multiplets

If we take the Exchange Transformation that goes all the way, to generate an Un-Chiral Multiplet, we get a theory with no Scalar and some interesting Zinn terms, and a mass just for the Spinor. That case is actually simpler than the above, and we write it down in Section 20. In that case the interaction term would just add to the Zinn Source sector, and the theory would still be a free massive theory. To get interactions there requires Gauge theory.

The Exchange Transformation can be applied twice, so that both Scalars are removed from the Lagrangian. The new Lagrangian gains two terms with Antighosts while the Fermions remain as quantized Fields. Start with the Chiral Multiplet with the Action in Equation (32). Now consider using an Exchange Transformation generated as follows. Instead of the Exchange Transformation in Section 12 we now note that the old 'Fields' $(S, \overline{S})$ are conjugate to the old 'Zinn Sources' $(\Gamma, \overline{\Gamma})$, and we want an Exchange Transformation that takes us to the new 'Fields' $(\eta, \overline{\eta})$ which are conjugate to the new 'Zinn Sources' $(J, \overline{J})$. We choose a generating functional of the new Zinn Sources $(J, \overline{J})$ and the old 'Zinn Sources' $(\Gamma, \overline{\Gamma})$. This is

$$\mathcal{G}_{\text{MUC}} = \int d^4x \left\{\Gamma J + \overline{\Gamma J}\right\} \qquad (44)$$

and we have

$$S \to \frac{\delta\mathcal{G}}{\delta\Gamma} = J;\ \overline{S} \to \frac{\delta\mathcal{G}}{\delta\overline{\Gamma}} = \overline{J}; \qquad (45)$$

and

$$\Gamma = \frac{\delta \mathcal{G}}{\delta J} \to \eta; \; \overline{\Gamma} = \frac{\delta \mathcal{G}}{\delta \overline{J}} \to \overline{\eta}; \tag{46}$$

We get the following transformed Action:

$$\begin{aligned} \mathcal{A}_{\rm MUC} = \int d^4x \Big\{ & -\phi^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}^{\alpha} + J\Box\overline{J} \\ & + \overline{\eta}\left(\overline{\phi}_{\delta}C^{\delta} + \xi\cdot\partial\overline{J}\right) + \eta\left(\phi_{\dot\delta}\overline{C}^{\dot\delta} + \xi\cdot\partial J\right) \\ & + Z_{\dot\alpha}\left(\partial^{\alpha\dot\alpha}JC_{\alpha} + \xi\cdot\partial\phi^{\dot\alpha}\right) + \overline{Z}_{\alpha}\left(\partial^{\alpha\dot\alpha}\overline{J}\overline{C}_{\dot\alpha} + \xi\cdot\partial\overline{\phi}^{\alpha}\right) \\ & - Z_{\dot\alpha}C^{\alpha}\overline{Z}_{\alpha}\overline{C}^{\dot\alpha} \Big\} \end{aligned} \tag{47}$$

which yields zero for the new Master Equation

$$\begin{aligned} \mathcal{P}_{\rm MUC}[\mathcal{A}] = \int d^4x \Big\{ & \frac{\delta\mathcal{A}}{\delta Z^{\dot\alpha}}\frac{\delta\mathcal{A}}{\delta\phi_{\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta\overline{Z}^{\alpha}}\frac{\delta\mathcal{A}}{\delta\overline{\phi}_{\alpha}} + \frac{\delta\mathcal{A}}{\delta\overline{\eta}}\frac{\delta\mathcal{A}}{\delta\overline{J}} + \frac{\delta\mathcal{A}}{\delta J}\frac{\delta\mathcal{A}}{\delta\eta} \Big\} \\ & + \frac{\partial\mathcal{A}}{\partial h_{\alpha\dot\alpha}}\frac{\partial\mathcal{A}}{\partial\xi^{\alpha\dot\alpha}} \end{aligned} \tag{48}$$

This looks very similar to (32), but the theory is not at all the same. The J are not quantized and the η are quantized. So the complex quantized Scalar Field $(S, \overline{S})$ is gone from the theory along with its Zinn Source $(\Gamma, \overline{\Gamma})$, while the quantized Fermion $(\phi, \overline{\phi})$ remains, and the new Zinn Sources $(J, \overline{J})$ and quantized Antighosts $(\eta, \overline{\eta})$ appear. We are assured that the new Action satisfies the new Master Equation, because the old Action satisfied the old Master Equation.

This procedure does not correspond to any known starting Action like the Half-Chiral Multiplet discussed above in section 10. Equation (47) is a new Action. We started with the Half-Chiral Multiplets found by BRST recycling, and then found the Exchange Transformations that took those theories to Chiral Multiplets. Now by generalizing those Exchange Transformations we have discovered new theories that have no physical Scalars at all, just physical Fermions.

20. Mass term for the Un-Chiral Multiplet

In Section 17 above, we discussed the mass term for the Half-Chiral Multiplet. It has a mass term that only goes half-way towards that of a Chiral Multiplet. Now let us start again with the Chiral Multiplet with the Action in Equation (32), with a mass term, so that we get Equation (36), and then use the Exchange Transformation in Section 19 on Equation (36). This yields the following Action (we are setting $g_1 \to 0$):

$$\begin{aligned} & \mathcal{A}_{\text{Majorana UnChiral Massive}} \\ & = \int d^4x \Big\{ -\phi^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}^{\alpha} + J\Box\overline{J} + \overline{\eta}\left(\overline{\phi}_{\delta}C^{\delta} + \xi\cdot\partial\overline{J}\right) \\ & + \eta\left(\phi_{\dot\delta}\overline{C}^{\dot\delta} + \xi\cdot\partial J\right) \\ & + Z_{\dot\alpha}\left(\partial^{\alpha\dot\alpha}JC_{\alpha} + \xi\cdot\partial\phi^{\dot\alpha}\right) + \overline{Z}_{\alpha}\left(\partial^{\alpha\dot\alpha}\overline{J}\overline{C}_{\dot\alpha} + \xi\cdot\partial\overline{\phi}^{\alpha}\right) \\ & - m_1\frac{1}{2}\phi^{\dot\alpha}\phi_{\dot\alpha} - m_1\frac{1}{2}\overline{\phi}^{\alpha}\overline{\phi}_{\alpha} \\ & - \left(m_1 J + Z_{\dot\alpha}C^{\alpha}\right)\left(m_1\overline{J} + \overline{Z}_{\alpha}\overline{C}^{\dot\alpha}\right) \Big\} \end{aligned} \tag{49}$$

The Scalar Mass terms $S\left(\Box - m_1\overline{m}_1\right)\overline{S}$ in Equation (36) have become the Zinn Source term $J\left(\Box - m_1\overline{m}_1\right)\overline{J}$ in Equation (49). So Equation (49) is an Action which has a Fermionic Mass term and no Scalars. Note that it is quite a lot simpler than the Half-Chiral Multiplet. We are guaranteed that it will satisfy the appropriate Master Equation for the Un-Chiral Multiplet, and so SUSY is still preserved. But the SUSY Charge has been suppressed in this Un-Chiral Multiplet Quantum Field theory.

It is clear that this is not a representation of the SUSY algebra on the physical states.

This is the kind of SUSY multiplet that we will use for the Quarks and Leptons in [34], except that we need to have the Dirac version for that. A similar exercise to the above would show that there is only a massive Fermion left in the Dirac Unchiral Multiplet.

21. Conclusion

In this paper we have shown that, for any chosen Chiral Multiplet, one can easily derive and write down three more theories.[12] The chosen Chiral Multiplet can be coupled to SUSY Gauge theory and other Chiral Multiplets, and the interactions of the new theories follow directly and simply from those interactions. But the three new Actions are quite different in their behaviour from the original chosen one. This can be done for each Chiral Multiplet in the theory independently.

It is in the Chiral form that it is easy to write down the couplings. Then one simply implements the appropriate set of Exchange Transformations, which result in a new Action and a new Master Equation. The new Action yields zero for the new Master Equation. This Exchange Transformation takes all or part of the Scalar Fields S and replaces them with Zinn Sources J, and also takes the related Zinn Sources Γ and replaces them with Antighost Fields η. These Exchange Transformations are closely related to Canonical Transformations, as is explained in Section 14.

These new Actions were discovered by generating the Irreducible Chiral Dotted Spinor Superfield in [15], in the hope of coupling the BRST cohomology of the Chiral Multiplet to it. Once the auxiliary W was integrated in that theory, it was noticed that the resulting Half-Chiral Action could be generated by an Exchange Transformation from a Chiral Action that has its auxiliary F integrated. So, in a sense, that coupling of the cohomology has now been done, and the result is that we have discovered new ways to realize SUSY in local Actions, and those new Half-Chiral Actions can be coupled to Gauge theory and each other (and even Supergravity).

But the result here goes farther, because we also have discovered Un-Chiral Multiplets that do not arise by way of the Irreducible Chiral Dotted Spinor Superfield of [15]. The Un-Chiral Multiplet Action arises simply by taking the full version of the Exchange Transformation that was suggested by the existence of the Half-Chiral Multiplet.

Supercharges are not constructible in the new theories (except in some sectors), because when the Zinn Sources become involved, no divergenceless SUSY current exists. This happens because the Zinn sources are not quantized, and they do not satisfy Equations of Motion.

In Sections 15, 16 and 17 we set out the details for the massive Half-Chiral Multiplet case, which shows explicitly how the effect of a SUSY Charge acts like it is 'half-present'. In Section 18 we observed the interactions of the Half-Chiral Multiplet.

Section 20 discusses mass for the Un-Chiral Multiplet case and shows that the SUSY Charge is completely gone there. In that case

[12] For a Dirac Multiplet this is limited by the conserved global U(1) phase, which must be conserved.

we need to couple the theory to SUSY Gauge Theory to get an interaction.

The result is that these new theories, when calculated in the renormalized Feynman expansion, iteratively, loop by loop, should be as valid as is the original purely Chiral theory [41]. But the theories are very different, of course, and the Half-Chiral and Un-Chiral theories are not subject to the SUSY algebra, as described in [36], because they do not have conserved SUSY Charges. They are lacking in Scalar Fields compared to Chiral Multiplets. The Dirac case [40] is needed to describe Leptons and Quarks, since they have conserved Baryon and Lepton numbers. It is not very different from the Majorana case which is discussed above. We shall use some of these new Half-Chiral Multiplets and Un-Chiral Multiplets, coupled to SUSY Gauge theory and to each other, to write down a new kind of SSM, in [34].

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