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Non-dominated Sorting Genetic Algorithms- II Based on Multi-objective Optimization Model in the Water Distribution System

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Abstract

In order to make the water distribution network design more scientific and reasonable, the economy and reliability analysis should be considered. Firstly, the cost of pipeline construction is used as one objective. The sum of node surplus head and the node surplus head variance are used as another two objective functions. A multi-objective optimization model in water distribution system is established using a joint. MATLAB and EPANET platform, along with a non-dominated sorting genetic algorithm (NSGA-II) are applied to solve the optimization problem. Finally, a water supply network is employed to demonstrate to the application of this method. It is concluded that the multi-objective optimization model based on the non-dominated sorting genetic algorithm-II performed reasonably and effectively to solve optimization problem for water distribution system.

1 Introduction

Water distribution system is playing an important role in urban construction and development. They are...
the essential substance which protect people's lives and the economic development[1]. However, in the water supply systems, node head are often higher than the minimum head required at that point. In other words, there is surplus head. It not only increases the cost of pipeline construction, but also is a waste of energy. Besides, the uneven distribution of surplus head will result in high and low pressure areas in supply network system, which will accelerate the leakage and affect the reliability of water supply network system.

It has been widely used in various fields to apply mathematical models to optimization design. Recently, it has also become an important method to optimize the water distribution system[2]. The development of water distribution system optimization models goes through two stages, from single objective to multi-objective. Previous optimization models have objective functions as following: (1)The cost of pipeline construction. This method focuses on the economy and regards the reliability as a constraint in the network design[3]; (2) Weighted average of the node surplus head. The smaller weighted average of the node surplus head, the greater reliability of water supply network system[1]; (3)Pipeline information entropy. Entropy of the water supply system reflects its redundancy, and also reflects the ability of the water supply network to withstand accidents[3]. (4)Flow rate. It reports the reliability of the water supply network from another point of view[3]; (5)The elasticity of the network. This method can show the size of the residual energy in pipeline network and the ability to resist accidents[3]. Objective functions above focus on different aspects. However, the node surplus head has not been considered in previous research of the water distribution system, which will result in the low reliability problem.

Therefore, this paper presents a new objective function to evaluate the pressure distribution of the system, and to optimize the diameters of the pipe.

2 Methodology

The multi-objective optimization model of the water supply network is established. The cost of pipeline construction is defined as the economy objective function. The sum of node surplus head and the node surplus head variance are used as reliability objective functions.

2.1 The economy objective function of the water supply network

$$Z = \sum_{j=1}^{P} (a + bD_j^\alpha) L_j ; \quad j = 1, 2, \ldots, P$$

Where:  
\[ Z \] — The cost of the network, RMB;  
\[ P \] — The total number of pipes, \( a \);  
\[ D_j \] — The j pipe diameter, mm;  
\[ L_j \] — The j pipe length, m;  
\[ a, b, \alpha \] — Statistics constant and index.

2.2 The sum of node surplus head

Node surplus head is the minimum free head which exceeds the head required. Each node surplus head:

$$I_i = H_i - H_{min} ; \quad i = 1, 2, \ldots, I$$

The sum of node surplus head in the water distribution system:
\[ I_s = \sum_{i=1}^{I} (H_i - H_{\text{min}}) ; \quad i = 1, 2, \ldots, I \]  

Where:  
1. \( I \) — The number of nodes in the network system, \( a \);  
2. \( I_{si} \) — Node surplus head in the network system, \( m \);  
3. \( I_s \) — The sum of node surplus head in the network system, \( m \);  
4. \( H_i \) — Pressure of \( i \) node in the network system, \( m \);  
5. \( H_{\text{min}} \) — The minimum free node head in the network system, \( m \).

### 2.3 The node surplus head variance

Node surplus head is defined as:

\[ \bar{I}_s = \frac{I_{s1} + I_{s2} + \cdots I_{sl}}{I} \]  

Node surplus head variance means:

\[ S = \sum_{i=1}^{I} (I_{si} - \bar{I}_s)^2 ; \quad i = 1, 2, \ldots, I \]  

Where:  
1. \( I_{si} \) — Meaning as above;  
2. \( I_s \) — Node surplus head mean in the network system, \( m \);  
3. \( S \) — Node surplus head variance in the network system, \( m^2 \).

Therefore, the objective functions of the optimization problem can be summarized as:

\[
\begin{align*}
\min \quad & Z = \sum_{j=1}^{P} \left( a + bD_j^q \right) L_j ; \quad j = 1, 2, \ldots, P \\
\min \quad & I_s = \sum_{i=1}^{I} (H_i - H_{\text{min}}) ; \quad i = 1, 2, \ldots, I \\
\min \quad & S = \sum_{i=1}^{I} \left( I_{si} - \bar{I}_s \right)^2 ; \quad i = 1, 2, \ldots, I \\
\text{s.t.} \quad & Aq + Q = 0, \quad Lh = 0, \quad h = sq^n, \quad H_{\text{min}} \leq H_i \leq H_{\text{max}}, \quad D_k \in \{ D_1, D_2, \ldots, D_z \}. 
\end{align*}
\]

### 3 The Non-dominated sorting genetic algorithm

GA (Genetic Algorithm) has emerged and developed owing to the biological evolution theory and genetic theory. It is a random search algorithm which simulates the biological natural selection and natural genetic mechanism[2].

NSGA(Non-dominated Sorting Genetic Algorithm) was proposed by Srinivas and Deb in 1995, and the improved algorithm put forward in 2000. Congestion and congestion comparison operator are used in this algorithm, and the elite strategy is also introduced to maintain good individuals into next generation. In this way, population level is increased rapidly[2]. The calculation flow chart is shown in Figure 1.
4 Application and discussion

The Net.2 example network is employed to demonstrate to the application of this method[4]. As shown in figure 2: There is a pump station, 36 nodes and 40 pipes in the network. Optional diameter range [100, 150, 200, 250, 300, 350 mm]. Initial population $P_0 = 100$. Crossover probability $P_c = 0.8$. Mutation probability $P_m = 0.02$. The number of iterations $N = 200$. After Optimizing, the results are shown in Figure 3 and Figure 4.

Figure 3(a) is a surface which shows the relationship among the cost of pipeline construction, the sum of the node surplus head and node surplus head variance. Figure 3(b) shows the relationship between the cost of pipeline construction and the sum of the node surplus head. As the cost of pipeline construction increases, the sum of node surplus head in the network system becomes lower, which improves the reliability of water supply system. Figure 4(a) shows the relationship between the cost of pipeline construction and node surplus head variance. As the node surplus head variance reduces, it increases the reliability of water supply system, and it also increases the cost of pipeline construction. Figure 4(b) shows the relationship between the sum of node surplus head and the node surplus head variance. As the sum of node surplus head reduces, the node surplus head variance reduces. They both increase the reliability of the water supply system.

In practice, management experience and the specific situation should be considered in decision-making. First make sure the basic reliability of the water supply system. Then try to seek out a reasonable investment. When the cost is the major issue, a program that has a larger sum of node surplus head and node surplus head variance should be taken. However, when the cost is not the key problem, a program that has a smaller sum of node surplus head and node surplus head variance should be taken.
5 Conclusions

① A new method that evaluates the pipe pressure distribution is brought and a mathematical model is established. It turns out that this model can better reflect the relationship among the cost of pipeline construction, the sum of node surplus head and node surplus head variance. When both the economy and reliability are taken into account, it has a better optimization result.

② Node surplus head variance has an effect on both the reliability and economy in the network system. Both reliability and economy should be met according to the practice, and choose the best program.

References


