Computers and Mathematics with Applications 64 (2012) 2806–2815

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

A new approach to the electrostatic pull-in instability of nanocantilever actuators using the ADM–Padé technique☆

Aminreza Noghrehabadi*, Mohammad Ghalambaz, Afshin Ghanbarzadeh

Department of Mechanical Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran

ARTICLE INFO

Article history:
Received 26 July 2011
Received in revised form 8 December 2011
Accepted 11 April 2012

Keywords:
Nanocantilever
Pull-in
Electromechanical switches
Adomian
Padé

ABSTRACT

In this paper, the Adomian decomposition method and Padé approximants are integrated to study the deflection and pull-in instability of nanocantilever electromechanical switches. In a distributed parameter model, intermolecular forces, including Casimir forces, are taken into account considering their range of application. A closed form power series solution based on Adomian polynomials is obtained. The obtained analytic results are compared with numerical solution. The Adomian method is accurate for small deflections, but the results of a pull-in instability study demonstrate that the accuracy of the Adomian solution is not as good for small deflections. Thus, to increase the accuracy of the Adomian solution for the pull-in instability, the Adomian power series is converted to Padé approximants. The results of the present method are compared with the numerical results as well as those of the Adomian decomposition method and other methods reported in the literature. The results obtained using the ADM–Padé are remarkably accurate compared with the numerical results. The proposed technique can be easily extended to solve a wide range of instability problems. Finally, the minimum initial gap and the detachment length of the actuator that does not stick to the substrate due to the intermolecular attractions, which is an important parameter for the pull-in instability of a nanocantilever actuator, are calculated using Adomian–Padé approximants.

1. Introduction

Micro-and nano-fabrication processes are planar technologies. Therefore, many micro- and nano-devices consist of beams and plates suspended horizontally over a substrate. On the microscale, suspended beams or plates serve as the active component of accelerometers, rate gyroscopes, pressure sensors, chemical sensors, electrical switches, optical switches, adaptive optical devices, resonators, electrostatic actuators, valves and pumps [1].

Conductive cantilever nano-actuators are one of the common components in nano-electromechanical system (NEMS) switches in nanotechnology, which are widely used in manufacturing systems with critical dimensions on the order of nanometers [2,3].

A typical form of NEMS actuator is a nanobeam that is suspended above a conductive flat ground (substrate). Applying a voltage difference between the nanobeam and the ground plane causes the nanobeam to deflect downward and be attracted to the substrate. At a critical deflection/voltage, which is known as pull-in deflection/voltage, the nanobeam pulls onto the substrate, and instability occurs [3]. The intermolecular forces have a significant influence on the instability of the nanobeam for nanoscale separations [3–8].

☆ The paper has been evaluated according to old Aims and Scope of the journal.

* Corresponding author. Tel.: +98 611 3330010x5678; fax: +98 611 3336642.
E-mail addresses: a.r.noghrehabadi@scu.ac.ir (A. Noghrehabadi), m.ghalambaz@gmail.com (M. Ghalambaz), ghanbarzadeh.a@scu.ac.ir (A. Ghanbarzadeh).

0898-1221/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2012.04.013

When the separation is large enough (typically greater than 20 nm), retardation appears. In the presence of retardation effects, the intermolecular interaction between the nanobeam surface and substrate surface can be described by the Casimir force [3,5,9]. Considering the ideal case, the Casimir interaction is proportional to the inverse fourth power of the separation [10,11]. Some researchers [3,10,12–19] have studied the effect of the Casimir force on the instability of NEMS. To study the deflection and pull-in parameters of nanocantilever beams, some investigators assumed that the electrostatic and intermolecular forces are uniform along the beam [10,11,13–16], and some of them used distributed models [3,4,6,8, 17–22]. Because of the nonlinearity of the governing equations of the distributed model of the deflection of nano-beams, obtaining an analytical solution for this type of nanoswitch is difficult and complicated. Some researchers [3,4,6] have tried to find semi-analytical solutions for this problem using Adomian polynomials, and others have tried to find approximate solutions using Green's function and a second order polynomial as the shape function [3,7,12]. The power series methods also, are widely used in other engineering applications [23–27].

Ramezani et al. [7,12] and Koochi et al. [6] applied a distributed parameter model to study the instability of nanocantilevers. However, their solution did not satisfy all boundary conditions. Soroush et al. [3] and Koochi et al. [4,6] considered the effect of Casimir attractions on the electrostatic pull-in instability of nano-actuators using the Adomian decomposition method. Although the results of the Adomian decomposition method are acceptable compared with numerical results, their accuracy near the pull-in instability is not perfect. The results show the Adomian decomposition method with eight terms in the series underestimated the deflection of nanocantilevers at the onset of pull-in stability [19,3], while Green's methods overestimated the deflection of nanocantilevers [7,12,19].

The results of the present work, however, show that increasing the number of Adomian series terms above eight increases the accuracy of the Adomian solution for small deflections, but its accuracy in predicting pull-in stability is not as good for small deflections. However, the combination of any series solutions with the Padé approximants provides a powerful tool for handling initial or boundary value problems [28–32]. Thus, the shortcoming on the accuracy of Adomian decomposition method is overcome in the present study using Padé approximants. Therefore, an integration of the Adomian power series with Padé approximants (ADM–Padé) is introduced as a new approach to study the deflection and pull-in behavior of nano-actuators using a distributed parameter model. A fair comparison is made between the presented method and the Adomian decomposition method results and numerical results as well as the results of other methods reported in the literature.

2. Mathematical model

Fig. 1 shows a nanocantilever beam of length L with a uniform rectangular cross section of thickness h and width w. The initial gap between the movable beam and the ground plane is g. The constitutive material of the nanocantilever is assumed to be linear elastic, and only the static deflection of the nano-beam is considered. The effect of finite kinematics is negligible when $L > 10\,g$ [2]. Thus, finite kinematics are not considered. This simplification is acceptable for most cases [3,7,12]. Considering the first order fringing field correction, the electrostatic force per unit length of the nanocantilever, (electrical force) can be defined as [3,7,12]

$$f_{\text{elec}} = \frac{\varepsilon_0 w V^2}{2(g-y)^2}\left(1 + 0.65\frac{(g-y)}{w}\right). \tag{1}$$

The intermolecular force per unit length of the beam (f_{mole}) including the Casimir force is defined as [3,5,7,12].

$$f_{\text{mole}} = \frac{\pi^2 \hbar c w}{240(g-y)^4} \tag{2}$$

where $\hbar = 1.055 \times 10^{-34}$ is Planck's constant divided by 2π and $c = 2.2998 \times 10^8$ m/s is the speed of light. Eq. (2) provides acceptable results for cantilevers that are sufficiently wider than the separation space [2,3]. Thus, only the cantilevers that are wider than the separation $g/w \leq 1$ are considered in this study [3,7,12].

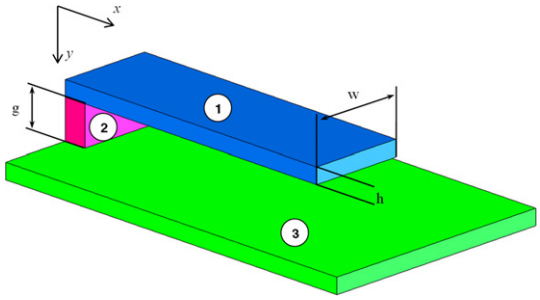

Fig. 1. Schematic representation of a cantilever actuator. 1: Nanocantilever actuator 2: Dielectric spacer, 3: Ground plane.

The appropriate approximation of the beam deflection can be found by applying the virtual work principle. In the absence of non-conservative forces and by considering only the static elastic small deflection of the nanocantilever beam, we can write

$$\delta W = \delta E_{\text{elast}} - \delta W_{\text{elec}} - \delta W_{\text{mole}} = \int_0^L \left(E_{\text{eff}} I \frac{d^2y}{dX^2} \delta \frac{d^2y}{dX^2} - f_{\text{elec}} \delta y - f_{\text{mole}} \delta y \right) dX. \tag{3}$$

After integrating (3), we have

$$\delta W = E_{\text{eff}} I \frac{d^2y}{dX^2} \delta \frac{dy}{dX}\bigg|_0^L - E_{\text{eff}} I \frac{d^3y}{dX^3} \delta y\bigg|_0^L + \int_0^L \left(E_{\text{eff}} I \frac{d^4y}{dX^4} - f_{\text{elec}} - f_{\text{mole}} \right) \delta y dX. \tag{4}$$

As there is no deflection and rotation at the fixed end and also because of the absence of the bending moment and shear force at the free end of the beam, the boundary value problem for a cantilever nanobeam can be defined as

$$E_{\text{eff}} I \frac{d^4y}{dX^4} = f_{\text{elec}} + f_{\text{mole}} \tag{5a}$$

where the geometrical boundary conditions at the fixed end are

$$y(0) = y'(0) = 0. \tag{5b}$$

The natural boundary conditions at the free end are

$$y''(L) = y'''(L) = 0 \tag{5c}$$

where y is the deflection of the beam, X is the position along the beam measured from the clamped end, and prime denotes differentiation with respect to X. E_{eff} is the effective Young's modulus, which is equal to $wh^3/12$, and I is the moment of inertia of the beam cross section [33]. For convenience, the model is parameterized in the nondimensional form.

By substituting (1) and (2) into (5) and introducing the nondimensional variables

$$\alpha = \frac{\pi^2 \hbar c w L^4}{240 g^5 E I}, \tag{6a}$$

$$\beta = \frac{\varepsilon_0 w V^2 L^4}{2 g^3 E I}, \qquad \gamma = 0.65 \frac{g}{w}, \qquad x = \frac{X}{L}, \qquad u = \frac{y}{g}. \tag{6b}$$

The nondimensional differential equation is obtained as follows:

$$\frac{d^4u}{dx^4} = \frac{\alpha}{(1-u(x))^4} + \frac{\beta}{(1-u(x))^2} + \frac{\gamma\beta}{(1-u(x))} \tag{7a}$$

subject to the following boundary condition

$$u(0) = u'(0) = 0, \quad \text{at } x = 0 \text{ at the bended end} \tag{7b}$$

and

$$u''(1) = u'''(1) = 0, \quad \text{at } x = 1 \text{ at the free end.} \tag{7c}$$

The nondimensional parameters α, β and γ, correspond to the values of the Casimir force, applied voltage and fringing field, respectively.

3. Analytical solution

3.1. Adomian decomposition method

The basic idea of the Adomian decomposition method has been explained [33–37]. Now, by substituting $z(x) = 1 - u(x)$, Eq. (1) can be transformed to a system of integral equations

$$\begin{aligned}
z_1(x) &= z_1(0) + \int_0^x z_2(t)dt, \\
z_2(x) &= z_2(0) + \int_0^x z_3(t)dt, \\
z_3(x) &= z_3(0) + \int_0^x z_4(t)dt, \\
z_4(x) &= z_4(0) + \int_0^x \left(\frac{\alpha}{z_1(t)^4} + \frac{\beta}{z_1(t)^2} + \frac{\beta\gamma}{z_1(t)} \right) dt.
\end{aligned} \tag{8}$$

The functions $z_i(x)$ can be written as the following summation [3]:

$$z_i = \sum_{n=0}^{\infty} z_{1,n}, \quad i = 1, 2, 3, 4. \tag{9}$$

By substituting Eq. (9) into Eq. (8) and using the boundary condition at the bended end (i.e., (7b)), the Adomian decomposition of system (8) can be written recursively as

$$z_{1,0} = 1, \qquad z_{2,0} = 0, \qquad z_{3,0} = A_1, \qquad z_{4,0} = A_2 \tag{10a}$$

$$\begin{aligned} z_{1,k+1} &= \int_0^x z_{2,k}dt, \\ z_{2,k+1} &= \int_0^x z_{3,k}dt, \\ z_{3,k+1} &= \int_0^x z_{3,k}dt, \\ z_{1,k+1} &= \int_0^x \left(\alpha C_k^4 + \beta C_k^2 + \gamma\beta C_k^1\right) dt \end{aligned} \tag{10b}$$

subject to the following constraints, which come from (7c):

$$z''(1) = 0, \qquad z''' = 0 \tag{11}$$

where undetermined coefficients A_1 and A_2 correspond to the 2nd and 3rd derivatives of beam deflection with respect to x at $x = 0$, respectively. These new coefficients can be evaluated using natural boundary conditions at the free end (i.e., (11)). The functions C_k^m, which approximate the nonlinear term y_k^{-m}, are determined through the Adomian's polynomials [3,36]:

$$C_k^m = \frac{1}{k!}\frac{d^k}{d\lambda^k}\left[\left(\sum_{i=0}^{k}\lambda^i y_i\right)^m\right]_{\lambda=0}. \tag{12}$$

Therefore, the polynomial solution of Eq. (7) is obtained (see the Appendix). The obtained polynomial solution can be summarized to

$$\begin{aligned} u(x) = {} & A_1\frac{x^2}{2!} - A_2\frac{x^3}{3!} + (\alpha+\beta+\beta\gamma)\frac{x^4}{4!} - (4\alpha+2\beta+\gamma\beta)\frac{A_1x^6}{6!} - (4\alpha+2\beta+\gamma\beta)\frac{A_2x^7}{7!} \\ & + \left(A_1^2(10\alpha+3\beta+\gamma\beta) + \frac{(4\alpha+2\beta+\gamma\beta)(\alpha+\beta+\gamma\beta)}{6}\right)\frac{6x^8}{8!} + O(x^9). \end{aligned} \tag{13}$$

3.2. The Padé approximants

As known, series solutions usually have a finite range of convergence and therefore are not always practical for large values of x. Some researchers have formally explained that since the radius of convergence of the power series may not be large enough to contain the boundaries of the domain, thus such series in isolation are not adequate for handling boundary value problems. However, the objective of Padé approximants is to seek a rational function for the series because Padé approximants converge on the entire real axis if the series solution is free of singularities on the real axis [38,39].

Any power series represents a function $f(x)$ in the form of

$$f(x) = \sum_{i=0}^{\infty} a_i x^i. \tag{14}$$

This expansion of (14) is the fundamental starting point of any analysis using Padé approximants. A Padé approximant is a rational fraction that provides more information about the mathematical behavior of the solution. The notation for such a Padé approximant can be written as [40]

$$a_0 + a_1x + a_2x^2 + \cdots = \frac{p_0 + p_1x + p_2x^2 + \cdots + p_Mx^M}{1 + q_1x + q_2x^2 + \cdots + q_Lx^L}. \tag{15}$$

By cross-multiplying (15) and comparing the coefficients of both sides, one can find that

$$a_l + \sum_{k=1}^{M} a_{l-k}q_k = p_l, \quad l = 0, \ldots, M \tag{16}$$

$$a_l + \sum_{k=1}^{L} a_{l-k}q_k = 0, \quad l = M+1, \ldots, M+L. \tag{17}$$

Solving linear equation (17) provides q_k, where $k = 1, \ldots, L$. Then by substituting q_k into (16), one may easily obtain p_l, where $l = 0, \ldots, M$. The coefficients in (17) form a Toeplitz matrix, which easily can be solved by the Gaussian elimination method. If $M \leq L \leq M + 2$, where M and L are the degree of numerator and denominator in the Padé series, respectively, then this Padé series gives a stable formula for an ordinary differential equation [38,40]. For more details, the readers are referred to [28–30,38–41].

To verify the convergence of the obtained series and Padé approximants, the deflection of a typical nano-actuator is computed analytically, and the solutions are compared with the numerical results. Numerical results are obtained using Runge–Kutta–Fehlberg method [42,43]. Table 1 presents the variation of the cantilever tip deflection (u_{tip}) computed using different selected terms of Adomian power series and different sizes of Padé approximants. This table ensures the convergence of the power series and Padé approximants. Greater accuracy can be obtained by evaluating more terms of the solution. The relative error is computed from

$$\text{Error} = \left| \frac{u_{\text{Analytical}} - u_{\text{Numerical}}}{u_{\text{Numerical}}} \right| \tag{18}$$

where $u_{\text{Analytical}}$ is the tip deflection computed from analytical method (i.e., ADM or ADM–Padé), $u_{\text{Numerical}}$ is the tip deflection computed using numerical method and *Error* represents relative error.

Table 1
The variation of the tip deflection of a typical beam using different selected terms of power series and different selected size of Padé approximants for $\alpha = 0.3$, $\beta = 0.6$, and $g/w = 1$ in the presence of the Casimir effect.

Series size	Tip deflection Adomian	Error	Padé size	Tip deflection ADM–Padé	Error
4	0.16125	2.761E−01	[2, 2]	0.1884526	2.319E−01
6	0.245455	1.019E−01	[3, 3]	0.1258229	4.871E−01
7	0.19375	1.302E−01	[4, 4]	0.2438808	5.928E−03
8	0.26501	1.897E−01	[5, 5]	0.2451924	5.817E−04
9	0.198313	1.097E−01	[6, 6]	0.2432816	8.370E−03
10	0.261546	1.741E−01	[7, 7]	0.2453241	4.496E−05
11	0.202195	9.232E−02	[8, 8]	0.245335	3.281E−07
12	0.256924	1.534E−01	[9, 9]	0.2453359	3.117E−06
13	0.20555	7.726E−02			
14	0.252442	1.332E−01			
15	0.20814	6.563E−02			
16	0.247855	1.127E−01			
17	0.210282	5.602E−02			
18	0.243607	9.358E−02			
19	0.212104	4.784E−02			
20	0.239901	7.695E−02			
Numerical 0.245335113					

By selecting an ADM–Padé size of [5, 5], which is built using 13 terms of an Adomian power series, the relative error between analytical and numerical results is less than 0.06%, as shown in Table 1. Comparing this error with the same series size of the Adomian method (i.e., 13 terms and a relative error of 15.3%) shows the ADM–Padé method could compute the deflection of nano-beams with greater accuracy than the Adomian method. The result of the ADM–Padé method with a size of [5, 5] with 0.06% error is within the acceptable range for most engineering applications. Therefore, the size of [5, 5] is selected in the following section for convenience.

Fig. 2 shows the deflection of narrow nanocantilevers using different sizes of Adomian series in the presence of the electrostatic forces. In this figure, the Adomian solution is compared with numerical results and those of an ADM–Padé with a size of [5, 5].

4. Instability study

To study the instability of the nano-actuator, Eq. (7) is solved numerically and simulated, and the results are compared with those of Eq. (13). For any given α, β and g/w, the cantilever tip pull-in deflection can be obtained from Eq. (13) by setting $du(1)/d\beta \to \infty$. No physical solution exists for $u(x)$ by increasing β beyond β_{PI}. For freestanding cantilevers, pull-in deflection can be obtained by setting $du(1)/d\alpha \to \infty$ in Eq. (13). In addition, the obtained results are compared with numerical results and those of the Adomian decomposition method [3] as well as those of Green's method [12].

Figs. 3 and 4 compare nanocantilever tip deflection obtained by numerical method with ADM–Padé as well as other methods and neglect intermolecular forces for wide beams (i.e., $g/w = 0$) and narrow beams (i.e., $g/w = 1$), respectively. As seen in these figures, the ADM–Padé provides results with very good agreement with the numerical results.

The effect of the fringing field on the nondimensional electrostatic pull-in parameter of β and tip deflection of nanocantilevers (u_{tip}) in the absence of intermolecular forces is shown in Figs. 5 and 6, respectively. In these figures, the results of the numerical method, Adomian method and ADM–Padé are compared with those of the lamped method [3] and the Green method [12].

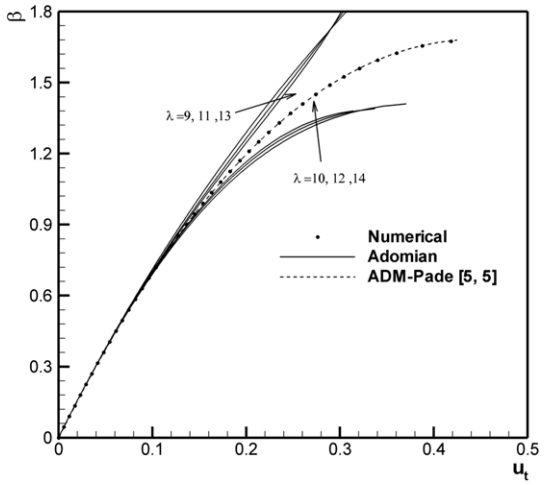

Fig. 2. Effect of Adomian series size on the nanocantilever tip deflection compared to the numerical results and an ADM–Padé size of [5, 5] for $g/w = 0$; intermolecular forces were neglected.

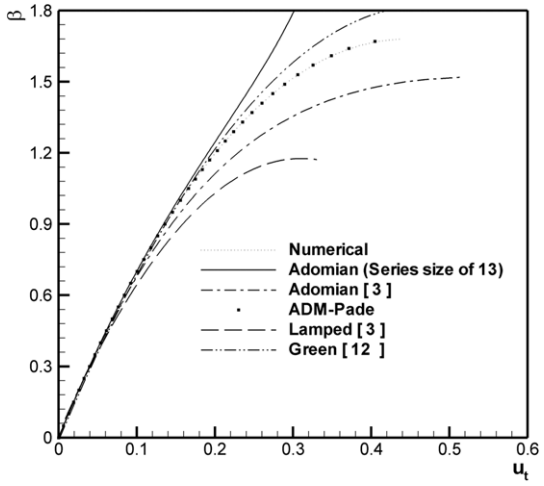

Fig. 3. A comparison of the nanocantilever tip deflection for wide beams (i.e., $g/w = 0$) obtained by a numerical method with ADM–Padé as well as other methods; intermolecular forces were neglected.

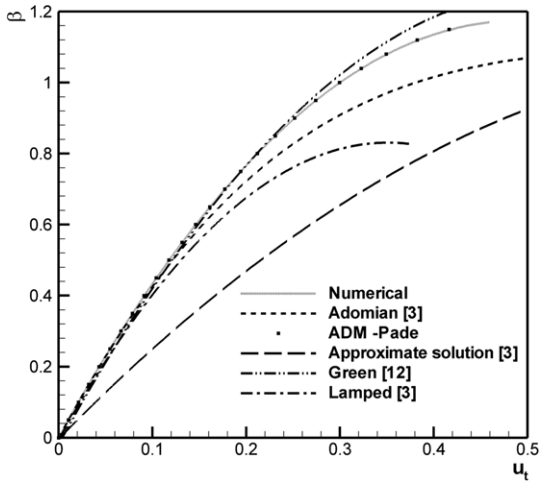

Fig. 4. A comparison of the nanocantilever tip deflection for narrow beams (i.e., $g/w = 1$) obtained by the numerical method with ADM–Padé as well as other methods; intermolecular forces were neglected.

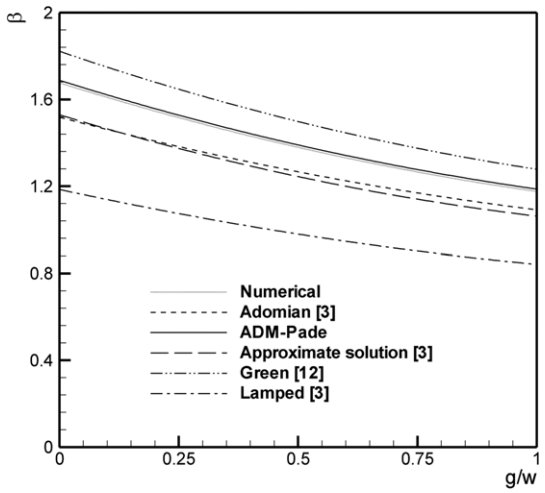

Fig. 5. Effect of the fringing field on the nondimensional electrostatic pull-in parameter of cantilevers, neglecting the intermolecular force.

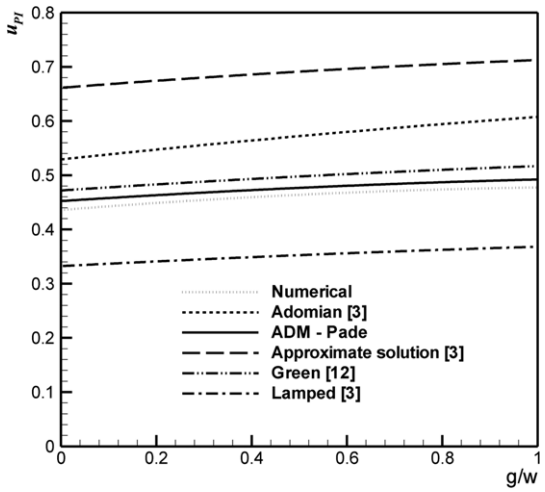

Fig. 6. Effect of the fringing field on the nondimensional tip deflection of cantilevers at the onset of pull-in, neglecting the intermolecular force.

4.1. Intermolecular force at nanoscale separations

When the gap between the fixed and movable beams is small enough, the movable beam might collapse onto the substrate without applying a voltage due to the intermolecular force. The relation between α and u_{tip} in the absence of a voltage difference (freestanding) is presented in Fig. 7. When α exceeds the critical value α_{CP}, no solution exists for u_{tip}, and the instability occurs even without any applied voltage. Using the ADM–Padé, $\alpha_{\text{CP}} = 0.94$, and the numerical method obtained $\alpha_{\text{CP}} = 0.939$; the Adomian method obtained $\alpha_{\text{CP}} = 0.814$ [3]. The maximum length of the actuator that does not stick to the fixed ground plane without applying voltage difference, ($L_{\max}$), is called the detachment length [17]. Alternatively, for a known switch length, there is a minimum gap, ($g_{\min}$), between the switch and the substrate to ensure that the switch does not adhere to the substrate as a result of intermolecular forces [17]. The detachment length and minimum gap of the actuator are basic design parameters for MEMS/NEMS and can be obtained by the critical value of α, i.e., α_{CP}. Substituting the value of α_{CP} into the definition of α (i.e., (6a)), the detachment length and minimum gap are obtained as

$$L_{\max} = \sqrt[4]{18.8\frac{g^5 E_{\text{eff}} h^3}{\pi^2 \hbar c}}, \qquad g_{\min} = \sqrt[5]{\frac{\pi^2 \hbar c L^4}{18.8\, E_{\text{eff}} h^3}}. \tag{19}$$

It is worth to mention that Lin and Zaho [15] obtained the following critical length and gap for the lamped model of nanocantilever actuators,

$$L_{\max} = \sqrt[4]{10.54\frac{g^5 E_{\text{eff}} h^3}{\pi^2 \hbar c}}, \qquad g_{\min} = \sqrt[5]{\frac{\pi^2\, \hbar c L^4}{10.54\ E_{\text{eff}} h^3}}. \tag{20}$$

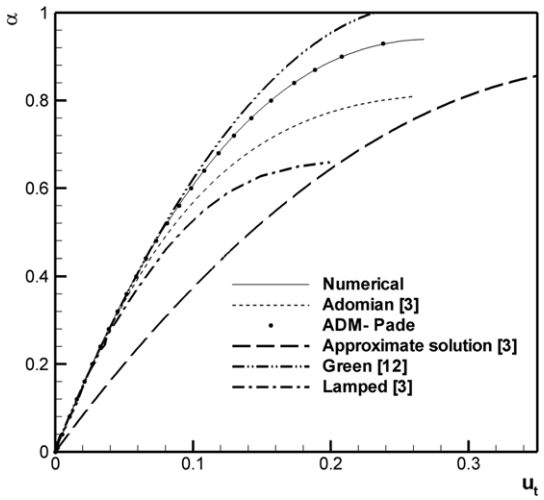

Fig. 7. Relationship between nondimensional intermolecular force (α) and the cantilever tip deflection when no voltage is applied considering the Casimir force. Collapse occurs for α values above the critical value of α ($\alpha_{CP} = 0.94$).

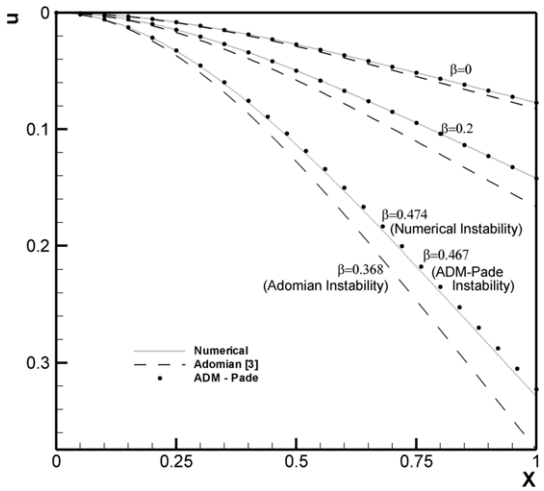

Fig. 8. Deflections of the cantilever for different values of β when $\alpha = 0.5$ and $g/w = 1$. Collapse occurs when β reaches values greater than the critical value ($\beta = 0.467$).

Comparison between Eqs. (19) and (20) show that the lamped model over estimated the minimum gap of cantilever beams that will not adhere with the substrate due to Casimir force.

4.2. Electrostatic and intermolecular force at nanoscale separations

Fig. 8 shows the centerline deflection of a typical nano-beam under both electrostatic and intermolecular loading for $\alpha = 0.5$ and $g/w = 1$ when β increases from zero to instability point. The obtained Padé solution at the onset of instability is

$$u(x) = \frac{-1136376435000x^2 + 67441705750x^3 + 461521946800x^4 - 165453613800x^5}{-1904069401000 - 947218284500x + 414777813000x^2 + 37749775000x^3 + 1351168644x^4 + 1836002523x^5}. \quad (21)$$

Fig. 8 reveals that the beam has an initial deflection due to the presence of the intermolecular force even in the absence of any applied voltage ($\beta = 0$). As seen, the ADM–Padé solution compared with that of the numerical solution is remarkably accurate for computing the deflection and pull-in instability of nanocantilever beams.

5. Conclusions

Pull-in parameters and deflection of nanocantilevers were computed using an integration of the Adomian power series and the Padé approximants method. The Padé approximants improved the convergence and accuracy of Adomian series. The ADM–Padé solution satisfies all boundary conditions. The intermolecular forces decrease the pull-in deflection and voltage of

nano-beams. The minimum initial gap and detachment length of the freestanding nano-actuator were determined, which are the useful design parameters for nano-electromechanical switches. We also compared the solution of the proposed method with that of the Adomian solution and the numerical results as well as those of the Green function method and the lamped model. The ADM–Padé overcame the shortcomings of the lumped parameter model in underestimating the pull-in voltage of nano-beams and was more accurate than the Adomian decomposition method and other analytical methods.

Appendix

$$
\begin{aligned}
&C_0^m = z_{1,0}^{-m},\\
&C_1^m = -m\, z_{1,0}^{-m-1} z_{1,1},\\
&C_2^m = \frac{1}{2} m\,(m+1)\; z_{1,0}^{-m-2} z_{1,1}^2 - m z_{1,0}^{-m-1} z_{1,2},\\
&\vdots = \vdots.
\end{aligned}
\tag{A.1}
$$

Substituting (A.1) in (10b), one may obtain

$$
\begin{aligned}
&z_{1,0} = 1, \quad z_{2,0} = 0, \quad z_{3,0} = A_1, \quad z_{4,0} = A_2\\
&z_{1,1} = 0, \quad z_{2,1} = A_1 x, \quad z_{3,1} = A_2 x, \quad z_{4,1} = -(\alpha+\beta+\gamma\beta)\, x,\\
&z_{1,2} = \frac{A_1 x^2}{2}, \quad z_{2,2} = \frac{A_2 x^2}{2}, \quad z_{3,2} = -(\alpha+\beta+\gamma\beta)\,\frac{x^2}{2}, \quad z_{4,2} = 0,\\
&z_{1,3} = \frac{A_2 x^3}{6}, \quad z_{2,3} = -\frac{(\alpha+\beta+\gamma\beta)}{6} x^3, \quad z_{3,3} = 0, \quad z_{4,3} = A_1\,(4\alpha+2\beta+\gamma\beta)\,\frac{x^3}{6},\\
&z_{1,4} = -(\alpha+\beta+\gamma\beta)\,\frac{x^4}{24}, \quad z_{2,4} = 0, \quad z_{3,4} = A_1\,(4\alpha+2\beta+\gamma\beta)\,\frac{x^4}{24}, \quad z_{4,4} = A_2\,(4\alpha+2\beta+\gamma\beta)\,\frac{x^4}{24},\\
&z_{1,5} = 0, \quad z_{2,5} = A_1\,(4\alpha+2\beta+\gamma\beta)\,\frac{x^5}{120}, \quad z_{3,5} = A_2\,(4\alpha+2\beta+\gamma\beta)\,\frac{x^5}{120}, \quad z_{4,5} = -Q\,\frac{x^5}{20},\\
&z_{1,6} = A_1\,(4\alpha+2\beta+\gamma\beta)\,\frac{x^6}{720}, \quad z_{2,6} = A_2\,(4\alpha+2\beta+\gamma\beta)\,\frac{x^6}{720}, \quad z_{3,6} = -Q\,\frac{x^6}{120}, \ldots\\
&z_{1,7} = A_2\,(4\alpha+2\beta+\gamma\beta)\,\frac{x^7}{5040}, \quad z_{2,7} = -Q\,\frac{x^7}{840}, \ldots\\
&z_{1,8} = -Q\,\frac{x^8}{6720}, \ldots\\
&\cdots = \cdots,
\end{aligned}
$$

where

$$
Q = A_1^2\,(10\alpha+3\beta+\gamma\beta) + \frac{(4\alpha+2\beta+\gamma\beta)\;(\alpha+\beta+\gamma\beta)}{6}.
$$

Therefore, the polynomial solution of (7) is obtained.

References

[1] C.H. Mastrangelo, C.H. Hsu, Mechanical stability and adhesion of microstructures under capillary force-part I: basic theory, J. Microelectromech. Syst. 2 (1993) 33–43.
[2] C.H. Ke, H.D. Espiona, Nanoelectromechanical systems (NEMS) and modeling, in: M. Rieth, W. Schommers, P.D. Gennes (Eds.), Handbook of Theoretical and Computional Nanotechnology, American Scientific Publishers, 2006, (Chapter 121).
[3] A. Soroush, A. Koochi, A.S. Kazemi, A. Noghrehabadi, H. Haddadpour, M. Abadyan, Investigating the effect of Casimir and van der Waals attractions on the electrostatic pull-in instability of nano-actuators, J. Phys. Scr. 82 (2010) 045801.
[4] R. Soroush, A. Koochi, A.S. Kazemi, A. Noghrehabadi, H. Haddadpour, M. Abadyan, Investigating the effect of Casimir and van der Waals attractions on the electrostatic pull-in instability of nano-actuator, Phys. Scr. 82 (2010) 045801.
[5] N.J. Israelachvili, Intermolecular and Surface Forces, Academic Press, London, 1992.
[6] A. Koochi, A.S. Kazemi, A. Noghrehabadi, A. Yekrangi, M. Abayan, New approach to model the buckling and stable length of multi walled carbon nanotube probes near graphite sheets, Mater. Des. 32 (2011) 2949–2955.
[7] A. Ramezani, A. Alasty, J. Akbari, Closed-form approximation and numerical validation of the influence of van der Waals force on electrostatic cantilevers at nano-scale separations, Nanotechnology 19 (2008) 15501–15511.
[8] A. Noghrehabadi, M. Ghalambaz, Y. Tadi Beni, M. Abadyan, M. Noghrehabadi, M. Noghrehabadi, A new solution on the buckling and stable length of multi wall carbon nanotube probes near graphite sheets, Procedia Engrg. 10 (2011) 3733–3741.
[9] S.K. Lamoreaux, The Casimir force: background, experiments, and applications, Rep. Progr. Phys. 68 (2005) 201–236.
[10] W.H. Lin, Y.P. Zhao, Dynamic behavior of nanoscale electrostatic actuators, Chin. Phys. Lett. 20 (2003) 2070–2073.
[11] A. Gusso, G.J. Delben, Dispersion force for materials relevant for micro and nanodevices fabrication, J. Phys. D: Appl. Phys. 41 (2008) 175405 (11 pp).

[12] A. Ramezania, A. Alasty, J. Akbari, Pull-in parameters of cantilever type nanomechanical switches in presence of Casimir force, Nonlinear Anal. Hybrid Syst. 1 (2007) 364–382.
[13] F.M. Serry, D. Walliser, G.J. Maclay, The role of the Casimir effect in the static deflectionand stiction of membrane strips in MEMS, J. Appl. Phys. 50 (1998) 2501–2506.
[14] H.B. Chan, V.A. Aksyuk, R.N. Kleiman, D.J. Bishop, F. Capasso, Quantum mechanical actuation of micro-electromechanical systems by the Casimir force, J. Science 291 (2001) 1941–1944.
[15] W.H. Lin, Y.P. Zhao, Casimir effect on the pull-in parameters of nanometer switches, Microsyst. Technol. 11 (2005) 80–85.
[16] W.H. Lin, Y.P. Zhao, Nonlinear behavior for nanoscale electrostatic actuators with Casimir force, Chaos Solitons Fractals 23 (2005) 1777–1785.
[17] M. Ghalambaz, A. Noghrehabadi, M. Abadyan, Y. Tadi Beni, A. Noghrehabadi, M. Noghrehabadi, A new power series solution on the electrostatic pull-in instability of nano cantilever actuators, Procedia Engrg. 10 (2011) 3716–3724.
[18] M. Ghalambaz, A. Noghrehabadi, A. Ghanbarzadeh, Investigating deflection of nano-cantilevers with Casimir effect using monotone solution, in: 19th Annual Conference on Mechanical Engineering, Birjand University, Iran, May 2011.
[19] M. Ghalambaz, A. Noghrehabadi, A. Vosoogh, A hybrid Power series—artificial bee colony algorithm to solve electrostatic pull-in instability and deflection of nano cantilever actuators considering Casimir attractions, Int. Rev. Mech. Engrg. 5 (2011) 586–593.
[20] M. Ghalambaz, A. Noghrehabadi, M. Abadyan, Y. Tadi Beni, A. Noghrehabadi, M. Noghrehabadi, A deflection of nano-cantilevers using monotone solution, Procedia Engrg. 10 (2011) 3725–3732.
[21] K. Jonnalagaddaa, S.W. Chob, I. Chasiotisa, T. Friedmannc, J. Sullivanc, Effect of intrinsic stress gradient on the effective mode-I fracture toughness of amorphous diamondlike carbon films for MEMS, J. Mech. Phys. Solids 56 (2008) 388–401.
[22] N. Pugno, B. Peng, H.D. Espinosa, Predictions of strength in MEMS components with defects—-a novel experimental–theoretical approach, Internat. J. Solids Struct. 42 (2005) 647–661.
[23] M.A. Noor, S.T. Mohyud-Dina, Variational iteration method for unsteady flow of gas through a porous medium using He's polynomials and Padenext term approximants, Comput. Math. Appl. 58 (2009) 2182–2189.
[24] A. Pozzi, E. Bassano, Application of Padé approximants to the study of plane jets, Comput. Math. Appl. 30 (1995) 107–123.
[25] Y.C. Jiao, Y. Yamamoto, C. Dang, Y. Hao, An aftertreatment technique for improving the accuracy of Adomian's decomposition method, Comput. Math. Appl. 43 (2002) 783–798.
[26] J. Lu, Variational iteration method for solving a nonlinear system of second-order boundary value problems, Comput. Math. Appl. 54 (2007) 1133–1138.
[27] O.D. Makinde, Hermite–Padé approach to thermal radiation effect on inherent irreversibility in a variable viscosity channel flow, Comput. Math. Appl. 58 (2009) 2330–2338.
[28] E. Celik, M. Bayram, Arbitrary order numerical method for solving differential–algebraic equation by Padé series, Appl. Math. Comput. 137 (2003) 57–65.
[29] E. Celik, M. Bayram, The numerical solution of physical problems modeled as a system of differential–algebraic equations (DAEs), J. Franklin Inst. 342 (2005) 1–6.
[30] M. Ghalambaz, A. Noghrehabadi, M. Ghalambaz, M. Abadyan, Y. Tadi Beni, M. Noghrehabadi, M. Noghrehabadi, A new solution for natural convection about a vertical cone embedded in porous media prescribed wall temperature using power series—Padé, Procedia Eng. 10 (2011) 3749–3757.
[31] O.D. Makinde, R.J. Moitsheki, On non-perturbative techniques for thermal radiation effect on natural convection past a vertical plate embedded in a saturated porous medium, Math. Probl. Eng. 2008 (2008) 689074 (11 pp).
[32] O.D. Makinde, On non-perturbative approach to transmission dynamics of infectious diseases with waning immunity, Int. J. Nonlinear Sci. Numer. Simul. 10 (2009) 451–458.
[33] S. Timoshenko, Theory of Plates and Shells, McGraw Hill, New York, 1987.
[34] G. Adomian, Inversion of nonlinear stochastic operators, J. Math. Anal. Appl. 91 (1983) 39–46.
[35] G. Adomian, R. Rach, Modified decomposition solution of linear and nonlinear boundary-value problems, Nonlinear Anal. 23 (1994) 615–619.
[36] W. Chen, Z. Lu, An algorithm for Adomian decomposition method, Appl. Math. Comput. 159 (2004) 221–235.
[37] R.C. Rach, A new definition of the Adomian polynomials, Kybernetes 37 (2008) 910–955.
[38] H. Simsek, E. Celik, The successive approximation method and Padé approximants for solutions the non-linear boundary value problem, Appl. Math. Comput. 146 (2003) 681–690.
[39] H. Chu, Y. Zhao, Y. Liu, A MAPLE package of new ADM-Padé approximate solution for nonlinear problems, Appl. Math. Comput. 217 (2010) 7074–7091.
[40] A.M. Wazwaz, Analytical approximations and Padé approximations for Volterras population model, Appl. Math. Comput. 100 (1999) 13–25.
[41] N. Guzel, M. Bayram, Numerical solution of differential–algebraic equations with index-2, Appl. Math. Comput. 174 (2006) 1279–1289.
[42] E. Fehlberg, Low-Order Classical Runge–Kutta Formulas with Step Size Control and their Application to Some Heat Transfer Problems, NASA, 1969.
[43] E. Fehlberg, Klassische Runge–Kutta-Formeln vierter und niedrigerer Ordnung mit Schrittweiten-Kontrolle und ihre Anwendung auf Wärmeleitungsprobleme, Computing (Arch. Elektron. Rechnen) 6 (1970) 61–71.