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11th International Conference on Modern Building Materials, Structures and Techniques, MBMST 2013 Analysis of the Time-Dependent Reliability Index

for Deteriorating Load-Carrying Members

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Abstract

The probability-based prediction and analysis of time-dependent survival probabilities and reliability indexes of non-deteriorating and deteriorating structural members exposed to extreme gravity and climate action effects are presented. A degradation function of reinforced concrete members under corrosion actions and sulphate attacks is discussed. The time-dependent safety margin of particular members as a random finite sequence of auto structural members is analyzed. Their time-dependent survival probability and reliability index values are expressed in simplified analytical manner. The merit of the method of transformed conditional probabilities based on more accurate conventional correlation factors is demonstrated by numerical examples.

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Keywords: degradation function, deteriorating members, correlation factor, safety margin, survival probability, reliability index.

1. Introduction

The structural members (beams, slabs, columns, joints) of buildings and civil engineering works are represented in the design practice by their particular members (normal or oblique sections, connections) for which the only possible failure mode exists. The robustness and safety requirements of design codes on the structural members should be satisfied for all particular and structural members.

Structural members of load-carrying structures may be treated as auto systems of correlated particular members. However, multicriteria failure modes and survival probabilities of structural members under different limit states may be objectively assessed and predicted only knowing structural safety parameters of particular members. Using the traditional deterministic design concepts and methods of partial safety factors it is hard to translate into reality probabilistic design formats for a prediction of the safety degree of deteriorating members subjected to maximum service and climate action effects.

The technical service life as the lifetime at preset target reliability index of deteriorating members is the period for which it can actually perform, according to the service requirements based on an intended purpose, without major repairs. Target service life of buildings and construction works should be specified by a client or owner in accordance with general assumptions recommended in design codes and standards as principles and application rules. However, they must have unsophisticated approaches for probability-based recommendations on generate search directions in the random processes of the parameters related to extreme loading and degrading resistance of deteriorating structures.

The probabilistic reliability prediction of the structural quality in design practice seems fairly limited. At present it is hard to revise structural design regulations based on the principle of probability optimization, nevertheless the practical activity of engineers to use probabilistic reliability methods in their design practice. The analysis of emergency situations

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shows, that in most cases the reliability estimation of structural members is inevitable, and namely in cases of then aggressive environment actions, and random short-time overloading by static or dynamic action effects, etc.

A possibility to predict objectively structural failures and collapses of deteriorating members of buildings and construction works subjected to extreme service loads, wind gusts, snow pressures and wave surfs is one of significant concerns of researchers. But simplified and quite exact data on methodology and mathematical formats as component parts of the structural reliability analysis should be implanted also into engineering design practice.

Probability-based approaches allow us explicitly predict safety and durability parameters of structural members. According to Rackwitz [1], it is possible to introduce probabilistic approaches and formats in design practice of deteriorating structures. But the engineering modeling of time-variant safety indexes of deteriorating structural members is still unsolved and not translated in design codes and standards.

This paper deals with the same design concepts on probability-based modeling and time-dependent reliability index for particular and single structural members of deteriorating and non-deteriorating reinforced concrete structures subjected to recurrent single or joint extreme variable loads and climate actions as random rectangular pulse renewal processes. The method of transformed conditional probabilities (TCP) as the safety verification format is based on the more accurate values of the conventional correlation factor (CCF).

2. Time-dependent analysis model

As it is known, aggressive actions are the main factors of deterioration processes of steel and reinforced concrete structures. The hierarchical models assume that the resistance of particular members changes in time, i.e. it must be treated as a time-dependent random quality R(t). The artificial ageing and deterioration of materials caused by aggressive environmental actions is very dangerous for load-carrying structures. Aggressive actions induced by concrete carbonation, chloride penetration and other chemical or physical attacks are the basic factors of deteriorating reinforced concrete structures. Corrosion protection of steel reinforcement bars and connections depends on the density, quality and thickness of concrete covers and its cracking intensities.

It is expedient to divide the life cycle of deteriorating concrete members as their physical and mechanical degradation processes [2] into the initiation, t_{in} , propagation, t_{pr} , and attack, t_{al} , phases (Fig. 1).



Fig. 1. Degradation function $\varphi(t)$ (a) and dynamic model (b) for time-dependent reliability analysis: 1 – unloaded members, 2 – loaded columns, 3 – loaded beams

A length of the initial phase, t_{in} , are a random variable depending on features of degradation process, environment aggressiveness and quality of protected covers. The unvulnerability of reinforced concrete structures may be characterized by the duration of this phase. When the degradation process of members is caused by intrinsic properties of materials, the phase $t_{in}\approx 0$. The corrosion initiation period comes to the end when a carbon dioxide front reaches the surface of reinforcement or the concentration of chloride ions becomes equal to its effective level. The resistance of members in

initiation period, R_{in} , is presented as a fixed stationary random process. Its numerical values are random only at the beginning of this process.

The concrete spalling process during the attack phase, t_{at} , is related to the concentration of corrosion products in normal and longitudinal cracks of members. These cracks are undesirable due to bond-anchorage losses and cross-section diminishes of reinforcing steel bars (rebars). The propagation and attack periods are delayed duration for structures protected by coats.

According to consecutive deterioration law, the resistance of members in the period $t > t_{in}$ is treated as a non-stationary process

$$R(t) = R_{in}\varphi(t) = R_0\varphi(t), \qquad (1)$$

where $\varphi(t)$ denotes the degradation function depending on the rate of the resistance decrease induced by artificial ageing and degradation of materials.

The degradation function of particular members for structures may be presented in the form

$$\varphi(t) = 1 - \alpha (t - t_{in})^b, \qquad (2)$$

where α is a random degradation intensity factor, *b* depends on a degradation mechanism and defines a non-linearity of this function. For typical degradation mechanism of reinforced concrete members exposed to corrosion actions and sulphate attacks, the parameter *b* is between 1.0 and 2.0 [3–5].

Action effects of structures are caused by permanent loads g, variable live loads q(t), snow loads s(t) and wind actions w(t). When the actions are of different nature, they sometimes may show quite complex physical interactions. If snow and wind act together, the result may be that the wind reduces the accumulated snow load on the roof. For some building configuration, however, the combined action by wind and snow may result in much higher loads on some specific locations. This dependency between wind and snow presents even if wind and snow loading are statistically completely independent processes.

The sustained part of imposed long-term live loads, $q_s(t)$, in civil and industry buildings contains the weight of furniture and heavy equipments, respectively. The short-term extraordinary part of live loads, $q_e(t)$, is caused by special events and mobile equipments. Their statistical parameters depend on the user category of buildings and works. Renewal rates of annual extreme floor and climate actions are equal to $\lambda = 1/year$.

The mean values and standard deviations of additional variables for reinforced concrete resistance models and load effects models are: $\theta_{Rm} = 0.99 - 1.04$, $\sigma \theta_R = 0.05 - 0.10$ and $\theta_{Em} \approx 1.0$, $\sigma \theta_E \approx 0.1$ [2], [6].

3. Time-dependent safety margin

According to probability-based approaches, the time-dependent safety margin of deteriorating particular members exposed to extreme action effects may be defined as their random performance process and presented as follows

$$Z(t) = g\{\mathbf{X}(t), \mathbf{\theta}\} = \theta_R R(t) - \theta_g E_g - \theta_q E_q(t) - \theta_{cl} E_{cl}(t), \qquad (3)$$

where $\mathbf{X}(\mathbf{t})$ and $\boldsymbol{\theta}$ are the vectors of basic and additional variables, representing respectively random components and their model uncertainties.

Taking into account model uncertainties, for deteriorating particular members affected by extreme action effects the variance of their safety margins may be expressed as follows

$$\sigma^{2}Z(t) = \sigma^{2} \left[\theta_{R}R(t) \right] + \sigma^{2} \left[\theta_{g}E_{g} \right] + \sigma^{2} \left[\theta_{q}E_{q}(t) \right] + \sigma^{2} \left[\theta_{cl}E_{cl}(t) \right].$$
(4)

According to ISO 2394 [7] and EN 1990 [8] recommendations, Gaussian and lognormal distribution laws are to be used for member resistances. The permanent actions and concrete strengths can be described by a normal distribution law. Therefore, for the sake of simplified but quite exact probabilistic analysis of deteriorating members, it is expedient to present Eq. (3) in the form

$$Z_k = R_{ck} - E_k \,, \tag{5}$$

where

$$R_{ck} = \theta_R R_k - \theta_g E_g \,, \tag{6}$$

is the conventional resistance of a member,

$$E_k = \theta_q E_{qk} + \theta_{cl} E_{cl,k} \,, \tag{7}$$

is the conventional bivariate distribution process of two stochastically independent annual extreme effects with the mean $E_{km} = \theta_{qm}E_{q,km} + \theta_{cl,m}E_{cl,km}$ and variance $\sigma^2 E_{km} = \sigma^2 (\theta_q E_{qk}) + \sigma^2 (\theta_{cl} E_{cl,k})$ [9]. For building floor and roof members, the extreme annual variable action effects are caused by live or by snow loading, respectively. For these structural members, Eq. (7) changes into $E_{1k} = \theta_q E_{ak}$ or $E_{2k} = \theta_{cl} E_{cl,k}$.

The imposed sustained, $q_s(t)$, and extraordinary, $q_e(t)$, variable service loads may be modeled as time-variant continuous and intermittent rectangular renewal processes, respectively. Lognormal, Weibull and gamma distributions are used for sustained action effects [7]. Intermittent short-term extraordinary loads may be used to be distributed by exponential and extreme value laws [2], [10]. The maximum sum of these imposed load components may be modeled by Gumbel or extreme value type I distribution law [11].

The means and variances of component action effects E_{q_s} and E_{q_e} are closely related with statistical properties of residential, commercial, office and industrial buildings. The annual extreme sum and its coefficient of correlation of variable imposed action effects, respectively, are

$$E_{qm} = E_{q_sm} + E_{q_em} = \frac{E_{q_sk}}{1 + k_{0.95}^{q_s} \times \delta E_{q_s}} + \frac{E_{q_ek}}{1 + k_{0.95}^{q_e} \times \delta E_{q_e}},$$
(8)

$$\delta E_q = \frac{\left[\left(\delta E_{q_s} \times E_{q_s m} \right)^2 + \left(\delta E_{q_e} \times E_{q_e m} \right)^2 \right]^2}{E_{q m}}, \tag{9}$$

where E_{q_sk} and E_{q_ek} are the characteristic (nominal) values of live action effect components, $k_{0.95}^{q_s}$ and $k_{0.95}^{q_e}$ are the characteristic fractiles of their probability distributions, $\delta E_{q_s} = 0.75 - 2.5$ and $\delta E_{q_e} = 1.6 - 3.5$ are the coefficients of variation of these components [2].

It is proposed to model annual extreme snow and wind climate action effects by a Gumbel distribution with the mean values equal to $E_{sm} = E_{sk}/(1+k_{0.98}\delta E_s)$ and $E_{wm} = E_{wk}/(1+k_{0.98}\delta E_w)$, where E_{sk} and E_{wk} are the characteristic (nominal) values of action effects and $k_{0.98}$ is the characteristic fractiles factor of these distributions [12], [13], [7], [2], [10]. The coefficients of variation of snow and wind extreme loads depend on the feature of geographical area and are equal to $\delta s = 0.3 - 0.7$ and $\delta w = 0.2 - 0.5$. The durations of extreme climate actions are $d_s = 14 - 28$ days and $d_w = 8 - 12$ hours.

When dangerous action effects may be caused by two statistically independent variable loads or actions, three stochastically dependent safety margins should be considered as follows

$$Z_{1k} = R_{ck} - E_{1k}, \ \exists \ k \in [1, n_1], \tag{10}$$

$$Z_{2k} = R_{ck} - E_{2k}, \ \exists \ k \in [1, n_2],$$
(11)

$$Z_{12k} = R_{ck} - E_{12k}, \ \exists \ k \in [1, n_{12}],$$
(12)

where n_{12} is the average recurrence number of joint extreme loads E_1 and E_2 [14].

4. Time-dependent survival probability

In many cases, the conventional resistance $R_{ck}=R_k-E_g$ and the action effect E_k of members may be treated as statistically independent variables. Thus, the instantaneous survival probability of a member at k-th extreme situation, assuming that it were safe at the situations 1, 2, ..., k-1, is calculated by the formula

$$\boldsymbol{P}(S_k) = 1 - \boldsymbol{P}\left(F_k \bigcap_{i=1}^{k-1} S_i\right) = \boldsymbol{P}(Z_k > 0) = \boldsymbol{P}\left\{R_{ck} > E_k \exists k \in [1,n]\right\} = \int_0^\infty f_{R_{ck}}(x)F_{E_k}(x)dx, \qquad (13)$$

where $P\left(F_k \bigcap_{i=1}^{k-1} S_i\right)$ is the instantaneous failure probability of a member at k-th sequence cut, $f_{R_{ck}}(x)$ is the density function

of conventional member resistance R_{ck} and

$$F_{E_k}(x) = \exp\left[-\exp\left(\frac{E_{km} - x}{0.7794 \times \sigma E_k} - 0.5772\right)\right]$$
(14)

is the cumulative distribution function of the action effect E_k the mean and standard deviation of which are E_{km} and σE_k .

Usually, the decreasing stochastically dependent instantaneous survival probabilities of particular and single members form random decreasing sequences as series auto systems. The time-dependent survival probability of members as series systems may be calculated by Monte Carlo simulation and numerical integration methods. However, it is more reasonable to use the unsophisticated simplified but quite exact method of transformed conditional probabilities (TCP) [15].

According to the concept of conventional safety margin processes, the total survival probability of particular or real structural deteriorating member during a given period t_n may be written in the form

$$P(T \ge t_n) = P\left(\bigcap_{k=1}^n S_k\right) = P\left[\bigcap_{k=1}^n (Z_k > 0)\right] = P\left[\bigcap_{k=1}^n (R_{ck} - E_k > 0) \ \forall \ k \in [1, n]\right] = \prod_{k=1}^n P(S_k) \left[1 + \rho_{12}^{x_2} \left(\frac{1}{P(S_1)} - 1\right)\right] \times \dots \times \left[1 + \rho_{1,2,\dots,k-1}^{x_k} \left(\frac{1}{P(S_{k-1})} - 1\right)\right] \times \dots \times \left[1 + \rho_{1,2,\dots,k-1}^{x_k} \left(\frac{1}{P(S_{k-1})} - 1\right)\right] \times \dots \right] \times \left[1 + \rho_{1,2,\dots,n-1}^{x_n} \left(\frac{1}{P(S_{n-1})} - 1\right)\right],$$
(15)

where R_{ck} by Eq. (6) and E_k by Eq. (7) are the time-dependent conventional resistance and extreme annual (joint or single) action effect modeled by Gaussian and Gumbel distribution laws, respectively; $P(S_k)$ by Eq. (13) is the instantaneous survival probability of a member at *k*-th extreme situation.

The coefficient of auto correlation of cuts of rank safety margin process, Z_k , by Eqs. (10–12) can be represented as

$$\rho_{kl} = Cov(Z_k, Z_l) / \sigma Z_k \times \sigma Z_k \approx \varphi_k \varphi_l / \left[1 + \sigma^2(E_k) / \sigma^2(R_{ck}) \right],$$
(16)

where $Cov(Z_k, Z_l)$ and $\sigma Z_k, \sigma Z_k$ are an auto covariance and standard deviations of random process cuts.

The bounded correlation factor of random deteriorating series system elements or decreasing sequence cuts may be expressed as

$$\rho_k^{x_k} = \rho_{1,2,\dots,k-1|k}^{x_k} = \left[\left(\rho_{k} + \rho_{2k} + \dots + \rho_{k-1,k} \right) / (k-1) \right]^{x_k}.$$
(17)

According to the concept of conventional correlation factor (CCF), the bounded index x_k of this factor for auto and real series systems may be expressed as

$$x_k \approx \left[\left(4.5 + 4\rho_k \right) / \left(1 - 0.98\rho_k \right) \right]^{\nu_k},$$
 (18)

where

$$v_k = \sum_{i=2}^k P(S_i) / ((k-1)\sqrt{n+2}),$$
(19)

is the parameter of sequence cuts.

This parameter and the TCP method with more accurate values of indexed conventional correlation factors (CCF) may be used also as main approaches in probabilistic safety analysis and prediction of series systems consisted of correlated structural members.

5. Time-dependent reliability index

The survival probability of deteriorating particular and structural members may be also introduced by the generalized reliability index

$$\beta(t) = \Phi^{-1} \left[P(T \ge t_n) \right], \tag{20}$$

where $\Phi[\bullet]$ is the cumulative distribution function of the standard normal distribution. The fitness of Eqs. (18) and (20) in structural reliability design is defined by an identity of curves 1 and 2 demonstrated in Fig. 2 [15], [17]. Fig. 2 illustrates that the values of total survival probability by Eq. (15) and reliability index by Eq. (20) are close to structural safety parameters calculation which is based on complicated mathematical models.

The reliability classes (RC) for load carrying structures may be defined by the reliability index, β , concept. According to the EN1990 [8], for persistent design situations, the target value, β_T , is equal to 3.3, 3.8 and 4.3 for reliability classes RC1, RC2 and RC3 of structural members. The values of β_T for particular members should be not less.



Fig 2. The reliability index, β , of equicorrelated series systems presented (a) ($\beta = 4$, n = 5, 50) and (b) ($\beta = 3$, 5, n = 7, 27) versus their correlation factor ρ_k and is calculated by numerical integration (1) and TCP (2) methods

Three reliability classes RC1, RC2 and RC3 are associated with the three consequences classes CC1, CC2 and CC3 (EN [8]. A classification into consequences classes is based on the ratio ρ as the ratio between construction plus failure costs and construction costs. However, it is more expedient to relate the calibrated values of target reliability index of deteriorating structures to their failure consequences and functional working classes (Table 1).

Consequences classes	Functional working classes		
	FC1	FC2	FC3
CC1	3.1	3.3	3.8
CC2	3.3	3.8	4.3
CC3	3.8	4.3	4.7

Table 1. Relation between consequences and functional working classes

When structural members are easily and not easily repairable or replaceable, they belong to FC1 and FC2 classes, respectively. Not repairable and not replaceable members must be treated as FC3 class structures.

The calibrated values of lifetime reliability index, β_T , recommended by design codes and standards belong to members of non-deteriorating structures. However, the consequences of failure of deteriorating structural systems and their members depend on different in nature initiation and propagation phases of degradation process and may lead to unexpected results.

The practice of design, construction, erection and maintenance activities for structures shows that their reliability indexes vary within wide limits. That is why these indexes of structures along with those of labour, materials and power consumption give more chances to evaluate the fitness of the structural solution of buildings and constructions and enable the selection of the economically advantageous level of their structural safety.

6. Numerical illustration

We discuss the time-dependent reliability index of non- deteriorating and deteriorating single reinforced concrete beams of a flat roof resisting to bending moments E_g and E_s caused by dead and extreme snow loads.



Fig. 3. The instantaneous (a) and long-term (b) reliability indexes, β , of non-deteriorating (1), low (2), medium (3) and high (4) deteriorating reinforced concrete members calculated by TCP method versus their service life *t*

The corrosion of reinforcing steel bars (rebars) is caused by internal chloride ions. Therefore, an initiation corrosion period $t_{in} \approx 0$. The degradation function of beams is: $\varphi(t) = 1 - a \cdot t$, where $a_m = 0.00125$; 0.0025; 0.00375. Therefore, the values of bounded correlation factors of particular members for non-deteriorating and deteriorating beams, respectively, are: $\rho_{k=1}^{x_k} \approx 0.30$, $\rho_{k=50}^{x_k} \approx 0.74$ and $\rho_{k=1}^{x_k} \approx 0.30$, $\rho_{k=50}^{x_k} \approx 0.72 - 0.68$.

The mean and variance of initial resistance are: $(\theta_R R_{in})_m = 521 \text{ kNm}, \sigma^2(\theta_R R_{in}) = 2714 (\text{kNm})^2$, respectively.

The means and variances of bending moments are: $(\theta_E E_g)_m = 140 \ kNm$, $\sigma^2(\theta_E E_g) = 196 \ (kNm)^2$; $(\theta_E E_s)_m = 60 \ kNm$, $\sigma^2(\theta_E E_s) = 576 \ (kNm)^2$.

The results of time-dependent reliability indexes of non-deteriorating and deteriorating reinforced concrete beams, when according to the values of degradation function their deterioration level is low, medium and high, are presented in Fig. 3.

The diminution of initial reliability index of non-deteriorating beams may be explained to a great effect of recurrent annual extreme snow loads. It demonstrates that a decrease of reliability indexes of load-carrying members depends on intensities of aggressive and extreme loading actions.

7. Conclusions

The unsophisticated but quite exact time-dependent safety margin process, Z(t), and its bounded conventional correlation factor (CCF), $\rho_k^{x_k}$, granted possibilities to assess and predict objectively in a simple manner instantaneous and long-term survival probabilities and reliability indexes of particular (sections, connections) and real structural members.

For the deteriorating concrete members, a distribution of the conventional resistance process, R_{ck} , may be modeled by the normal or bivariate distribution law. The conventional extreme annual action effect, E_k , may be treated as an intermittent rectangular renewal pulse process which distribution may be modeled by Gumbel law.

The survival probability of random particular members as auto series system elements or safety margin sequence cuts of non-deteriorating and deteriorating series structural systems of buildings and construction works subjected to recurrent extreme annual static or dynamic service and climate action effects may be calculated using the transformed conditional probabilities (TCP) method with more accurate values of indexed conventional correlation factors (CCF).

In this case, the more accurate and simplified method of transformed conditional probabilities may be treated as full probabilistic. It helps not only structural, but also resident and management engineers, having minimum appropriate skills, to use probability-based approaches and formats in their design and inspection practice.

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