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Numerical Experiment to Analyse the Reliability of Coarse Grid based Numerical Methods for Modelling Groundwater Flow by Drainage Objects like Wells in Confined Aquifer

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Abstract

At the neighbourhood of several drainage objects types in shallow aquifer like fully penetrated wells, partially penetrated wells, well with laterals, horizontal drainages etc. groundwater flow forms contain frequently 2D/3D singular behaviours (e.g. logarithmical or polar singularities), which require special attention by coarse grid simulations Groundwater reservoir modelling. The first objective of the paper is to test the accuracy of Finite Volume Methods with specific reference to one of the most used standard reservoir modelling software in groundwater modelling PMWIN/MODFLOW (i.e. Processing MODFLOW for Window), Chiang (2001). The numerical experiment will be accomplished mainly for fully penetrating well (W) and partially penetrating well (ppW) using various discretization size.

It will be shown that the standard Well Index method (Peaceman 1983), currently used by coarse grid simulations, allows the local correction in well blocks only for hydraulic head (or pressure) in the wellbore obtained by coarse grid simulation for imposed global discharge/recharge rate of the well. In the paper an extension of the WI based method will be proposed which allows also the well discharge rate correction by coarse grid modelling for imposed hydraulic head (imposed pressure) in the well. The proposed method is proved with several numerical simulation examples for fully wells and ppWells.

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1. *Keywords:* coarse grid simulation, MODFLOW, Well Block Index, well discharge correction;

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1. Introduction

An optimal groundwater management requires reliable mathematical modelling approaches which take into account correctly all important physical aspects of the flow system.

In this regard can be mentioned that for the planning of complex groundwater catchment plant (GWCP) is necessary to analyse several modelling scales [6,7]: a large extended spatial region (1st order modelling scale) which includes rivers, lakes, zonal inhomogeneity, natural groundwater recharge zones and the (GWCP) as a global effect; the neighbourhood region (2nd order modelling scale) including the GWCP components in which the real flow conditions at the components like fully penetrating wells, partially penetrating wells, well with laterals, drains etc. have a great importance. The local flow condition at these components leads mathematically to singular behaviour (e.g. 2D logarithmical singularity). The same spatial scale problem appears by exploitation of petroleum resources [5].

Analytical or semi analytical methods allows the correctly modelling of such singular flow behaviours in near zone of drainage objects [10], [2], [9], [6], [7], 10] but these are generally applicable in limited cases like simplified geometry and boundary condition, homogeneous aquifer etc.

The method of decoupled overlapping grid modelling [5] or the coupled analytical modelling for the neighbourhood region of drainage objects with the coarse grid simulations for the regional reservoir [6], [7], [8] are also alternatives to model correctly the local flow conditions at drainage objects**.** An important disadvantage of these methods is that these involve development and using specialized programs/software which are less common as the known coarse grid simulations programs like PMWIN/MODFLOW [11,12].

Another possibility for modelling can offer the FDM and FEM based numerical programs, using a highly refined mesh in the neighbourhood of drainage object [12]. It is to mention that these methods would significantly increase computational cost especially for 3D field-scale simulations with a large number of wells. The most attractive alternative to above described method represent the well-known well index (WI) method [3], [14] which is widely accepted as standard method for involving well blocks in coarse grid simulations. This method consists in performing the coarse grid reservoir simulation (e.g. PMWIN/MODFLOW) in which wellbore are modelled as point sours in well blocks followed by correction of the resulted hydraulic head in the well block.

In the paper there will be shown that the above mentioned widely accepted standard WI-method [3] allows only the correction of steady-state well bore pressure (or hydraulic head) in well by imposed discharge/recharge rate of the well. Consequently more frequently technical applications for which the well discharge is searched by given hydraulic head, the correction with the currant WI method described above not applicable. In the paper an extension of the standard WI method will be proposed which allows also the well discharge correction by imposed head (or pressure) in the well. The efficiently of the method will be confirm by means of several numerical tests for partially penetrating wells.

2. Performing of error estimation by coarse grid simulation of wells

For examination the reliability of FVM (e.g. PMWIN/MODFLOW) by modelling of extraction wells, a numerical experiment was performed for partially penetrating wells (pWell) located in a homogeneous confined aquifer (Fig.1). In the standard coarse grid reservoir simulation programs (e.g. PMWIN/MODFLOW) extraction/injection wellbore is modelled as point source placed in discretization blocks (well-blocks) which are crosses from the wellbore. The modelling errors occur because the real form of solution function between hydraulic head and the distance from well (e.g. logarithmic) is replaced with simplified relations (e.g. linear) currently used in the numerical methods. To perform the numerical experiment for error estimation several sizes of space discretisation are used ($\Delta x=\Delta y=\delta=10$m, 5m, 2m and 1m and $\Delta z=\delta_z$). It was analysed two representative cases of the wellbore modelling: Case 1 when is searched the hydraulic head (h_W) or drawdown (i.e. $\Delta H_W=h_R-h_W$) in the well by given well discharge Q_W and Case 2: when is searched the well discharge Q_W by given drawdown or hydraulic head in the wellbore. In both cases the hydraulic head (h_R) at the external boundary of the reservoir is given.

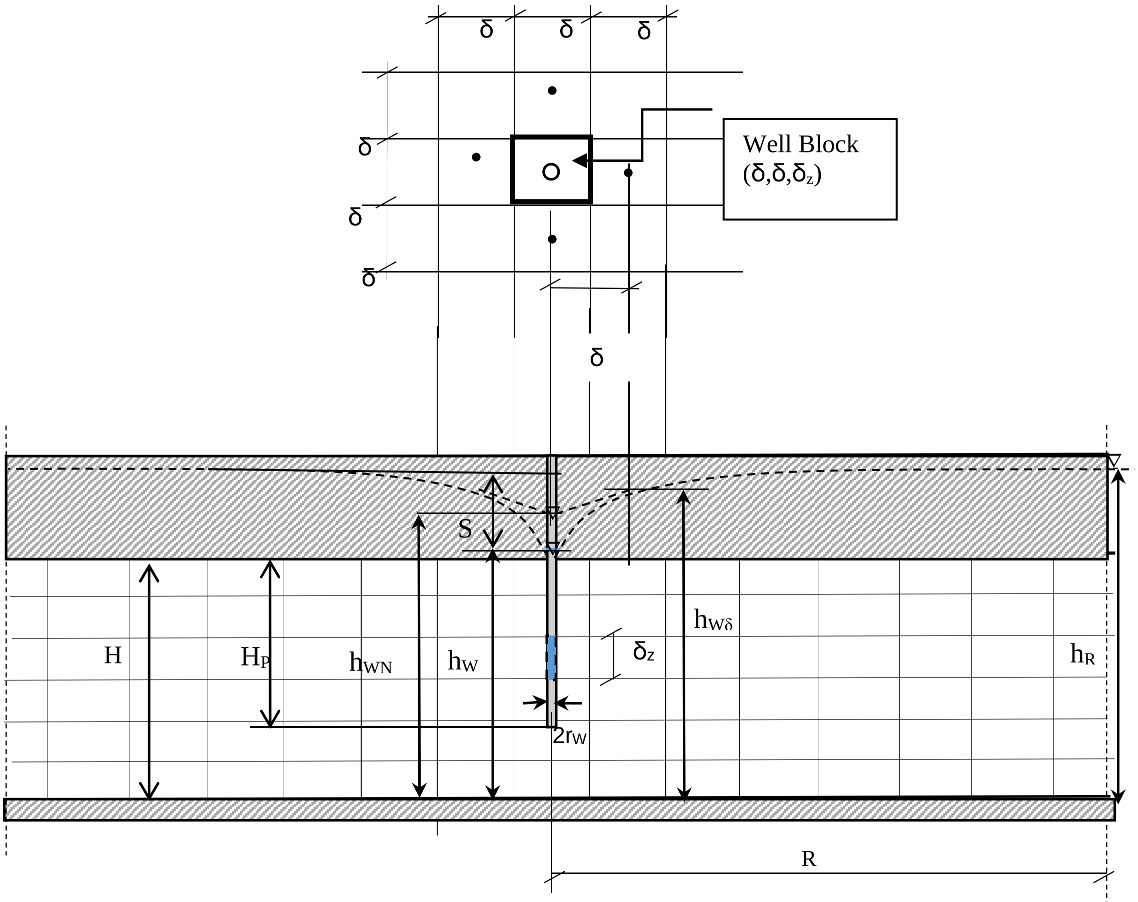

Fig. 1 Scheme of a partially penetrating well (ppW); notations and spatial discretization.

The numerical estimation of the discharge (Q_{WN}) calculated numerical using MODFLOW, will be compared with the discharge (Q_{WA}) calculated by means of an analytical solution using the Muscat formula (1973):

$$Q_{pW} = \frac{2\pi k_f \cdot H \cdot \Delta h_{RW}}{ln\frac{R}{r_W} + \left(\frac{H}{H_P} - 1\right) ln\frac{4H}{r_W} - \frac{T}{2H_P}\left(f\left(\frac{H_P}{H}\right)\right)} \qquad (1)$$

For fully penetration $H_P = H$ on obtain from (1) the well-known case of fully penetrating well.

The values of function f (H_P/H) used in (1) are given in Table 1.

Table 1. Function values for Muskat formula.

H_P/H	0.05	0.1	0.20	0.30	0.40	0.50	0.60	0.80	1.0
$f(H_P/H)$	8.0	6.5	5.2	4.2	3.6	3.0	2.4	1.4	0.0

Results for drawdown errors by given well discharge for different grid size and well radius are presented in Fig. 2.

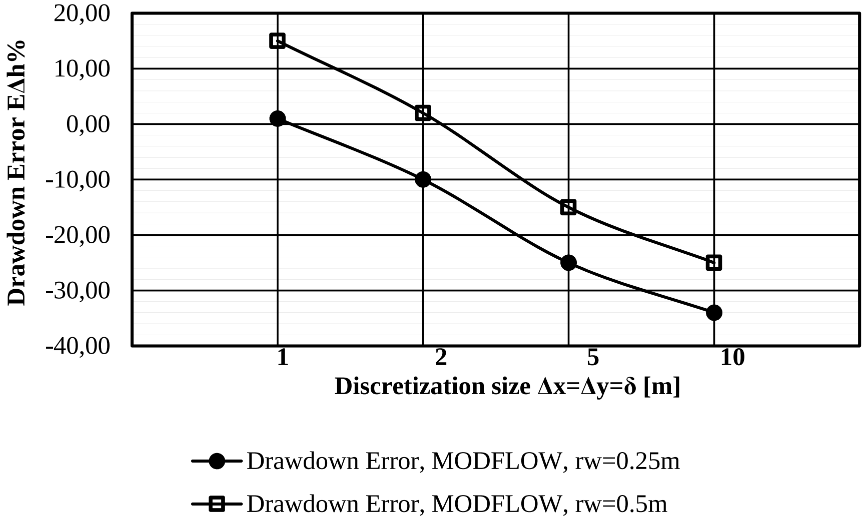

Fig. 2. . Relative error (Eh %) of the Hydraulic Head Drawdown in a fully penetrated Well by given Discharge for parameter: R=100m, H=20m, S=2m, k=0,000278m/s

In Fig.3 there are depicted the well discharge errors for ppW by given hydraulic head (drawdown) in well for different grid size and well radius.

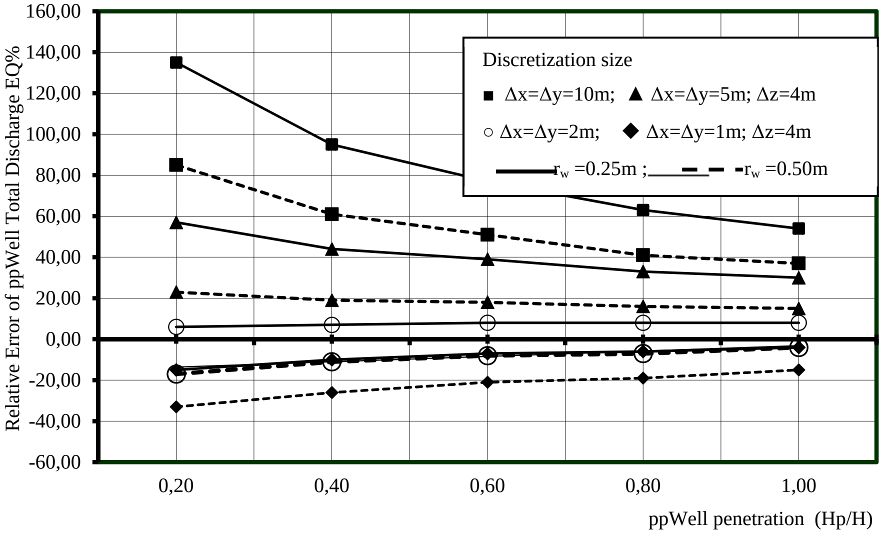

Fig. 3. Relative error (E_Q %) of ppW - Discharge by given hydraulic head, calculated analytical (1) and numerical (PMWIN/MODLOW software) for different discretization size and ppW radius for Parameter: R=100m, H=20m, h_R=24m; h_W=22m, S=2m, k=0.000278m/s

The relative errors of the hydraulic head drawdown in well *($E_{\Delta h}$%)* and of the well discharge E_Q% are defined as follow:

$$E_{\Delta h}\% = \frac{\Delta h_{RWN} - \Delta h_{RWA}}{\Delta h_{RWA}} 100 \quad ; \quad E_Q\% = \frac{Q_{WN} - Q_{WA}}{Q_{WA}} 100 \qquad (2)$$

where Δh_{RWN} and Δh_{RWA} denote the drawdown in well calculated numeric indexed as ($_N$) respectively analytic indexed as ($_A$).

One can see that spatial discretization size significantly influences the hydraulic head drawdown in wells for given well discharge (Fig. 2), error range about (-35% ÷ +15%). The error range of the well discharge by given hydraulic head is even higher (Figure 3) about (-35 % ÷ +135%).

These numerical tests shown that standard coarse grid reservoir modelling software in groundwater modelling (e.g. PMWIN/MODFLOW) don’t be applied for solving wellbore problems, without local corrections in well blocks. In the next paragraph the WI correction method is presented including also its extension.

3. Well-Block-Index based correction of well drawdown and its extension for well discharge correction

The correction of the numerical results obtained using coarse grid simulation for hydraulic head (or drawdown) in the well by given discharge, can be made using the well-known Well-Index method (WI) introduced in [3] and widely accepted as standard method [14]. The basic relations for a well element of the length $\Delta l_W = \delta_z$, situated in a well-block (δ, δ, δ_z), Fig.1 are:

$$h_{WA} - h_{WN} = \Delta h_{WAN} = \frac{\Delta Q_{WN}}{WI} \quad \text{where} \quad WI = \frac{2\pi k_f \delta_z}{ln\left(\dfrac{\delta}{r_W\, exp(\pi/2)}\right)} \qquad (3)$$

h_{WA}, h_{WN} are hydraulic heads in wellbore (index *A* analytic respectively *N* numeric), *WI* is the so called Well Index resulting from water rate balance for the well-block, Δh_{WAN} is the hydraulic head or drawdown (*S*) correction (head difference between analytic-numeric) in well-block corresponding to an well discharge of ΔQ_{WN}, r_w is the wellbore radius and k_f is the aquifer hydraulic conductivity. This relation allows only the correction of the numerical calculated hydraulic head *(h_{WN})* in the wellbore to obtain of the real hydraulic head *(h_W)* or drawdown (S) in the well. It is to mentioned that for a fully penetrating well in (3) ΔQ_{WN} will be substituted with the total well discharge Q_{WN} and δ_z with the total thickness H of the aquifer (Fig.1)

In Fig.4 there are represented comparatively the calculated drawdown in wellbore before correction and after WI correction for fully penetrating well. We can see a very god efficiency of the WI based correction: errors have decreased from 35% at values lower than 2%.

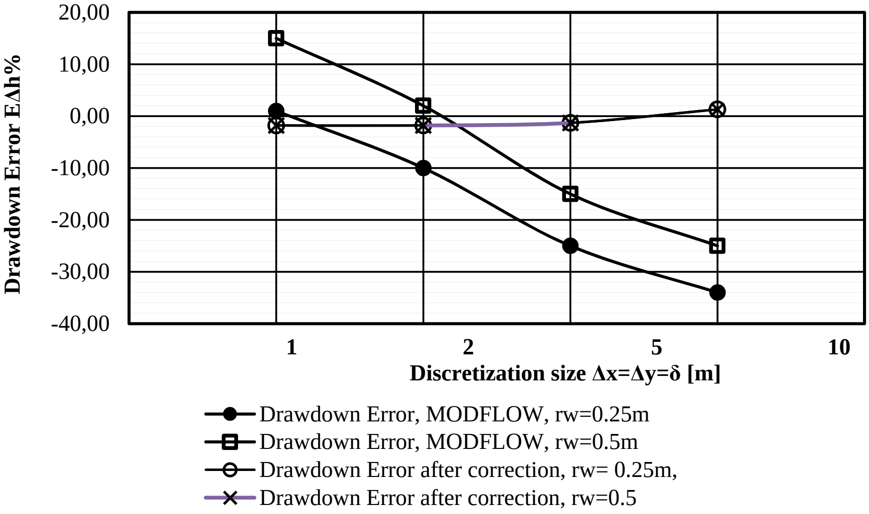

Fig. 4. Relative error ($E_{\Delta h}$ %) of the Hydraulic Head/Drawdown in a fully penetrated Well by given Discharge, before and after Well Index based correction.

The above presented WI correction only for hydraulic head is applicable. So it is logical to be named Well-Head-Index (WHI).

For technical application of wellbore modelling to solve groundwater or petroleum reservoir exploitation problems the correction in case 2 (e.g. correction of well discharge values obtained numerical by using coarse grid simulation for given hydraulic head in wellbore) is also very important because generally the well discharge distribution along the wellbore is beforehand unknown. Furthermore, on propose an extension of the available WHI method by introducing a new correction parameter which we name Well-Discharge-Index (WDI).

To define the proposed WDI-method we observe that for the ppW element well-block (Fig.1) or generally for a drainage element object is valid the following analytical relation between the total drawdown ($S=h_R-h_W$) and discharge ΔQ_{WA} in wellbore element:

$$S = h_R - h_W = \Delta Q_{WA} \cdot F_A(R, k_f, ...) \tag{4}$$

As example for mathematical expression of the function F_A we can see the case of ppW or fully well (formula (1)). On the other hand, between the imposed head in wellbore (h_{WNi}) and the numerical obtained discharge (ΔQ_{WN}) exist a similar relation bot with another function F_N, different from F_A, which depends on the discretization size δ used in coarse grid simulation:

$$h_R - h_{WNi} = \Delta Q_{WN} \cdot F_N(\delta, R, k_f, ...) \tag{5}$$

We note that ΔQ_{WN} (numerical obtained value) is different from ΔQ_{WA} (searched real discharge equal to analytical value) because it is depending from the discretization size δ. We observe that the same discharge ΔQ_{WN} can be expressed formally as analytical solution i.e. using the function form F_A instead of F_N when we take into account the supplementary head difference (Analytical-Numerical) in wellbore element (i.e. Δh_{WAN}) as additional drawdown:

$$h_R - h_{WNi} + \Delta h_{WAN} = \Delta Q_{WN} \cdot F_A(R, r_W, k_f, ...) \tag{6}$$

The supplementary head difference in wellbore Δh_{WAN} can be determine from (3-1)

For usually numerical modelling the imposed head in wellbore is equal to the given head in well i.e. $h_{WNi} = h_{WA}$. Replacing h_{WNi} in (6) with h_{WA} and Δh_{WAN} from (3-1) we obtain

$$h_R - h_{WA} + \frac{Q_{WN}}{WI} = Q_{WN} \cdot F_A(R, r_W, k_f, ...) \quad (7)$$

From the ratio of the two relationships (4) and (7) we obtain:

$$WDI = \frac{Q_{WA}}{Q_{WN}} = \frac{1}{1 + \frac{Q_{WN}}{S \cdot WI}} \quad (8)$$

That is the *proposed correction coefficient for discharge* which we name **Well Discharge Index (WDI).** We can see that WDI allows the correction of numerical calculated well discharge ΔQ_{WN} from coarse grid simulation (e.g. PMWIN/MODFLOW) obtaining the corrected discharge ΔQ_{WA}. By adding the wellbore elements, we can be obtaining the corrected total well discharge $Q_{WA.}$ The proposed method was tested for a ppW using the following parameter: R=100m, H=20m, S=2m, k=0.000278m/s, r_w=0.25m, r_w=0.5m, h_R=24m.

Some test results are presented in Fig. 5. On can see a very god efficiency of the proposed WDI correction: discharge errors have decreased from 135% at values of few %.

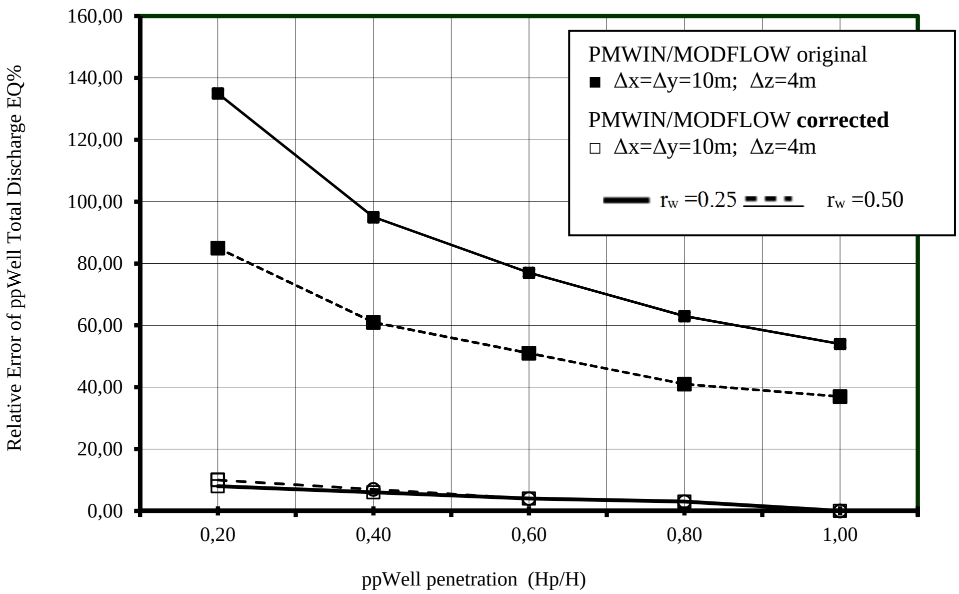

Fig. 5. Comparative errors of the total discharge (EQ %) for a ppW calculated before and after correction using the proposed method.

4. Conclusion

In the paper on analyse the reliability of coarse grid based simulation in neighbourhood of wellbore means several numerical experiments performed for partially penetrating wells and particularly fully penetrating wells. The performed numerical tests shown that the discretization size influence significantly the numerical results: for hydraulic head by given discharge errors up to 35% and much drastically for well discharge by given head errors more as 130%.

These numerical tests shown that standard coarse grid reservoir modelling software for groundwater modelling (e.g. PMWIN/MODFLOW) for solving wellbore problems require local corrections in well blocks. The available standard Well Index method allows the local correction in well blocks only for hydraulic head (or pressure) in the wellbore resulting for imposed global discharge/recharge rate of the well. But in several practical example such as partially penetrating wellbores or horizontal drainage the discharge cannot known in advance. In the paper an extension of the WI based method is proposed, which we name Well-Block-Discharge-Index (WDI), allows also the correction of the well discharge rate obtained numerically for imposed hydraulic head (imposed pressure) in the partially penetrated well.

The proposed method was tested for partially penetrated well. The numerical tests performed for various parameters confirms a very good efficiency of the proposed correction method: discharge errors have decreased from 135% at values of few %.

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