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Preface

This special issue is devoted to Computational Complex Analysis, i.e., to topics dealing with various computational aspects of complex analysis such as approximation and quadrature in the complex plane, numerical conformal mapping and its applications, complex variable methods for the numerical solution of partial differential equations, etc. This is a subject area that has grown rapidly in the last fifteen years, thanks especially to the influence of the late Peter Henrici. Additional important stimuli to the subject are furnished by the recurring Oberwolfach meetings on Constructive Methods of Complex Analysis organized by Dieter Gaier and Richard Varga as well as the meetings on Computational Methods and Function Theory organized primarily by Stephan Ruscheweyh.

The papers presented here offer a broad view of the current research in the area and include several articles on numerical conformal mapping and related matters. This is, we believe, appropriate because of the practical relevance of conformal transformations, and also because the very first special issue of this journal was on numerical conformal mapping [15]. The papers divide roughly into three parts: (i) papers that deal with conformal mapping and associated matters; (ii) papers on approximation and quadrature; (iii) papers that deal with other topics, including complex variable methods for differential equations.

The first paper, by Howell, describes a method (based on solving numerically the Schwarzian differential equation) for the conformal mapping of the upper half-plane onto circular arc polygons, i.e., onto Jordan domains bounded by straight lines and circular arcs. Also included are two alternative versions of the transformation, for mapping from the unit disc and (for dealing with elongated regions) from an infinite strip.

Paper 2, by Hough, Levesley and Chandler-Wilde, describes a numerical method for the conformal mapping of the interior or exterior of a closed piecewise analytic Jordan curve onto the interior or exterior of the unit circle. The method under consideration is a collocation method based on the much studied integral equation formulation of Symm [14]. The objective of the approach adopted here is to develop a robust and reasonably fast algorithm which, by providing for some treatment of corner singularities, also produces approximations of good accuracy.

Paper 3, by DeLillo and Elcrat, addresses the problem of choosing an appropriate canonical domain for overcoming the crowding difficulties that occur when, in a conformal mapping problem, the given physical domain Ω is elongated. The proposed method seeks to overcome these difficulties by taking as canonical domain the interior $\mathscr{E}$ of an ellipse which has the same aspect ratio as Ω. The method is based on representing the conformal map, from $\mathscr{E}$ onto Ω, as a truncated Chebyshev series whose coefficients are computed in $O(N \log N)$ operations,

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using techniques similar to those of the method (for conformal mapping from the unit disc onto Ω) proposed by Fornberg [7].

Paper 4, by Papamichael and Saff, contains a study of the local behaviour of the errors in the Bergman kernel method (with polynomial basis) for the numerical conformal mapping of a Jordan domain Ω onto a disc D. Let B be any (arbitrarily small) subdomain of Ω. Then, the results of the paper concern the behaviour of the local errors (with respect to $L^2(B)$) of the approximations to the kernel function and to the derivative of the mapping function. These results show that if ∂B contains a subarc of $\partial\Omega$, then the rates of convergence of the local errors are not substantially different than those of the corresponding global errors with respect to $L^2(\Omega)$. Although somewhat surprising, this fact is consistent with the second author's *principle of contamination* in best approximation [13].

Paper 5, by Floryan and Zemach, describes a method for computing the conformal map of the upper half-plane onto a semi-infinite region with a periodic boundary. The method is based on the use of a generalized Schwarz–Christoffel formulation (which was developed by the two authors in an earlier paper [6]), and can deal with periodic boundaries of arbitrary form (i.e., with boundaries made up of straight lines and curved segments meeting at corners). The paper gives full computational details, and contains several numerical examples illustrating the application of the method to various geometries, including highly deformed configurations.

Paper 6, by DeLillo and Pfaltzgraff, contains a study of various relations from the theory of harmonic measure and extremal length, for the purpose of providing geometric characterizations and estimates of the boundary distortions that occur under a conformal map from the unit disk onto a Jordan domain Ω. In particular, the authors use a theorem due to Pfluger [11] to give an estimate of the "crowding" that occurs when the domain Ω is elongated, and present examples illustrating the significance of their various estimates.

Paper 7, by Greenhow, presents a review of work done both by the author and by others (the paper contains some fifty references) in connection with the use of a complex variable method for the solution of various two-dimensional problems of free-surface hydrodynamics. Particular emphasis is given to problems involving rigid bodies in the flow. The basic tool of the method is the Cauchy theorem, the application of which leads to various boundary integral equation formulations. The review covers both the derivation of these formulations and the numerical schemes used for the solution of the associated boundary integral equations. This is (alas) the only paper in the issue that treats Computational Complex Analysis from the truly practical (i.e., the applications) point of view.

Paper 8, by Starke, is concerned with a minimization problem for rational functions in the complex plane. This minimization problem (which is known as the *rational Zolotarev problem in the complex plane*) arises in connection with the determination of optimal parameters for use with the ADI iterative method applied to nonsymmetric linear systems. An almost optimal solution to the problem can be given in terms of the *Fejér–Walsh points* which, in turn, are known in terms of a mapping function Ψ that maps conformally a circular annulus onto a doubly-connected region. In the present paper the author uses the generalization (to doubly-connected polygonal domains) of the Schwarz–Christoffel transformation, as was implemented by Däppen [3], to compute Ψ and hence the associated Fejér–Walsh points and ADI parameters.

Paper 9, by Wegert, presents iterative methods for the computation of functions $w = u + \mathrm{i}v$ which are holomorphic in the unit disc and satisfy nonlinear boundary conditions of the form

$F(t, u(t), v(t)) = 0$ on the unit circle. The methods are of Newton type and involve, in each step of the iteration, the solution of a discrete linear Riemann–Hilbert problem. It is shown that for sufficiently smooth data the resulting discrete Newton iteration converges locally quadratically to a (polynomial) solution of the corresponding discrete nonlinear Riemann–Hilbert problem. The results of this paper extend the area of applicability of earlier results due to Wegmann [17].

Paper 10, by Wegmann, considers a class of extremal problems for orientation-preserving harmonic mappings from the unit disc onto bounded convex regions. In particular, the author gives a simple and precise geometric characterization of the extremal mappings, and uses his results to obtain sharp estimates for the Fourier coefficients of boundary values of harmonic mappings. The work of this paper extends earlier work by the author [16] and is connected to the work of Duren and Schober [4,5].

Paper 11, by Gragg, on "*Positive definite Toeplitz matrices, the Arnoldi process for isometric operators, and Gaussian quadrature on the unit circle*", was first presented in 1981 but, so far, only a Russian translation of the paper was available. Although not widely known, the paper has already had great impact on subsequent work in the area. It is hoped that the publication of the original English version will now bring this important work to the attention of a much wider audience.

Paper 12, by Berrut, is concerned with the problem of determining the quadrature weights, associated with given quadrature points z_j, for approximating the integral of a function f along a given path in the complex plane. More specifically, the author considers the problem of obtaining a formula for optimal integration in H^2 (i.e., in the space of all functions which are analytic in the unit disc and square integrable on the unit circle), and derives a closed-form expression for the corresponding optimal weights. He also discusses various computational matters connected with the case where the points z_j are equally distributed on a circle, and compares numerically the accuracy of the optimal formula with that of some well-known integration formulae.

Paper 13, by Ammar, Gragg and Reichel, describes an algorithm for the evaluation of Szegő polynomials, i.e., of polynomials that are orthogonal with respect to an inner product defined on (part of) the unit circle. The proposed algorithm is analogous to that of Clenshaw [2] for the computation of linear combinations of orthogonal polynomials that satisfy a three-term recurrence relation. The algorithm proposed here presents certain computational advantages over an algorithm which was proposed for the same purpose by the three authors in an earlier paper [12].

Paper 14, by Jones, Thron, Njåstad and Waadeland, is concerned with a frequency analysis problem, i.e., the problem of determining unknown frequencies from an observed discrete set of N signal values. For the solution of this problem the authors consider the use of the Wiener–Levinson method formulated in terms of Szegő polynomials orthogonal on the unit circle with respect to a distribution function defined by the N signal values. The usefulness of this method is established by obtaining results related to a conjecture (made by the first and third authors and Saff in [10]) concerning the convergence of the zeros of the Szegő polynomials.

In paper 15, Brezinski and Matos introduce the notion of *least-squares orthogonal polynomials*; a class of polynomials which (despite their name) are not really orthogonal but biorthogonal in the sense of the definitions of [1,9]. The authors study the question of existence and uniqueness of such polynomials, describe a recursive scheme for their computation, and derive some of their properties. They also discuss possible applications in connection with Padé-type approximation and Gaussian quadrature methods.

In paper 16, Jagels and Reichel consider two procedures for computing recursion coefficients for Szegő polynomials. These procedures are the analogues (for inner products defined on the unit circle) of the well-known Chebyshev and Stieltjes algorithms for inner products defined on part of the real axis. The main result of the paper shows that the analogue of the Chebyshev procedure (i.e., the algorithm for the coefficients of the Szegő polynomials) is much better conditioned than the corresponding Chebyshev algorithm for inner products on the real axis.

Paper 17, by Baratchart and Zerner, is concerned with the problem of recovering an analytic function in a Hardy–Sobolev class in the disc from a denumerable set of boundary values on the unit circle. In particular, the authors provide convergence results and estimates for a linear triangular interpolation scheme, and highlight some of the important questions that remain unanswered.

In paper 18, Barnard, Pearce and Varga show how certain numerically motivated results on the zeros of partial sums of e^z, in classical one complex variable theory, can be used to answer a question (posed by Graham in [8]) in several complex variables. The paper also contains a brief, but very informative, review regarding the extensions of various results of classical one complex variable function theory (such as the Distortion, Growth and Koebe Covering theorems) to several complex variables.

Paper 19, by J.M. Borwein and P.B. Borwein, is concerned with a class of remarkable series for $1/\pi$ which were first studied by Ramanujan. The series arise by computing singular invariants for j and, in each particular example, the constants involved are of degree equal to the class number of the associated imaginary quadratic field. In the present paper the authors compute all class number three examples, the largest of which adds 37 additional digit accuracy per term.

Let $\mathscr{E}_r$ denote the ellipse with foci ± 1 and semiaxes $\frac{1}{2}(r \pm 1/r)$. Also, let $f_j(z) := w_j(z)F(z)$, $j = 1, 2, 3, 4$, where F is analytic in the interior of $\mathscr{E}_r$ and continuous in its closure, and $w_1(z) := 1$, $w_2(z) := (z+1)^{1/2}$, $w_3(z) := (z^2-1)^{1/2}$, $w_4 := (z-1)^{1/2}$. In paper 20, Mason and Elliott consider the problem of approximating functions of the form f_j, $j = 1, 2, 3, 4$, respectively by $w_j(z)p_n^{(j)}(z)$, where $p_n^{(j)}$ denotes the nth partial sum of the expansion of F in Chebyshev polynomials of the jth kind, $j = 1, 2, 3, 4$. In each of these four cases, the authors derive estimates for the norms of the corresponding projections and show that all resulting approximations are near-minimax.

In paper 21, Iserles introduces a formal computational algorithm for determining the solution g of the Schröder equation, i.e., the equation $f'(0)g(z) = g(f(z))$, where f is a function analytic in an open neighborhood of the origin 0 and such that $f(0) = 0$ and $f'(0) \neq 0$. The computational method is based on evaluating a generating function of certain recursively defined polynomials, and the techniques used are also applicable to the analysis of convergence acceleration of functional iteration in the complex plane.

Finally, paper 22, by McCoy, concerns the solution of the Dirichlet problem for the Helmholtz equation on a disc. The specific problem under consideration is that of determining a Chebyshev approximation, i.e., an approximate solution that minimizes the error relative to a particular norm. The approach adopted here involves placing the problem in a geometrical setting, by characterizing the best approximating function in terms of the properties of an error curve and, in particular, in terms of a *near-circularity* property of this curve. The resulting analysis leads to a constructive procedure for determining near-optimal approximations.

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Guest Editors

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