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Dissipative force on an external quark in heavy quark cloud

Shankhadeep Chakrabortty

Institute of Physics, Bhubaneswar 751 005, India

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ABSTRACT

Within the finite temperature $\mathcal{N}=4$ strongly coupled super-Yang–Mills, we compute the dissipative force on an external quark in the presence of evenly distributed heavy quark cloud. This is computed holographically by constructing the corresponding gravity dual. We study the behaviour of this force as a function of the cloud density. Along the way we also analyze the stability of the gravity dual for vector and tensor perturbations.

Collisions of heavy nuclei are believed to produce QGP, a strongly coupled thermal state of matter [1,2,5]. This motivated many researchers to compute QGP observables using gauge/gravity correspondence. Though the gauge theory in question is quite different from QCD, this correspondence gives us a systematic prescription to compute various quantities associated with a strongly coupled thermal system [3]. These include viscosity, entropy production, transport coefficients to name a few [4,8,6,15]. In this respect, an interesting quantity is the dissipative force experienced by an external heavy quark moving in a thermal plasma [7–9]. Within the framework of gauge/gravity duality, an external heavy quark is modeled by a fundamental string attached to the boundary of an AdS black hole. For $\mathcal{N}=4$ super-Yang–Mills, the end point of the string carry a fundamental $SU(N)$ charge. The string extends along the radial direction of the AdS–Schwarzschild metric. This external quark, with a mass proportional to the length of the string, loses its energy as the string trails back imparting a drag force.

Our aim in this Letter is to carry out a simple calculation of the drag force for thermal $\mathcal{N}=4$ super-Yang–Mills on R^3 in the following scenario. We consider an *uniformly* distributed heavy quark cloud in this hot plasma. We then ask: how does the drag force on an external quark change with the density of the quark cloud? Like other drag force calculations, we compute it holographically. First, we construct the gravity dual of the quark cloud. In the bulk, this represents a black hole in the presence of a string cloud. These strings, assumed to be non-interacting, are aligned along the radial direction of the bulk geometry and are distributed uniformly over R^3. We then study the thermodynamics of black hole and verify the thermodynamical stability. We also analyze it's gravitational stability under vector and tensor perturbation in a gauge invariant way. Subsequently, we introduce a probe string with an end on the boundary and calculate the dissipative force on the heavy quark following the usual approach. At this point, though, the relevance of this work in the light of recent quark–gluon-plasma experiment is not immediately obvious. Nonetheless, we would like to make the following comment. The dynamics of a heavy quark (say charm) passing through the plasma is usually described by considering it's interaction with the medium and the resulting energy loss is calculated. In such calculations, any possible effects of other heavy quark due to the back-reaction of the plasma are neglected. In the context of $\mathcal{N}=4$ SYM, our work can perhaps serve as an attempt to compute such back-reaction effects. Within the gauge/gravity correspondence, such effects can be modeled in terms of the deformation of the geometry due to finite density string cloud. This work shows that the back-reacted gravity background is explicitly computable. There are several works, starting with [7], where drag force on an external quark has been calculated by introducing D7-brane as a probe in D3 background. External quark on the D7-brane comes from the end point of an open string stretching between the D7 and the horizon of the D3-brane. The usual probe approximation here is justified because the free energy of the external quark goes as $\mathcal{O}(N_c)$ whereas the plasma, being in adjoint representation, contributes $\mathcal{O}(N_c^2)$ to the free energy. So, in this sense, in the large colour limit, with $N_c \to \infty$, external quark can be treated as a probe. However, when large number of external quarks are introduced, the background geometry may get modified as our work indicates. Before we go into our computations, here is a note of caution. We study the motion of an external quark in an uniformly dense external quark cloud. However, in general, this motion is expected to back-react and change the uniformity. Such effects will be neglected in this work and, therefore, it is within this approximation that our results should be interpreted.

E-mail address: sankha@iopb.res.in.

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1. Gravity dual for external quark cloud

We consider the $(n+1)$-dimensional gravitational action given by

$$\mathcal{S} = \frac{1}{16\pi G_{n+1}} \int dx^{n+1} \sqrt{-g}(R - 2\Lambda) + S_m, \tag{1}$$

where S_m represents the matter part of the action. We represent the matter part as

$$S_m = -\frac{1}{2} \sum_i \mathcal{T}_i \int d^2\xi \sqrt{-h} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}, \tag{2}$$

where we considered $g^{\mu\nu}$ and $h^{\alpha\beta}$ are the space–time metric and world-sheet metric respectively with μ, ν represents space–time directions and α, β stands for world sheet coordinates. S_m is a sum over all the string contributions with i'th string having a tension $\mathcal{T}_i$. The integration in (2) is over the two-dimensional string coordinates.

Varying this action with respect to the space–time metric leads to

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_{n+1} T_{\mu\nu}, \tag{3}$$

with

$$T^{\mu\nu} = -\sum_i \mathcal{T}_i \int d^2\xi \frac{1}{\sqrt{|g_{\mu\nu}|}} \sqrt{|h_{\alpha\beta}|} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \delta_i^{n+1}(x - X). \tag{4}$$

In the above, the delta function represents the source divergences due to the presence of the strings. In the following, we will consider the space–time metric of the form

$$ds^2 = g_{tt}(r)\, dt^2 + g_{rr}(r)\, dr^2 + r^2 \delta_{ab}\, dx^a\, dx^b, \tag{5}$$

where (a, b) run over $n-1$ space directions. We will further consider strings with uniform tensions T and use the static gauge $t = \xi^0$, $r = \xi^1$. The non vanishing components of $T^{\mu\nu}$, following from (4), are

$$T^{tt} = -\frac{a g^{tt}}{r^{n-1}}, \qquad T^{rr} = -\frac{a g^{rr}}{r^{n-1}}. \tag{6}$$

Here we have assumed that the strings are uniformly distributed over $n-1$ directions such that the density is[1]

$$a(x) = T \sum_i \delta_i^{(n-1)}(x - X_i), \quad \text{with } a > 0. \tag{7}$$

For negative a, $T_{\mu\nu}$ will cease to satisfy the weak and the dominant energy conditions.[2] We look for a solution of (3) in AdS space and parametrize the metric accordingly treating a as a constant,[3]

$$ds^2 = -V(r)\, dt^2 + \frac{dr^2}{V(r)} + r^2 h_{ij}\, dx^i\, dx^j. \tag{9}$$

Here h_{ij} is the metric on the $(n-1)$-dimensional boundary. As for the matter part we will focus on to the string cloud for which the non-zero $T_{\mu\nu}$ components are[4] given by

$$T^t_t = T^r_r = -\frac{a}{r^{n-1}}, \quad \text{with } a > 0. \tag{10}$$

The solution which satisfy the Einstein's equation can be easily constructed. It is given by[5]

$$V(r) = K + \frac{r^2}{l^2} - \frac{2m}{r^{n-2}} - \frac{2a}{(n-1) r^{n-3}}. \tag{11}$$

Here $K = 0, 1, -1$ depending on whether the $(n-1)$-dimensional boundary is flat, spherical or hyperbolic respectively, having curvature $(n-1)(n-2)K$ and volume V_{n-1}. In writing down $V(r)$ we have also parametrized cosmological constant as $\Lambda = -n(n-1)/(2l^2)$. With

[1] To define this properly, we need to think of an IR cutoff in $n-1$ directions.

[2] For earlier discussions on string cloud/fluid models see [10–12].

[3] Clearly a in (7) depends on x. However, in Eqs. (9), (10) and in (11), a is treated as constant. To do this, we have replaced $a(x)$ by an average density as

$$a = \frac{1}{V_{n-1}} \int a(x)\, d^{n-1}x = \frac{T}{V_{n-1}} \sum_{i=1}^{N} \int \delta_i^{(n-1)}(x - X_i)\, d^{n-1}x = \frac{T}{V_{n-1}} \sum_{i=1}^{N} 1 = \frac{TN}{V_{n-1}}. \tag{8}$$

Here, V_{n-1} is the volume in $(n-1)$-dimensional space after imposing an IR cut-off. Now we consider the limit V_{n-1} going to infinity along with the number of strings N, keeping N/V_{n-1} constant.

[4] It turns out that replacement of δ_{ij} by h_{ij} in (5) keep the components of the stress-tensor same.

[5] This is a slight generalization of the metric in [13].

Eq. (11), the metric (9) represents a black hole with singularity at $r = 0$ and the horizon is located at $V(r) = 0$. The horizon has a topology of flat, spherical or hyperbolic depending on the value of K. However, our interest in this work, lies in the $K = 0$ case. In this case of flat horizon, the integration constant m is related to the ADM (M) mass of the black hole as follows,

$$M = \frac{(n-1)V_{n-1}m}{8\pi G_{n+1}}. \tag{12}$$

The horizon radius, denoted by r_+, satisfies the following equation

$$\frac{r_+^2}{l^2} - \frac{2m}{r_+^{n-2}} - \frac{2a}{(n-1)r_+^{n-3}} = 0. \tag{13}$$

This allow us to write m in terms of horizon radius as

$$m = \frac{(n-1)r_+^n - 2al^2 r_+}{2(n-1)l^2}. \tag{14}$$

The temperature of the black hole is given by

$$T = \frac{\sqrt{g^{rr}}\partial_r\sqrt{g_{tt}}}{2\pi}\bigg|_{r=r_+} = \frac{n(n-1)r_+^{n+2} - 2al^2 r_+^3}{4\pi(n-1)l^2 r_+^{n+1}}. \tag{15}$$

Note that the zero mass black hole has a non-zero temperature and is given by

$$T_0 = \frac{a}{2\pi}\left(\frac{n-1}{2al^2}\right)^{\frac{n-2}{n-1}}. \tag{16}$$

The black hole temperature increases with the horizon size and for large r_+, it behaves as $T \sim r_+/l^2$. The entropy is defined as

$$S = \int T^{-1}\,dM, \tag{17}$$

leading to the entropy density[6]

$$s = \frac{r_+^{n-1}}{4G_{n+1}}. \tag{18}$$

Note that s is finite even for black hole with zero mass. The specific heat associated with the black hole is

$$C = \frac{\partial M}{\partial T} = \frac{V_{n-1}(n-1)r_+^{n-1}(n(n-1)r_+^n - 2al^2 r_+)}{4G_{n+1}(n(n-1)r_+^n + 2(n-2)al^2 r_+)}. \tag{19}$$

Now we have a detail look at the thermodynamic quantities just evaluated. First of all, if we restrict the temperature to be non-negative, the black hole can have minimum radius

$$r_+^{\min} = \left(\frac{2al^2}{n(n-1)}\right)^{\frac{1}{n-1}}. \tag{20}$$

It can easily be checked that if we focus on to the region $T \geqslant 0$, there is a single positive real solution of (13). We also note from (20) and (14) that the mass becomes negative at zero temperature

$$m^{\min} = -\frac{a}{n}r_+^{\min}. \tag{21}$$

This is somewhat similar to the AdS–Schwarzschild with negative curvature horizon [14]. We note that, for mass $m \geqslant m^{\min}$, the specific heat (19) continues to be positive and is continuous as a function of r_+. This suggests thermodynamical stability of the black hole. Finally, we write down the free energy of this black hole

$$\mathcal{F} = -\frac{(n-1)r_+^n + 2al^2(n-2)r_+}{16\pi(n-1)}. \tag{22}$$

Before we go on to analyze gravitational stability of the black hole, we would like to make the following comment. Quite naturally, one may wonder if this black hole has a higher dimensional origin. In particular, can this arise, in some near horizon limit, from some brane configuration in ten or eleven dimensions after compactifying on spheres (with the cloud smeared over the compact manifold)? We indeed tried to get this from some bound state configurations of D-branes and strings but have not succeeded yet.

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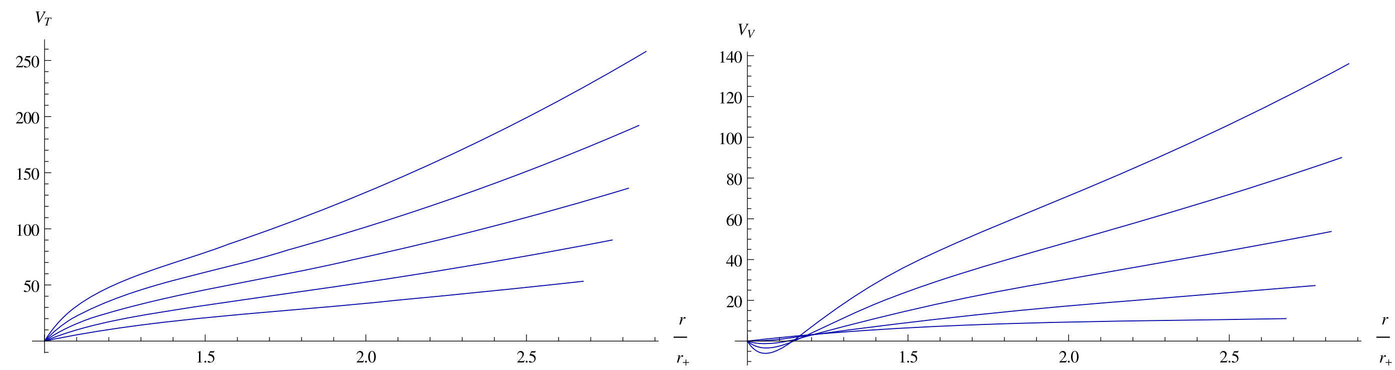
[6] Due to the nature of a dependent term in (11), our definition of ADM mass is perhaps ambiguous. However, with this definition, entropy of the black hole comes out to be one quarter of the horizon area, provided we assume that the equation $dS = M\,dT$ holds for this black hole.



(a) Plot-1 (b) Plot-2

Fig. 1. Plot-1 shows, for various dimensions, the effective potential V_T in tensor perturbation is positive beyond horizon radius. Plot-2 shows the effective potential V_V in vector perturbation is not always non-negative for $p > 3$. In both cases horizontal axis is normalized with respect to black hole horizon radius r_+.

2. Stability of the flat black hole

We now study the stability of the $K = 0$ black hole geometry using the gravitational perturbation in a gauge invariant way [16,17, 19–21,18]. We consider perturbation on a background space–time M^{2+p}

$$M^{2+p} = \mathcal{N}^2 \times \mathcal{K}^p, \tag{23}$$

where the space–time metric is,

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\delta_{ij}\,dx^i\,dx^j, \quad f(r) = \frac{r^2}{l^2} - \frac{2m}{r^{p-1}} - \frac{2a}{pr^{p-2}}. \tag{24}$$

We identify $\mathcal{N}^2$ as a two-dimensional space–time coordinatized by t and r, whereas $\mathcal{K}^p$ is a p-dimensional maximally symmetric space coordinatized by $x^i s$. Each perturbed tensor realized on $\mathcal{K}^p$ can be grouped into scalar, vector, and tensor components such that Einstein equations of motion respect the decomposition. Here we do stability analysis for tensor and vector perturbations. Scalar perturbation is somewhat more involved and will be reported else where.

2.1. Tensor perturbation

In the case of the tensor perturbation, the metric tensor and energy–momentum tensor become decomposed in scalar, vector, tensor part with respect to $\mathcal{K}^p$ in the following manner [17],

$$h_{ab} = 0, \qquad h_{ai} = 0, \qquad h_{ij} = 2r^2 H_T \mathbb{T}_{ij}, \qquad \delta T_{ab} = 0, \qquad \delta T^a_i = 0, \qquad \delta T^i_j = \tau_T \mathbb{T}^i_j, \tag{25}$$

$\mathbb{T}_{ij}$ is the tensor harmonic function defined on $\mathcal{K}^p$. It satisfies the following properties,

$$(\hat{\triangle} + k_T^2)\mathbb{T}_{ij} = 0, \qquad \mathbb{T}^i_i = 0, \qquad \hat{D}_j \mathbb{T}^j_i = 0. \tag{26}$$

Here we note that in $\mathcal{K}^p$ space, $\hat{\triangle}$ and $\hat{D}_j$ are realized as the Laplace–Beltrami self-adjoint operator and the covariant derivative respectively. For $K = 0$, k_T^2 can take non-negative real continuous values [19]. Gauge invariant quantities like H_T and τ_T are function of variables belong to $\mathcal{N}^2$ space–time [20].

Now substituting all the variations in the perturbed Einstein equation, we get the master equation of tensor perturbation [17].

$$\Box H_T + \frac{p}{r} Dr \cdot DH_T - \frac{k_T^2}{r^2} H_T = -\kappa^2 \tau_T. \tag{27}$$

We introduce a new variable Φ,

$$\Phi = r^{p/2} H_T, \tag{28}$$

and substitute it into the master equation. It takes following canonical form,

$$\Box\Phi - \frac{V_T \Phi}{f} = 0, \tag{29}$$

where V_T is defined as,

$$V_T = \frac{f}{r^2}\left[k_T^2 + \frac{prf'}{2} + \frac{p(p-2)f}{4}\right]. \tag{30}$$

According to (4) the energy–momentum tensor is constructed with the space–time vector $X^\mu(t,r)$ which does not contribute to the linear order of gauge invariant tensor perturbation. Therefore we set τ_T to be zero in (29). It is clear in plot-1 of Fig. 1 that for higher dimensions V_T is always positive beyond horizon. So the black hole geometry is stable under tensor perturbation.

2.2. Vector perturbation

In the case of vector perturbation, metric and energy–momentum tensor are decomposed in terms of vector harmonics V_i as well as vector harmonic tensor V_{ij} [17].

$$h_{ab} = 0, \qquad h_{ai} = r f_a V_i, \qquad h_{ij} = 2r^2 H_T V_{ij}, \qquad \delta T_{ab} = 0, \qquad \delta T^a_i = r\tau^a V_i, \qquad \delta T^i_j = \tau_T V^i_j. \tag{31}$$

The vector harmonics are defined as

$$(\hat{\Delta} + k_V^2) V_i = 0, \qquad \hat{D}_i V^i = 0. \tag{32}$$

From vector harmonics we can construct vector type harmonic tensor,

$$V_{ij} = -\frac{1}{2k_V}(\hat{D}_i V_j + \hat{D}_j V_i), \qquad (\hat{\Delta} + k_V^2) V_{ij} = 0, \qquad V^i_i = 0, \qquad \hat{D}_j V^j_i = \frac{k_V}{2} V_i. \tag{33}$$

The gauge invariant parameters for $K = 0$ are given by

$$F_a = f_a + r D_a\left(\frac{H_T}{k_V}\right), \tau_T, \tau^a. \tag{34}$$

Upon considering the perturbations of the Einstein equation and the conservation law of energy–momentum tensor, master equation arising from the gravitational perturbation with the source term takes the following form [19],

$$r^p D_a\left(\frac{1}{r^p} D^a \Omega\right) - \frac{k_V^2}{r^2}\Omega = -\frac{2\kappa^2}{k_V^2} r^p \epsilon^{ab} D_a(r\tau_b) \tag{35}$$

where,

$$r^{p-1} F^a = \epsilon^{ab} D_b \Omega + \frac{2\kappa^2}{k_V^2} r^{p+1}\tau^a. \tag{36}$$

Now introducing the change of variable

$$\Phi = r^{-p/2}\Omega \tag{37}$$

we recast the master equation in a canonical form, where the effective potential for vector perturbation comes out as [21],

$$V_V = \frac{f}{r^2}\left[k_V^2 + \frac{p(p+2)}{4} f - \frac{pr}{2} f'\right]. \tag{38}$$

The plot-2 in Fig. 2 implicates that beyond horizon, V_V is not always non-negative for $p > 3$. We follow S-deformation method [17] to construct modified effective potential

$$\tilde{V}_V = V_V + f\frac{dS}{dr} - S^2, \tag{39}$$

where S is an arbitrary function of r. If we choose $S = \frac{pf}{2r}$, we get the modified effective potential,

$$\tilde{V}_V = \frac{f k_V^2}{r^2} > 0. \tag{40}$$

Once again k_V^2 is the eigenvalue of a positive operator. So the above form of $\tilde{V}_V$ furnishes the sufficient condition for the stability of the black hole.

Having constructed this black hole geometry we compute the drag force on an external quark moving in external quark cloud

3. Dissipative force on an external quark moving in the heavy quark cloud

We now like to calculate the dissipative force experienced by the external heavy quark moving in the cloud of heavy quarks. Our aim is to study the force as a function of the cloud density. Calculational procedure to evaluate drag force on an external quark is by now standard. This can be found, for example, in [7–9]. We will follow the notations of [9]. The drag force on a very massive quark with fundamental $SU(N)$ charge at finite temperature is calculated holographically by studying the motion of a string whose end point is on the boundary. This end point represents the massive quark whose mass is proportional to the length of the string. We will consider here the gauge theory on R^3 coordinatized by x^1, x^2, x^3. This means, for the purpose of this computation, we only consider $K = 0$, $n = 4$ case of the black holes discussed previously.

Let us consider the motion of a string only in one direction, say x^1. In static gauge, $t = \xi^0$, $r = \xi^1$, the embedding of the world-sheet is given by the function $x^1(t, r)$. The induced action of the string in our case follows from a straightforward computation

$$S = -\frac{1}{2\pi\alpha'}\int dt\, dr\sqrt{1 + \frac{3r^4 - 2al^2 r - 6ml^2}{3l^4}(\partial_r x^1)^2 - \frac{3r^4}{3r^4 - 2al^2 r - 6ml^2}(\partial_t x^1)^2}, \tag{41}$$

where we have scaled x^1 by l.

The ansatz that describes the behaviour of the string with attached quark moving with constant speed v along x^1 is given by [9]

$$x^1(r,t) = vt + \xi(r), \tag{42}$$

for which (41) simplifies to

$$S = -\frac{1}{2\pi\alpha'}\int dt\, dr \sqrt{1 + \frac{3r^4 - 2al^2r - 6ml^2}{3l^4}(\partial_r\xi)^2 - \frac{3r^4}{3r^4 - 2al^2r - 6ml^2}v^2}. \tag{43}$$

The momentum conjugate to $\xi(r)$ is

$$\pi_\xi = -\frac{1}{2\pi\alpha'}\frac{(3r^4 - 2al^2r - 6ml^2)\partial_r\xi}{3l^4\sqrt{1 + \frac{3r^4-2al^2r-6ml^2}{3l^4}(\partial_r\xi)^2 - \frac{3r^4}{3r^4-2al^2r-6ml^2}v^2}}. \tag{44}$$

Equation of motion can be obtained by inverting this equation for $\partial_r\xi$. However, as in [9], to get a real ξ, the constant of motion π_ξ has to be set to

$$\pi_\xi = -\frac{1}{2\pi\alpha'}\frac{vr_v^2}{l^2} \tag{45}$$

where r_v is the real positive solution of the equation

$$3\big(1 - v^2\big)r_v^4 - 2al^2r_v - 6ml^2 = 0. \tag{46}$$

Though this equation can be solved explicitly, the solutions are not very illuminating. However, it is easy to check that there is only one real positive solution. Substituting this solution of r_v in (45), we get π_ξ. The dissipative force is then given by [9]

$$F = -\frac{1}{2\pi\alpha'}\frac{vr_v^2}{l^2}, \tag{47}$$

with r_v given by the positive real solution of (46). Now we wish to rewrite the expression of the dissipative force in terms of gauge theory parameters. Along this line, we solve (15) for r_+,

$$r_+ = \frac{l^2}{6}A(T,b), \tag{48}$$

where b is the scaled quark cloud density, $b = a/l^4$ and A is given by,

$$A(T,b) = \left[\left\{2\big(9b + 4\pi^3T^3\big) + 6\sqrt{b\big(9b + 8\pi^3T^3\big)}\right\}^{1/3} + 2\pi T\left\{1 + \frac{2^{2/3}\pi T}{\{(9b + 4\pi^3T^3) + 3\sqrt{b(9b + 8\pi^3T^3)}\}^{1/3}}\right\}\right]. \tag{49}$$

Substituting (48) and the following useful relation

$$\frac{l^4}{\alpha'^2} = g_{YM}^2N, \tag{50}$$

in the expression of the dissipative force (47), we get the modified form

$$F = -\frac{A^2}{72\pi}\sqrt{g_{YM}^2N}\,v\frac{r_v^2}{r_+^2}. \tag{51}$$

Here g_{YM} is the Yang–Mills (YM) gauge coupling and N is the order of the gauge group $SU(N)$. We are able to solve the ratio r_v^2/r_+^2 in a closed form by substituting (13) into (46). The relevant equation takes the following form,

$$\big(1 - v^2\big)\left(\frac{{r_v}^4}{{r_+}^4}\right) - \frac{144b}{(A(T,b))^3}\left(\left(\frac{r_v}{r_+}\right) - 1\right) - 1 = 0. \tag{52}$$

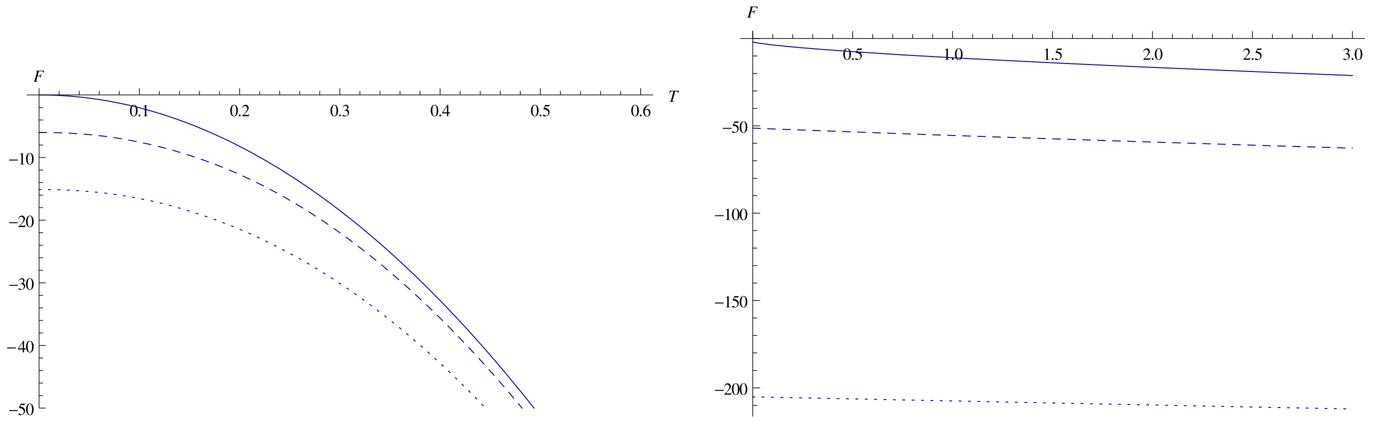
It turns out that the real positive solution of (52) is expressible in terms of $A(T,b)$ and b itself. Denoting the solution as $f(A,b)$ and plugging it back into (51) we achieve the form of dissipative force expressible in terms of gauge theory parameters

$$F = -\frac{A^2}{72\pi}\sqrt{g_{YM}^2N}\,vf(A,b)^2. \tag{53}$$

We note here that $f(A,b)$ is an explicitly computable function.

We would now study (53) for different values of heavy quark density and for fixed T. As for an example, it is interesting to check that if the temperature is fixed at the value T_0 as mentioned in (16) the dissipative force behaves as $F \sim -b^{2/3}$, where b is now the density of the quark cloud. Also for $T = 0$, $A(T,b)$ in (49) simplifies significantly resulting the dissipative force to vary as $F \sim -b^{2/3}$. For generic temperature and small b, it is possible to have a power series solution of (53) in b. However, for appreciable density, we find it more suitable to analyze F in terms of plots. In Fig. 2 plot-1 shows the behaviour of the drag force as a function of T for different b. For fixed T, we clearly see that the force becomes stronger with the quark density.[7]

[7] Note that the free energy (22) is perfectly well behaved at $T = 0$. In fact, it is $\mathcal{F} = -\frac{3ar_+}{32\pi}$. Substituting r_+, we find $\mathcal{F} \sim -b^{\frac{4}{3}}$. Furthermore computation of the drag force leads to $F \sim -b^{\frac{2}{3}}$ in this limit.



(a) Plot 1 (b) Plot 2

Fig. 2. Plot-1 shows the variation of F as a function of T for the values of quark density $b = 0$ (solid), 0.5 (dashed), 2 (dotted) respectively. Plot-2 shows the variation of F as a function of b for the values of $T = 0.1$ (solid), 0.5 (dashed), 1 (dotted) respectively. We see in both cases the larger the quark cloud density as well as temperature, the more is the dissipative force.

4. Conclusion

In this Letter, using the AdS/CFT duality, we have computed the dissipative force experienced by an external heavy quark with fundamental $SU(N)$ charge moving in the heavy quark cloud at finite temperature. In the dual theory, we have considered motion of a string (with one of it's end point at boundary) in a $(n+1)$-dimensional background with flat boundary pervaded with string cloud as the matter source.

The geometry implies existence of a black hole parametrized by it's mass m and the string cloud density a. The black hole turns out to be thermodynamically stable and it resembles AdS–Schwarzschild black hole with negative curvature horizon. We have been able to check the gravitational stability of the geometry for tensor and vector perturbations.

In the above scenario, we have computed the drag force exerted on the external quark. The most general form of the dissipative force turns out to be a complicated function of temperature of the boundary theory T and the re-scaled quark cloud density b. However for the temperature corresponding to massless black hole in the dual gravity theory, it behaves like $F \sim -b^{\frac{2}{3}}$. We have plotted it with respect to both T and b separately while keeping one of them constant at a time. Both plots exhibit an enhancement in the drag force in the presence of evenly distributed quark cloud.

There are few issues that we hope to look into. Firstly, what is the higher dimensional brane geometry whose near horizon geometry contains the black hole that we are considering? Secondly, is the gravitational background stable under scalar perturbation?

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