Cohesive zone laws for fatigue crack growth: Numerical field projection of the micromechanical damage process in an elasto-plastic medium

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Abstract

Cohesive zone failure models are widely used to simulate fatigue crack propagation under cyclic loading, but the model parameters are phenomenological and are not closely tied to the underlying micromechanics of the problem. In this paper, we will inversely extract the cohesive zone laws for fatigue crack growth in an elasto-plastic ductile solid using a field projection method (FPM), which projects the equivalent tractions and separations at the cohesive crack-tip from field information outside the process zone. In our small-scale yielding model, a single row of discrete voids is deployed directly ahead of a crack in an elasto-plastic medium subjected to cyclic mode I $K$-field loading. Damage accumulation under cyclic loading is captured by the growth of voids within the micro-voiding zone ahead of the crack, while the evolution of the cohesive zone law representing the micro-voiding zone is inversely extracted via the FPM. We show that the field-projected cohesive zone law captures the essential micromechanisms of fatigue crack growth in the ductile medium: from loading and unloading hysteresis caused by void growth and plastic hardening, to the softening damage locus associated with crack propagation via a void by void growth mechanism. The results demonstrate the effectiveness of the FPM in obtaining a micromechanics-based cohesive zone law in place of phenomenological models, which opens the way for a unified treatment of fatigue crack problems.

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1. Introduction

The fracture and fatigue behavior of a material depends on the micromechanical processes occurring at the process zone ahead of a crack tip. For example, crack propagation in a ductile metal occurs by void growth and coalescence in an extended process zone, while brittle fracture is associated with the mechanical rupture of inter and intra-granular bonds confined within a narrow process zone. A widely adopted approach to link the microscopic fracture process to the macroscopic failure behavior is to model the process zone with an equivalent cohesive zone law. This cohesive zone law constitutes the relationship between the cohesive-zone tractions in equilibrium with the stress fields of the surrounding body and the cohesive-zone separations compatible with the deformation fields of the surrounding body (Dugdale, 1960; Barenblatt, 1962). Ideally, all the micromechanical damage processes that occur within the process zone are embodied in the cohesive zone law. Since the area bounded by the cohesive zone law represents the energy release rate or cohesive energy of the material, crack propagation can then be simulated with the cohesive energy as the criterion for crack advance.

In the modeling of cohesive fracture, a general view is that cohesive strength (denoted by the peak traction in the cohesive zone law) and cohesive energy are two important material parameters to describe macroscopic fracture behavior. Often, a functional form of the cohesive zone law, such as the bilinear, trapezoidal, or exponential model, is assumed a priori, and the cohesive strength and cohesive energy are fitted to measurement data (Guo et al., 1999; Williams and Hadavinia, 2002; Valoroso and Fedele, 2010). However, it has been recognized that the functional form of the cohesive zone traction–separation relationship is sensitive to certain micromechanisms of fracture processes (Chandra et al., 2002; Li and Chandra, 2003; Murphy and Ivankovic, 2005; Olden et al., 2008; Hong et al., 2009; Chew et al., 2009). The functional form of the cohesive failure model is expected to play an even more critical role in predicting the fatigue life of a material, since the cohesive zone law for fatigue crack growth has to account for damage accumulation under cyclic loading. In one of the early models proposed, no distinction was made between the loading and unloading paths in the cohesive zone model, and the crack ceases to grow after a number of cycles due to plastic shake-down (de Andres et al., 1999). Therefore, a distinction needs to be made...
between the loading and unloading paths in the cohesive zone model so that subcritical crack growth becomes possible (Nguyen et al., 2001). Such loading and unloading hysteresis is intended to simulate dissipative mechanisms such as plasticity-induced void growth (Chew et al., 2006), crystallographic slip (Kanninen and Popelar, 1985), and frictional interactions between asperities (Gilbert et al., 1995). A vast number of cohesive zone models have since been proposed with different approaches of handling the loading and unloading hysteresis. Nguyen et al. (2001) assumed that the cohesive stiffness and the peak traction decrease proportionally with the unloading stiffness as the number of fatigue cycles increases. Roe and Siegmund (2001) introduced a history-dependent damage parameter which degrades the cohesive traction with cyclic loading. Maiti and Geubelle (2006) and Maiti et al. (2006) extended the cohesive modeling of fatigue crack growth to polymers, and developed an evolution law relating the cohesive stiffness, rate of crack opening displacement, and the number of cycles. In addition, they addressed the crack retardation and healing effects associated with crack closure. Ural et al. (2009) introduced a non-monotonically evolving damage variable that allows crack growth to be retarded and the material’s strength to be restored for self-healing materials. However, the physical connection between these damage evolution models and the actual dissipative mechanisms under fatigue crack growth is still not clearly understood.

Regardless of the assumed functional form of the cohesive traction–separation relationship and the damage evolution model for fatigue crack growth, all these models require calibration against some form of macroscopic measurement data such as the classical Paris power relation, i.e. crack advance rate per cycle (da/dN) vs. the range of applied stress intensity factor (ΔK). The cohesive zone model and associated damage evolution parameters are therefore regarded as phenomenological. Here, our motivation is to develop micromechanics-based cohesive zone laws for fatigue crack growth, where both the functional form of the cohesive zone law and the damage evolution model arise naturally from the underlying crack-growth mechanics in the process zone. However, direct in situ experimental measurement of the evolving cohesive traction–separation relationship under cyclic loading is highly nontrivial especially in materials where the fracture process zone is small (Que and Tin-Loi, 2002; Tan et al., 2005). One promising approach is the planar field projection method (FPM), which relates the cohesive tractions and separations of a crack-tip with a cohesive zone in a homogenous isotropic elastic solid and far-field measurement data (Hong and Kim, 2003). The FPM provides an inversion method to systematically uncover the shape of cohesive zone laws governed by different micromechanical fracture processes (Hong et al., 2009; Chew et al., 2009). The approach has been extended to cover nonlinear and elasto-plastic materials (Chew, 2013).

In this paper, the FPM will be used to inversely reconstruct the cohesive traction–separation relationship for a ductile material subjected to cyclic fatigue loading. In our model problem, a single row of cylindrical discrete voids is deployed directly ahead of a crack in an elasto-plastic medium, where damage accumulation in the form of void growth in the process zone can be quantitatively measured. The crack-tip cohesive zone law obtained by FPM will embody the full micromechanisms of void growth and coalescence during fatigue crack growth, which is in contrast to previously proposed phenomenological models. In Section 2, we describe the small-scale yielding model adopted for simulation of fatigue crack growth. We provide a brief overview of the FPM in Section 3 and discuss its implementation to extract cohesive zone laws for a ductile elasto-plastic material. The results for the cohesive zone law for fatigue crack growth are discussed in Section 4. Section 5 concludes this paper with a summary.

2. Problem formulation

Our small-scale yielding model consists of a homogenous material with a semi-infinite crack loaded remotely by the symmetric mode I K-field under plane strain conditions. Due to the overall geometrical symmetry, only one half of the model needs to be analyzed. As shown in Fig. 1a, the model comprises of three well-delineated zones: (i) a process zone which deposits a single row of cylindrical discrete voids ahead of the crack to account for microvoid growth and coalescence during monotonic or cyclic loading, (ii) an elasto-plastic background material to account for substantial plastic dissipation during ductile fracture, and (iii) a vanishingly thin elastic strip between the process zone and the elasto-plastic background material, within which the framework of the field projection method (FPM) remains valid. The properties of the elastic strip are denoted by Young’s modulus, E, and Poisson’s ratio ν, while the uniaxial tensile stress–strain behavior of (i) and (ii) are described by the true stress-logarithmic strain relation

\[ \varepsilon = \frac{\sigma}{E}, \quad \sigma < \sigma_0 \]

\[ \varepsilon = \frac{\sigma}{E} \left( \frac{\sigma_0}{\sigma} \right)^{1/N}, \quad \sigma \geq \sigma_0 \]  

(1)

Here, \( \sigma_0 \) is the initial yield stress in tension and N is the strain hardening exponent; \( N \to 0 \) corresponds to an elastic–ideally plastic solid. Generalization to multiaxial stress states assumes isotropic hardening and Mises yield condition.

Along the remote circular boundary of the small-scale yielding model, the elastic asymptotic (in-plane) mode I displacements are prescribed under plane strain conditions

\[ u_1(R, \theta) = K_1 \frac{1 + \nu}{E} \frac{R}{2\pi} (3 - 4\nu \cos \theta) \cos \frac{\theta}{2} \]

\[ u_2(R, \theta) = K_2 \frac{1 + \nu}{E} \frac{R}{2\pi} (3 - 4\nu \cos \theta) \sin \frac{\theta}{2} \]

where \( R^2 = x_1^2 + x_2^2 \) and \( \theta = \arctan(x_2/x_1) \). The mode I stress intensity factor \( K_1 \) is related to the J-integral by

\[ J = \frac{1 - \nu^2}{E} K_1^2 \]  

(3)

At various stages of the loading, the value of the J-integral is calculated on a number of contours around the crack using the domain integral method (Moran and Shih, 1987). The domain integral value was found to be in good agreement with the value given by (3) for the prescribed amplitude \( K_1 \). This consistency check assures that small-scale yielding conditions are satisfied.

Xia and Shih (1995) simplified the ductile fracture process by placing a single row of void-containing Gurson cells ahead of the crack-tip; this layer of computational cells was representative of the fracture process zone. In our study on fatigue crack growth, we represent the process zone with a single row of discrete cylindrical voids of initial radius \( R_0 \) and with void spacing \( D \) deployed ahead of the crack. Therefore, the process zone can be divided into an array of unit cells, each of dimensions \( D \) by \( D \) containing a single discrete cylindrical void. In our study, a total of 23 void-containing unit cells are deployed ahead of the crack (Fig. 1b). The initial void volume fraction of each unit cell is given by \( f_0 = \pi R_0^2 / D^2 \), with the macroscopic stress computed from

\[ \sigma_R = \frac{1}{V} \int_{V} \sigma_{ij} dV \]  

(4)

where \( \sigma_R \) represents the local Cauchy stress within a voided cell, and V is the cell volume in the current deformed configuration with unit thickness in the axial direction. The macroscopic mean stress is
given by $\Sigma_{\text{eq}} = \Sigma_{\text{eq}}/3$. The current void volume fraction is calculated from $f = V_f/V$ where $V_f$ represents the current deformed void volume obtained by numerical integration. The porosity evolution will be used to quantify the accumulation of damage caused by cyclic loads.

Our focus is on the fatigue crack growth of ductile materials, where a large plastic zone evolves under cyclic loading. As such, we adopt the material parameters $\sigma_0/E = 0.001$, $\nu = 0.3$, $N = 0.1$ with $f_0 = 0.001$ for all simulations. All computations are carried out under plane strain conditions using the general-purpose finite element program Abaqus Version 6.11. A close-up view of the finite element mesh is shown in Fig. 1c. The mesh contains 4-noded bilinear quadrilateral hybrid plane strain elements. The crack-tip has a small initial root radius $r_0$, with the distance between the crack-tip and the nearest void fixed at $D$. Previous studies have shown that the influence of the notch-tip radius $r_0$ is negligible for small $r_0/D$. Here, we fix $r_0/D = 0.18$ for all our calculations.

3. Field projection method

3.1. Overview

As introduced by Hong and Kim (2003), the cohesive zone representation of a fracture process in a single crack-tip can be
inversely determined from the elastic field surrounding the crack-tip process zone. An overview of the FPM is briefly summarized here. Designating the Greek subscript \( \alpha = 1 \) or \( 2 \) throughout this paper, the interaction \( J^\text{int} \left[ S, S^\text{in} \right] \) between the physical cohesive-crack field of interest \( S(\sigma_{xy}, u_{x,y}) \) and auxiliary proving fields \( S^\text{in}(\sigma_{xy}, u_{x,y}) \) for \( n=0,1,2,\ldots,N \), is defined as

\[
J^\text{int} \left[ S, S^\text{in} \right] = \int_\Gamma \left( \sigma_{xy}^\text{in} u_{y1} n_1 - \sigma_{xy} u_{y1} n_2 - \sigma_{xy}^\text{in} u_{y1} n_2 \right) \mathrm{d}s.
\]  

(5)

The distributions of the cohesive traction \( t_\delta(x_1) \) and the separation \( \delta_1(x_1) \) in the cohesive zone for \( 0 < x_1 < 2c \) can be represented as (Hong and Kim, 2003),

\[
t_\delta(x_1) = \left( \sqrt{\frac{N}{C}} \sum_{n=0}^{N} A_n U_n \left( \frac{x_1}{C} \right) - 1 \right) 
\]

(6)

\[
\delta_1(x_1) = \left( \sqrt{\frac{N}{C}} \sum_{n=0}^{N} B_n C \int_{x_1/C}^{2} \sqrt{z} U_n(\zeta - 1) \mathrm{d}\zeta \right) 
\]

(7)

where \( N \to \infty, U_0(\cdots) \) denotes the Chebyshev polynomials of the second kind, \( i \) the imaginary number \( \sqrt{-1}, \kappa = 3 - 4r \) for plane strain, \( \mu \) the shear modulus, \( \nu \) Poisson’s ratio, and \( A_n \) and \( B_n \) complex coefficients which are obtained from the surrounding elastic field as follows

\[
A_n = \frac{2}{\pi C} \left\{ J^\text{int} \left[ S, S^\text{in} \right] + i J^\text{int} \left[ S, S^\text{in} \right] \right\} 
\]

(8)

\[
B_n = \frac{2}{\pi C} \left\{ J^\text{int} \left[ S, S^\text{in} \right] - i J^\text{int} \left[ S, S^\text{in} \right] \right\} 
\]

(9)

The auxiliary elastic field \( S^\text{in}(\sigma_{xy}, u_{x,y}) \) with a subscript \( t_\delta \) or \( B_n \) stands for a set of stress and displacement fields corresponding to the complex elastic potential,

\[
\Phi_{t_\delta}^{(n)}(z) = \frac{i^2 \mu}{\kappa + 1} \sqrt{z - 1} U_n(z) 
\]

(10)

or

\[
\Phi_{B_n}^{(n)}(z) = -\frac{i^2 \mu}{\kappa + 1} \sqrt{z + 1} U_n(z) 
\]

(11)

respectively, with which the stress and the displacement gradient fields can be derived as

\[
\sigma_{22} - i \sigma_{12} = (z - \bar{z}) \Phi(z) + 2 \Phi(z) 
\]

\[
\frac{1}{2} (\sigma_{11} + \sigma_{22}) = \Phi(z) + \overline{\Phi(z)} 
\]

\[
u_{1,1} + i \nu_{2,1} = \frac{1}{2 \mu} \left\{ (z - \bar{z}) \Phi(z) + (\kappa - 1) \Phi(z) \right\} 
\]

\[
u_{1,2} + i \nu_{2,2} = -\frac{i}{2 \mu} \left\{ (z - \bar{z}) \Phi(z) - (\kappa - 1) \Phi(z) + 2 \Phi(z) \right\} 
\]

(12)

Here, \( z = x_1 + ix_2, \bar{z} = x_1 - ix_2 \) and \( \bar{z} = (z - c)/c \). Note that in our discrete void model, a total of 23 voids are placed directly ahead of the crack to represent the damage process zone. Accordingly, we assume a fixed process zone size of \( 2c = 23.5D \), which includes the cell containing the initial notch tip directly ahead of the crack (Fig. 1c).

3.2. Elastic strip approach

The above FPM was developed within the framework of continuum deformation kinematics of linear elastic (Hong and Kim, 2003). In ductile fracture, however, substantial plastic dissipation and nonlinear deformation is involved in the background material. We have recently extended the FPM to extract, from far-field measurements, the tractions along cohesive interfaces bound by nonlinear materials or elasto-plastic materials in the context of monotonic loading (Chew, 2013). Under repeated cyclic loading, however, the cohesive tractions and separations cannot simply be determined from the current stress state in the far-field, but will be a function of the entire loading history. For simplicity, we adopt a numerical procedure first developed in Chew et al. (2009), in which a vanishingly thin elastic strip is implemented between the process zone and the elasto-plastic background material. See shaded region in Fig. 1c. The interaction \( J^\text{int} \) in (5) is then taken along the fixed contour \( \Gamma \) within the elastic strip, from which the equivalent cohesive traction–separation relationship of the process zone is extracted. The elastic strip thickness \( e_b \) is fixed at 0.006D.

For such a model to be valid, the presence of the elastic strip must have negligible effects on both voiding within the process zone as well as plastic dissipation in the background. To confirm this, we examine the porosity \( f \) and mean stress \( \Sigma_{in} \) evolution ahead of the crack under monotonic loading for our small-scale yielding model with and without the elastic strip. See solid and dashed lines in Fig. 2a and b. Results show that the thin elastic strip surrounding the process zone has no observable effects on the porosity and mean stress evolution ahead of the crack. At initial applied loads of \( J(\sigma_D) = 0.7 \), voids adjacent to the crack-tip grow rapidly, with near-tip porosity reaching \( f = 0.1 \). At higher applied loads of \( J(\sigma_dD) = 1.5 \), the near-tip porosity exceeds \( f = 0.25 \), while stress relaxation associated with the zone of voiding shifts the peak stress location further ahead of the crack. This location of peak mean stress denotes the spatial extent of

**Fig. 2.** Distribution of (a) porosity \( f \) and (b) mean stress \( \Sigma_{in}/\sigma_0 \) ahead of the crack under monotonic loading, with and without the presence of the thin elastic strip surrounding the process zone.
damage which now spans over 6D, i.e. 6 active voids ahead of the crack, which implies that the crack growth mechanism under monotonic loading involves multiple void interactions ahead of the crack. All these trends are correctly captured by the elastic strip model qualitatively, as well as quantitatively. Fig. 3 compares the plastic strain contours at fixed load of \( J(\sigma_0 D) = 1.5 \), with and without the elastic strip. A close-up view of the plastic strains near the elastic strip is shown in the inset. Again, the presence of the elastic strip minimally disturbs both the plastic dissipation within the process zone, as well as in the background material, demonstrating the validity of this elastic strip approach.

The FPM is used to reconstruct the distributions of the cohesive tractions and separations ahead of the crack from the interaction \( J \)-integral contour \( \Gamma \) taken along the elastic strip surrounding the process zone. Fig. 4 shows the projected cohesive traction and separation distributions, as well as the cohesive zone law, under monotonic loading. As shown in Fig. 4a, the projected cohesive tractions rapidly converge after \( N = 4 \) terms in the Chebyshev polynomial representation. In fact, the cohesive tractions for \( N = 4 \) and \( N = 5 \) are almost the same, indicating that convergence in the series solution has occurred. At higher-order polynomial representation of \( N = 6 \), the cohesive tractions begin to diverge due to numerical errors associated with the inversion scheme. See Hong and Kim (2003) for detailed error analysis of the FPM. The projected cohesive separations in Fig. 4b show that convergence occurs after \( N = 5 \) terms; the cohesive separation distributions represented by \( N = 5 \) and \( N = 6 \) are very close. Herein, we use \( N = 5 \) terms in the Chebyshev series solution to represent the converged cohesive tractions and separations. The relationship between the cohesive traction and separation distributions then constitute the cohesive zone law in Fig. 4c. Previously, we have shown that cohesive zone laws for crazing in polymers have a convex traction–separation relationship (Hong et al., 2009; Chew et al., 2009). Here, our simulations also show that ductile metals, which undergo significant background plastic dissipation, also possess convex-shaped cohesive zone laws. The shape of this convex cohesive zone law is associated with the underlying micromechanisms of void growth and coalescence in the process zone, and is in contrast to previously proposed phenomenological models which have exponential, bilinear or trapezoidal traction–separation relationships.

4. Cohesive zone laws for fatigue crack growth

The self-similar damage mechanisms for crack-growth under monotonic loading allow the cohesive traction–separation relationship to be constructed from a single loading state. The cohesive zone laws under cyclic loading are far more complex. In addition to identifying the envelope of the cohesive traction–separation relationship, the loading and unloading hysteresis which is a function of the cyclic loading history will have to be correctly modeled. Inversely reconstructing the cohesive zone law for fatigue crack growth therefore requires high resolution extraction of the projected tractions and separations in the cohesive zone throughout the entire loading history up to the point of failure. Our focus will be on the cohesive traction–separation history at specific locations of \( x_1 D = 1–4 \); these locations are centered on the first four voids ahead of the crack, herein termed as void 1–4. We will subject the small-scale yielding model to cyclic \( K \)-field loading with applied energy change of \( \Delta J = (1 - v^2)\Delta K_I^2/E \) at each loading cycle, where \( \Delta K_I = K_{I,\text{max}} - K_{I,\text{min}} \) represents the range of applied stress intensity factor. In our simulations, we set \( K_{I,\text{min}} = 0 \) by fully unloading the model at the end of each cycle. Unlike constrained ductile problems where the maximum porosity can lie some distances ahead of the crack (Chew et al., 2007), our simulations show that the void closest to the current crack-tip consistently grows the largest. Once the porosity of this void reaches the critical void volume fraction of \( f_v = 0.25 \), a node release algorithm is applied along nodes bridging the current crack-tip and the critical void in the unit–cell to relax the residual forces linearly to zero over the cycle period. The crack then advances in discrete intervals of the void spacing \( D \).

4.1. Steady-state fatigue crack growth

Fig. 5a shows the porosity evolution \( f \) of voids 1–4 vs. number of cycles \( n \) subjected to cyclic loads of \( \Delta J/\sigma_0 D = 0.9 \). Observe that only \( n = 3 \) loading cycles are necessary for void 1 to reach the critical porosity of \( f_v = 0.25 \). At this instant, void 2 undergoes substantial growth, while void 4 remains benign. The growth rate of void 4 only increases when void 2 coalesces with the crack. These results imply that the damage extent under cyclic loads of \( \Delta J/\sigma_0 D = 0.9 \) is limited to the first 2 voids ahead of the crack. Compared to the...
growth reduces the stress-carrying capacity of the material ahead of the crack, void growth associated with cyclic plasticity build-up does not substantially lower the peak cohesive traction in Fig. 6. Despite intense void growth near the crack-tip, the peak cohesive traction at each loading cycle remains almost constant up to the point of void coalescence, since softening due to void growth is compensated by hardening of the elasto-plastic matrix. Each time a void coalesces with the crack-tip and the crack advances, the neighboring voids in the vicinity are subjected to higher stresses due to the configurational shift in the crack-tip stress-fields, and a step drop in the peak cohesive traction is experienced. The cohesive separations in Fig. 7 reflect the extent of damage in the process zone. The cohesive separations scale with the porosity of the individual voids, but undergo step jumps each time the crack advances. Interestingly for voids 2–4, initiation of void coalescence with the crack-tip consistently occurs at a fixed cohesive separation of $\delta_2/D = \sim 0.15$. For void 1, the critical separation to initiate void coalescence is slightly higher at $\delta_1/D = \sim 0.18$ presumably due to the blunter notch tip of $a_0 = 0.18D$, which is an order of magnitude larger than the initial void radius of $R_0 = 0.018D$ in the process zone.

The relationship between the cohesive tractions and separations in Figs. 6 and 7 constitutes the cohesive zone law for fatigue crack growth in Fig. 8. Previous approaches have phenomenologically modeled the loading and unloading hysteresis in the cohesive zone law to avoid plastic shake-down during fatigue crack growth (Nguyen et al., 2001; Roe and Siegmund, 2001; Maiti and Geubelle, 2006; Ural et al., 2009). Here, damage accumulation under cyclic loading arises naturally from the growth of voids in the process zone, which represents the actual micromechanisms during ductile fracture. Our field projected cohesive zone laws in Fig. 8 indeed reproduces the loading and unloading hysteresis previously assumed in phenomenological cohesive zone models. Instead of fitting the evolution law for damage variables such as the cohesive stiffness, rate of crack opening displacements, and number of cycles to macroscopic experimental data, the FPM allows the damage evolution law to be directly constructed from measurement data without any a priori assumptions of the crack growth mechanisms. Results in Fig. 8 demonstrate that the shape of the cohesive zone law reaches its steady-state profile for voids 3 and 4, which is in agreement with the self-similar plastic strain contours for $\Delta a = 3D$ and $\Delta a = 4D$ in Fig. 5b. For comparison purposes, we include the cohesive zone law for crack growth under monotonic loading as dashed curves in Fig. 8c and d. Observe that the cohesive zone law under monotonic loading forms an envelope over the damage locus for fatigue crack growth; the difference in the area under both cohesive zone laws quantitatively reflects the difference in driving force necessary for steady-state fatigue crack growth vs. crack growth under monotonic loading. However, the area enclosed by the cohesive zone law represents the intrinsic fracture toughness and will be different from the applied K-field loading ($J/\sigma_0D = 0.9$ and 1.5 for cyclic and monotonic loading respectively) which includes plastic dissipation in the background material. The damage accumulation mechanisms associated with void growth and plastic hardening under steady-state fatigue loading are reflected in the $\tau_2 - \delta_2$ profile up to $\delta_2/D = 0.15$ in Fig. 8c and d, beyond which the softening part of the cohesive zone law reflects the coalescence mechanism to advance the crack. In both regimes, the step drops in the traction–separation envelope are associated with crack advance due to coalescence of voids with the crack-tip. These step drops are particularly distinctive in cohesive zone laws for fatigue crack growth, since only a limited number of voids participate in the crack growth process. In contrast, multiple voids concurrently participate in the crack growth process under monotonic loading, leading to less well-defined drops in the stress-carrying capacity with void coalescence.

Fig. 4. Distribution of (a) cohesive traction $\tau_2/a_0$, (b) separation $\delta_2/D$, and (c) a cohesive zone law under monotonic loading, extracted by the field projection method with a polynomial of order N. The field-projected solution converges at $N = 5$. 

While our process zone comprises of discrete voids, the field-projected traction and separation distributions are constructed from polynomial functions which are continuous; these distributions therefore represent the equivalent homogenized response of the micro-voiding zone. Figs. 6 and 7 show the field-projected cohesive tractions and separations vs. number of cycles $n$ centered at voids 1–4 under cyclic loads of $J/\sigma_0D = 0.9$. Unlike crack growth under monotonic loading where softening due to void

multiple void interactions for crack growth under monotonic loading (damage extent of $\sim 6D$), crack growth under fatigue loading occurs by a void by void growth mechanism (Tvergaard and Hutchinson, 2002). Fig. 5b shows the associated plastic strain contours at four instances during crack growth. At $\Delta a = 3D$ and $\Delta a = 4D$, steady-state fatigue crack growth is reached as shown by the self-similar plastic strain contours. Observe that significant plastic strains occur at the wake of the current crack-tip, which is typical of fatigue crack growth. However, the spread of the plastic strain within the process zone during steady-state fatigue crack growth is limited to the first 2 voids ahead of the current crack-tip, which explains the limited damage extent ahead of the crack in Fig. 5a.
4.2. Near-threshold fatigue crack growth

Fig. 9a shows the porosity evolution \( f \) of voids 1–3 vs. number of cycles \( n \) subjected to cyclic loads of \( \Delta f/\sigma_D = 0.4 \). Compared to our results in Fig. 5a, a much larger number of cycles is required to initiate crack growth. While the void closest to the crack-tip grows steadily (void 1), the growth rate of void 2 saturates at \( f \approx 0.03 \). Once void 1 coalesces with the crack, void 2 resumes its growth but at a much faster rate until its porosity saturates near \( f_c \). At this stage (\( n = 220 \) cycles), void 3 still remains benign. After coalescence of void 2 with the crack, void 3 grows but saturates at porosity of \( f \approx 0.08 \), and the crack stops growing. The plastic strain contours before (\( n = 10 \)) and just after coalescence of void 1 (\( n = 20 \)) are shown in Fig. 9b. Observe that the spread of plastic dissipation in the process zone is confined to the nearest 1–2 voids closest to the tip, which is expected from the void by void growth mechanism in Fig. 9a. Significant background plastic dissipation builds-up after the growth and coalescence of voids 1 and 2, which could contribute to a shielding effect responsible for the growth saturation of void 3.

Fig. 10 shows the field-projected cohesive zone laws for voids 1–3 subjected to cyclic loads of \( \Delta f/\sigma_D = 0.4 \). We observe that a complete cohesive zone law can only be extracted for void 1, which exhibits a constant traction envelope during the regime of rapid void growth, undergoes initiation of void coalescence at \( \delta_2/D = 0.125 \), and achieves complete loss of cohesion at \( \delta_2/D = 0.25 \). Void 2 similarly undergoes void coalescence at \( \delta_2/D = 0.125 \) and subsequent softening, but never completely loses cohesive traction since the ligament between voids 2 and 3 remains intact. For void 3, the cohesive traction–separation relationship terminates at \( \delta_2/D = 0.1 \), and no further loading and unloading hysteresis is observed. At this point, the porosity of void 3 saturates, and the absence of further damage accumulation results in plastic shake-down.
Fig. 6. Evolution of cohesive tractions $t_{r}/\sigma_{0}$ vs. No. of cycles $n$ centered at voids 1–4 under cyclic loads of $\Delta J/(\sigma_{0}D) = 0.9$.

Fig. 7. Evolution of cohesive separations $s_{y}/D$ vs. No. of cycles $n$ centered at voids 1–4 under cyclic loads of $\Delta J/(\sigma_{0}D) = 0.9$. 
Fig. 8. Field-projected cohesive zone laws centered at voids 1–4 under cyclic loads of $\Delta J/(\sigma_0 D) = 0.9$. Cohesive zone law for crack growth under monotonic loading denoted by dashed curves in (c) and (d).

Fig. 9. (a) Evolution of porosity $f$ vs. No. of cycles $n$ for the first three voids ahead of the crack, under cyclic loads of $\Delta J/(\sigma_0 D) = 0.4$. Void coalescence occurs at the critical porosity $f_c = 0.25$. (b) Plastic strain contours before and just after the crack advances by coalescence of void 1 with the crack-tip.
5. Concluding remarks

We have previously developed an inverse method, termed the field projection method (FPM), to extract the cohesive zone laws for crack growth under monotonic loading from far-field measurement data (Hong and Kim, 2003; Hong et al., 2009; Chew et al., 2009). Here, we have used the FPM to determine the cohesive zone laws for fatigue crack growth in an elasto-plastic material, with void growth and coalescence as the mechanism for crack growth. Unlike fracture under monotonic loading, the cohesive zone laws have to account for damage accumulation caused by micro-voiding in the process zone under cyclic loading. Our simulations show that the micro-voiding mechanism for crack growth transitions from a multiple void interaction mechanism under monotonic loading, to a void by void growth mechanism under cyclic loading. Our field-projected cohesive zone law for fatigue crack growth displays loading and unloading hysteresis associated with concurrent microvoid growth and plastic hardening. In addition, step drops in the cohesive traction–separation envelope are observed, due to the void by void growth mechanism activated under cyclic loading. The field-projected cohesive zone law therefore embodies the underlying micromechanisms for fatigue crack growth.

The original FPM was developed within the framework of continuum deformation kinematics of linear elasticity. For fatigue crack growth in a ductile material, significant plastic dissipation occurs in the background material. In our model problem, we have introduced a vanishingly thin elastic strip surrounding the micro-voiding zone, which allows the FPM to reconstruct the equivalent cohesive zone law of the micro-voiding zone without including the contributions of background plasticity. Our simulations demonstrate that this elastic strip approximation has negligible effect on the modeling of fatigue crack growth, and the extracted cohesive zone law accurately captures the accumulation of damage during cyclic loading. We have recently developed a nonlinear field projection method (NFPM) as a more general inverse method capable of extracting the cohesive zone laws from the far-fields of elasto-plastic materials without the elastic strip approximation (Chew, 2013). The outcome of the present study will be useful for comparison purposes with future results obtained from the NFPM to quantify the accuracy of the latter in inversely identifying non-phenomenological cohesive zone models to be employed in fatigue problems.

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References