A New Particle Filter for Nonlinear Tracking Problems

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Abstract: A new particle filter is presented for nonlinear tracking problems. In practice, maneuvering target tracking systems are usually nonlinear and incompletely observed, and the main difficulty of maneuvering target tracking problem lies in the fact that the maneuverability at every step is of high uncertainties. Here a new smoothing particle filter algorithm is proposed, which combines the particle filter to tackle the nonlinear and non-Gaussian peculiarities of the problem, together with smoothing of the PDF of system modes and thus settles the estimate problem of the target maneuverability. The simulation comparison with the auxiliary particle filters shows that the approach has superiority and yields performance improvements in solving nonlinear tracking problems.

Keywords: particle filters; nonlinear/non-Gaussian model; maneuvering target tracking; smoothing technique

In practical target tracking environment, with the presence of uncertain target models and incomplete observations, nonlinear models in state equation and measurement relations as well as non-Gaussian noise assumptions are more suitable to high performance requirements and realistic applications. Traditionally, these problems are solved by using linearizing filters, mainly the extended Kalman filters (EKF)\(^1\), but these linearizing methods are not efficient enough in practice.

As computational power is increasing rapidly, the Monte Carlo method, or particle filters, for the general case of nonlinear and non-Gaussian systems has begun to draw a considerable degree of interest. There is now a substantial literature concerning with the simulation based filters in which the required probability density function (PDF) is represented by a scatter of particles that propagate and update through the state space\(^2\). The propagation and adaptation rules are chosen so that the combined weight of particles in a particular region will approximate the integral of the PDF. Some modified methods have also been proposed to improve the performance of the particle filters\(^3,5,6\).

The main advantage of the particle filters is to handle any functional nonlinearity and system of measurement noise with any distribution. And PDF of the state is based on all the available information. But the high uncertainty and incompleteness of the information in maneuvering target tracking problem will weaken this advantage. One main difficulty lies in the fact that the observation at a single step is always of high uncertainties and incomplete to some extent; and the utilization of the valid information in single step is not enough for effective estimates.

Kalsson and Bergman\(^6\) applied Auxiliary Particle Filters as smoothing filters to the maneuvering
target tracking problem. An auxiliary variable is used to smooth and resample the particles, which represent the PDF of the system states. In order to tackle the maneuverability of target, this paper proposes a new smoothing particle filters for tracking a maneuvering target, which draw valid information of system mode from successive observations, instead of the system state. Except for the PDF of system state, the PDF of the system modes is propagate and updated additionally. When the PDF of the system modes is smoothed with the observation of next time step, the maneuverability is well settled and better performance is achieved.

The merit of the method is approved in comparison with the auxiliary particle filters\(^\text{[5]}\) by the simulation results.

1 Particle Filters

A general problem consists of a nonlinear state equation and a non-linear measurement relation of the form

\[
x_{t+1} = f(x_t, v_t) \\
y_t = h(x_t, e_t)
\]

where \(x_t, x_{t+1}\) are states of system at time \(t\) and \(t+1\) respectively; \(y_t\) denotes measurement observed at time \(t\); the process noise \(v_t\) and measurement noise \(e_t\) are able to be non-Gaussian, but known.

The paper of Gordon et al.\(^\text{[3]}\) marks the onset of a rebirth for algorithms based on Monte Carlo simulation techniques for solving problems with both nonlinear and non-Gaussian characters in an optimal manner. The sequential Monte Carlo methods, or particle filters, provide an approximate Bayesian solution to discrete time recursive identification or filtering problems by updating an approximate description of the posterior filtering density.

The Monte Carlo approximation to a PDF \(p(x_t)\) at time \(t\) consists of a set of random nodes in the state space, \(\{s_{t,i}, i = 1, \ldots, N_P\}\), termed the ‘support’, and a set of associated weights \(\{q_{t,i}, i = 1, \ldots, N_P\}\), which sum to 1. The first subscript of \(s_{t,i}\) and \(q_{t,i}\) represents the time \(t\), the second subscript \(i\) of them represents the \(i\)th particle, and \(N_P\) denotes the number of the particles in the set. The supports and the weights together form a random measure \(\{s_{t,i}, q_{t,i}\} i = 1, \ldots, N_P\).

The objective is to choose a measure so that

\[
\sum_{i=1}^{N_p} g(s_{t,i}) q_{t,i} \approx \int g(x_t) p(x_t) \, dx_t
\]

for typical functions \(g(\cdot)\) of the state space. This is an approximation in the sense that the left-hand side converges (in probability) to the right-hand side as \(N_P \to \infty\).

With \(Y_t\) representing the measurement sequence through time \(t\), the non-Gaussian prediction density \(p(x_t \mid Y_{t-1})\) and filtering density \(p(x_t \mid Y_t)\) for the Bayesian interference are given by

\[
p(x_t \mid Y_{t-1}) = \int p(x_t \mid x_{t-1}) p(x_{t-1} \mid Y_{t-1}) \, dx_{t-1}
\]

\[
p(x_t \mid Y_t) \propto p(x_t \mid x_{t-1}) p(x_{t-1} \mid Y_{t-1})
\]

The basic particle filtering algorithm given by Gordon Salmond is as follows:

1. Initialization: Begin by simulating a sample \(\{s_{t,1}, i = 1, \ldots, N_P\}\) from \(p(x_{t-1})\). The algorithm starts from a random measure with equal weight on each of the \(N_P\) sample values.

2. Preliminaries (step \(t\)): Assume an equally weighted random measure \(\{s_{t,j}, j = 1, \ldots, N_P\}\) to approximate \(p(x_{t-1} \mid Y_{t-1})\).

3. Prediction: Estimate the density \(p(x_t \mid Y_{t-1})\), up to a normalizing constant \(K\), by the mixture

\[
p(x_t \mid Y_t) = K \sum_{i=1}^{N_p} p(x_t \mid s_{t-1,i}) p(y_t \mid x_{t-1,i})
\]

According to stratified sampling theory\(^\text{[5]}\), to calculate \(p(x_t \mid Y_t)\), take exactly one sample point from each of the \(N_P\) strata, by generating support points \(\tilde{s}_{t,i} = f(t-1) \{s_{t-1,1}, \ldots, s_{t-1,i}\}\) from the system model with corresponding weights

\[
q_{t,i} = \frac{p(y_t \mid \tilde{s}_{t,i})}{\sum_{j=1}^{N_P} p(y_t \mid \tilde{s}_{t,j})}, \quad i = 1, 2, \ldots, N_P
\]
then a new measure \( \left( \bar{s}_i, q_i, q_i \right)_{i=1, \ldots, N} \) is formed.

(4) Update. Resample form the random measure \( \left( \bar{s}_i, q_i \right)_{i=1, \ldots, N} \) to obtain an equally weighted random measure \( \left( s_{i}, N \bar{p} \right)_{i=1, \ldots, N} \).

2 Proposed Algorithm

This paper is interested in how to track a maneuvering target by utilizing the available information more efficiently, especially under nonlinear and non-Gaussian condition. There are two main difficulties: the first is the unknown target motion models, the second is the uncertainties and incompleteness of observation information. Here a new smoothing technique is introduced to particle filters to cope with the maneuverability as well as the nonlinear and non-Gaussian characters.

A general maneuvering target tracking problem consists of a nonlinear state equation and a nonlinear measurement relation of the form, and

\[
x_{t+1} = f(x_t, m_{t,j}, w_t)
\]

\[
y_t = h(x_t, m_{t,j}, v_t)
\]

where the process noise \( w_t \) and measurement noise \( v_t \) are zero mean, white noise and independent of past and current states and their PDF is assumed to be known; \( M_s = \{ m_j, j = 1, \ldots, N \} \) represents the set of all possible system modes, called mode space and \( N_s \) is the number of the modes in \( M_s \); \( m_{t,j} \) denotes that the \( j \)th mode of \( M_s \) at time \( t \). The mode transition is governed by a first homogeneous Markov chain \[7\],

\[
P_x \begin{pmatrix} m_{t+1,j} \mid m_{t,i} \end{pmatrix} = P_x \forall i, \ j \in \{1, \ldots, N_s\}
\]

where \( P_x \) is the Markov transition probability from mode \( i \) to mode \( j \).

For maneuvering target tracking, a key problem is to estimate the system mode, \( m_{t,j} \). When the particle filter is applied to estimate the system state, it is natural to combine the mode transition probabilities into the distribution of particle set which represents the PDF of the system state.

Let \( P \left( m_{t,j} \mid Y_t \right) \) and \( P \left( m_{t,j} \mid Y_{t+1} \right) \), termed mode probabilities, denote the probabilities when \( m_{t,j} \) is in effect at time \( t \) of the given measurements \( Y_t \) and \( Y_{t+1} \) for \( j = 1, \ldots, N_s \). So \( \left\{ P \left( m_{t,j} \mid Y_j \right), j = 1, 2, \ldots, N_s \right\} \) approximates the probability distribution function of system modes at time step \( t \) over mode space \( M_s \), conditioned on \( Y_t \).

At time \( t \), the particle set \( \left\{ x_{t,i} \right\}_{i=1}^{N} \) and its corresponding weights \( \left\{ q_i \right\}_{i=1}^{N} \) form the following approximations

\[
p \left( x_{t+1} \mid Y_t \right) = \sum_{i=1}^{N} p \left( x_{t+1} \mid x_t, i \right) q_{t,i} \quad (5)
\]

\[
p \left( x_{t+1} \mid Y_t \right) \propto \sum_{i=1}^{N} p \left( x_{t+1} \mid x_t, i \right) q_{t,i} \quad (6)
\]

Considering the probability of system mode \( P \left( m_{t,j} \mid Y_t \right) \), for every particle \( x_{t,j} \), there exist

\[
p \left( x_{t+1} \mid x_{t,j} \right) = \sum_{j=1}^{N_s} \sum_{i=1}^{N_s} p \left( x_{t+1} \mid x_{t,i}, m_{t,j} \right) P \left( m_{t,j} \mid Y_t \right) \quad (7)
\]

\[
p \left( x_{t+1} \mid Y_t \right) \propto \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} p \left( x_{t+1} \mid x_{t,i}, m_{t,j} \right) P \left( m_{t,j} \mid Y_t \right) q_{t,j} \quad (8)
\]

Considering the stratified sampling theory mentioned in Ref. \[5\], if \( p \left( x \right) = \sum_{i=1}^{N} \beta_i p_i \left( x \right) \), where \( \beta_i \) is a proportion parameter corresponding to \( p_i \left( x \right) \), that is \( p \left( x \right) \) consists \( l \) strata, a population quantity \( \int g \left( x \right) p \left( x \right) dx \) can be estimated efficiently by sampling a fixed number \( N_i \) for \( i = 1, \ldots, l \), from each of the strata, with \( N_1 + \ldots + N_l = N \), \( N \) is total number of the samples. The greatest efficiency is attained with the Neyman allocation \( N_i \propto \beta_i \sigma_i \) for \( i = 1, \ldots, l \), where \( \sigma_i^2 \) is the variance of \( g \left( x \right) \) in the \( i \)th stratum. In practice, the proportional allocation \( N_i \propto \beta_i \) for \( i = 1, \ldots, l \) is frequently used.

So in the proposed particle filter for maneuvering target tracking problem, the particles are predicted according to stratified sampling theory with PDF of the system modes, in order to maintain the particle numbers while target maneuverability is considered.

So according to stratified sampling theory, in the prediction step, sample a number \( N_{m_{t,j}} \) from

\[ \frac{1}{N_s} \sum_{j=1}^{N_s} \sum_{i=1}^{N_s} p \left( x_{t+1} \mid x_{t,i}, m_{t,j} \right) P \left( m_{t,j} \mid Y_t \right) q_{t,j} \]
the \( j \) th stratum according to \( m_{i,j} \), with \( N_{m_{i,j}} \) 
\[ \mathbb{P}\left(m_{t+1,j} \mid Y_{t}\right) = \sum_{j=1}^{N_{m_{i,j}}} \mathbb{P}\left(m_{t,j} \mid Y_{t}\right) \]

and \( \sum_{j=1}^{N_{m_{i,j}}} \mathbb{P}\left(m_{t,j} \mid Y_{t}\right) = N_{t} \) for \( j = 1, \ldots, N_{m_{i,j}} \) will achieve a great efficiency. In practice, predicting \( x_{t+1,i} \) by \( x_{t+1,i} = f(x_{t,i}, m_{t+1,j}, w_{t+1,i}) \), for \( i = 1, \ldots, N_{p} \), where \( m_{t+1,j} \) is a random sample drawn from system mode set \( M_{s} \) with distribution probabilities \( \left\{ \mathbb{P}\left(m_{t,j} \mid Y_{t}\right), j = 1, 2, \ldots, N_{s}\right\} \)

For every system mode \( m_{t,j} \in M_{s} \), assign the number of the particles predicted with system mode \( m_{t,j} \) to \( N_{m_{t,j}} \), then \( N_{m_{t,j}} = N_{p} \mathbb{P}\left(m_{t,j} \mid Y_{t}\right) \).

So Eq. (8) can be rewritten by using stratified sampling technique, where \( x_{t,i} \) is the \( k \)th particle of the \( N_{m_{t,j}} \) particle to pass the system mode \( m_{t,j} \).

Then the probability is obtained,
\[ \mathbb{P}\left(x_{t+1} \mid M_{s}, j = 1, 2, \ldots, N_{s}\right) = \sum_{j=1}^{N_{s}} \mathbb{P}\left(x_{t} \mid m_{t,j}, y_{t}\right) \mathbb{P}\left(m_{t+1} \mid Y_{t}\right) q_{t,i} \approx \]
\[ \mathbb{P}\left(x_{t+1} \mid M_{s}\right) \sum_{j=1}^{N_{s}} \mathbb{P}\left(x_{t} \mid m_{t+1,j}, y_{t}\right) q_{t,i} \]

where \( x_{t,i} \) is the \( k \)th particle of the \( N_{m_{t,j}} \) particle that \( x_{t+1} \) is in \( m_{t+1,j} \) and \( y_{t} \) is the expected mean \( x_{t+1,1} \),
\[ = \mathbb{E}\left( x_{t+1,1} \mid m_{t+1,j}, y_{t}\right) \], and marginalize over \( x_{t+1} \).

This is the posterior PDF of the system modes at time \( t \) conditioned on \( Y_{t+1} \). By using this probability to predict the particles of time step \( t+1 \) again, the good performance can be achieved.

In a conclusion, the algorithm proposed in this paper is as following:

1. Initialization: Begin by simulating a sample \( \left\{ x_{t,i}, i = 1, \ldots, N_{p} \right\} \) from \( \mathbb{P}\left(x_{t} \mid M_{s}\right) \) with equal weights, and set initial mode probabilities \( \mathbb{P}\left(m_{t,j} \mid Y_{t}\right) = \mathbb{P}\left(m_{t,j}\right), j = 1, 2, \ldots, N_{s}\).

2. First predicting step: Compute \( \tilde{x}_{t+1,i} = f(x_{t,i}, m_{t+1,j}, w_{t+1,i}) \) for \( i = 1, \ldots, N_{p} \) and \( j_{i} \in \left\{ 1, 2, \ldots, N_{s}\right\} \), where \( m_{t+1,j} \) is a sample drawn from the system mode set \( M_{s} \) with distribution probabilities \( \left\{ \mathbb{P}\left(m_{t,j} \mid Y_{t}\right), j = 1, 2, \ldots, N_{s}\right\} \).

3. Compute the posterior system mode probabilities conditioned on \( Y_{t} \) and predict again: the expected mean \( \tilde{x}_{t+1,i} \) is a sample drawn from system mode set \( M_{s} \) with distribution probabilities \( \left\{ \mathbb{P}\left(m_{t,j} \mid Y_{t}\right), j = 1, 2, \ldots, N_{s}\right\} \).

4. Update step: Compute the likelihood weights and normalize, \( q_{t+1,i} = \frac{1}{c'} \mathbb{P}\left(x_{t+1} \mid M_{s}, j = 1, \ldots, N_{s}\right) \), and \( c' \) is used to normalize the weights to sum to 1, \( i = 1, \ldots, N_{p} \). Then compute the prior probabilities of the system mode at time step \( t+1 \) conditioned on \( Y_{t+1} \):

\[ \mathbb{P}\left(m_{t+1,j} \mid Y_{t+1}\right) \approx \sum_{i=1}^{N_{p}} \mathbb{P}\left(m_{t+1,i} \mid M_{s}\right) \mathbb{P}\left(m_{t,i} \mid Y_{t+1}\right) q_{t+1,i} \]

and normalize them to sum to 1.

5. Perform resampling and roughening procedure of the random measure \( \left( x_{t+1,1}, i = 1, \ldots, N_{p}\right) \) and obtain equally weighted measure \( \left( x_{t+1,1}, j, i = 1, \ldots, N_{p}\right) \), so as to overcome the impoverishment of the particle filter.

6. Increase \( t \) and iterate to step (2).

3 Simulation Results

A simulation study is performed here by using
the maneuvering target model presented in Ref. [6]. The target model in form of discrete state space model is given by

$$X_{t+1} = A(\omega)X_t + [B_v]v_t$$

$$X_t = (x_t, \dot{x}_t)^T, x_t = (\xi, \dot{\xi}, \eta, \dot{\eta})$$

where $\xi$ and $\eta$ are the Cartesian position coordinates, $\dot{\xi}$ and $\dot{\eta}$ are the velocity components.

$$A(\omega) = \begin{bmatrix}
1 & \sin(\omega T) & 0 & 1 - \cos(\omega T) \\
0 & \cos(\omega T) & 0 & -\sin(\omega T) \\
\frac{1 - \cos(\omega T)}{\omega} & 0 & 1 & \sin(\omega T) \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}$$

$$B_v = \begin{bmatrix}
\frac{T^2}{2} \\
0 \\
T \\
\frac{T^2}{2} \\
0 \\
T \\
0 \\
0 \\
\end{bmatrix}, B_v = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
\end{bmatrix}$$

where $T$ is the sample time of the system.

The turn rate is assumed to be related to the velocity according to the following model $\omega = a_{typ}\sqrt{\xi^2 + \eta^2}$, where $a_{typ}$ is the typical maneuvering acceleration which is modeled as a set of three discrete values and chosen in simulation as $a_{typ} \in \{4, 5, 0, -4, 5\}$ to represent the different target system modes. The radar measurements are modeled as

$$y_t = h(X_t) + e_t = \left[\sqrt{\xi_t^2 + \eta_t^2} \arctan(\eta_t/\xi_t)\right] + e_t$$

where $e_t$ is the measurements noise. Its PDF is assumed to be known. For simplicity, Gaussian noise is chosen with angular and distance standard deviations of 0.5° and 20m respectively. The sampling period is chosen to be $T = 4s$.

The proposed algorithm of section 2 and Auxiliary Particle Filter algorithm are applied, with a sample size of $N = 1000$. Fig. 1 shows that the smoothing of the PDF of the system mode improves the performance of the filter.

In the simulation procedure, roughening technique has been applied to overcome the impoverishment of particle filters. Fig. 2 shows the true maneuvering targets tracking trajectories together with the estimates of the filters.

By using $N_{mc} = 100$ Monte Carlo simulations for every filter, the simulation results are given below in form of the position Root Mean Square Error (RMSE) for proposed algorithm, APF and measurements, the unit of the quantities is meter.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Results of 100 Monte Carlo Simulations</th>
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<tbody>
<tr>
<td>RMSE/m</td>
<td>Proposed</td>
</tr>
<tr>
<td>RMSE/m</td>
<td>10.2362</td>
</tr>
</tbody>
</table>

The RMSE is defined as follow

$$E_{RMSE} = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{L} \left[ (\xi_{t,i} - \xi_{t,meas})^2 + (\eta_{t,i} - \eta_{t,meas})^2 \right]$$

where $L = 30$ is the simulation path length and $\xi_{t,i}$ and $\eta_{t,i}$ are the filter position estimates at time $t$ in
Monte Carlo run $i$.

In Fig. 3, the RMSE is presented for each time step, according to the following equation, for different methods.

$$E_{\text{RMSE}}(t) = \sqrt{\frac{1}{N_{\text{mc}}} \sum_{i=1}^{N_{\text{mc}}} \left( \xi_{y,i} - \xi_{y,\text{true}} \right)^2 + \left( \eta_{y,i} - \eta_{y,\text{true}} \right)^2}$$

![Fig. 3 Comparison of RMSE](image)

From the simulation result it is known that the proposed algorithm achieves better performance in comparison with APF, especially when the target performs high maneuverability. The smoothing of the PDF of the system modes has advantages more than the smoothing of the PDF of the system state for highly maneuvering target tracking problems.

4 Conclusions

This paper describes a new smoothing particle filter for applications to the nonlinear tracking problems. Compared with the Auxiliary Particle Filters, the proposed method has been shown to be a very powerful tool to tackle the maneuverability of target. In simulation results, the RMSE of the proposed filter is improved.

References


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