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A mathematical model of the pressure of an extreme ideology on a society

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Abstract

Extreme behavior is produced by small groups, but affects a large amount of people. The fear, the usual strategy of these groups, influences the decisions of the whole population. In this paper we propose a dynamical mathematical model to study the ideological evolution of the population in a region where some groups want to get political goals through violence. After a classification of the subpopulations using data from votes obtained by political parties in general elections, model parameters have been estimated and future tendencies are studied.

Keywords: Epidemiological model; Extreme ideologies; Transmission dynamics; Simulations

1. Introduction

Fanatical behavior is produced by small groups but affects large groups of people. The fear, the usual strategy of these groups, influences the decisions of the whole population. The understanding of the transmission dynamics of such behavior increases the knowledge of the mechanism behind the evolution of cultural norms and values. Also, it can give us tools to prevent the appearance of these groups, to know their *a priori* evolution, and how to achieve their disappearance.

The development of this paper has as a starting point, the Reference [1], to our knowledge the only antecedent of a type-epidemiological continuous mathematical model where the spread of fanatical behavior is considered. In [1] the dynamics of the spread of extreme behavior is studied as a type of epidemiology contact process (recruitment) that may be under the influence of friends, mates, environment, fear, menace, terrorism, propaganda, force of the law, etc. Then a mathematical model is built and its equilibrium points, thresholds and bifurcations studied.

Following the continuous model proposed in [1], in [2] a discrete model is developed and similar conclusions are reached.

Another interesting reference is [3] where type-epidemiological mathematical models are proposed to understand how ideas and rumors spread in certain populations.

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In this paper, our objective is to obtain a type-epidemiological mathematical model that help us to understand the situation in the Basque Country [4]. The Basque Country is a northern Spanish region where there exists an armed Basque nationalist organization ETA (Basque for "Basque Homeland and Freedom") [5]. ETA was founded in 1959 and evolved from a group advocating traditional cultural ways to an armed group using violence to demand Basque independence. Its ideology is Marxist–Leninist. Now, ETA is proscribed as terrorist by the Spanish authorities, the European Union, the United States and the United Nations.

The democratic system in the Basque Country and in the rest of Spain is affected by the terrorist acts of ETA (murders, kidnapping, vandalism, etc.). Therefore terrorism is one of the most important topics for Spanish public opinion.

In the Basque Country the population is divided into people that:

- agree with ETA in the objective of independence and the use of violence to get it,
- agree with ETA only in the objective of independence, without the use of violence,
- completely disagree with ETA.

Also, ETA supporters are about the 10% of population in the Basque Country and they help ETA members when needed.

The paper is organized as follows. From electoral manifestos and using statistical techniques, in Section 2, a classification of different political parties respect to the political goal "independence" is done in order to determine the ideological landscape. In Section 3 an epidemiological-type mathematical model where the pressure of the terrorism affects the ideology of the others is proposed. The model developed in Section 3 is not appropriate for data obtained in Section 2 (same units), hence Section 4 is devoted to scale the model properly to be fitted with classification data obtained in Section 5. Simulations to predict the short-term ideological evolution of population in the Basque Country are presented in Section 6. Finally, in Section 7, conclusions are presented.

2. Classification of ideological groups

Let us consider as source data results of the general elections to the Spanish Parliament in the Basque Country since June 15th 1977 to March 14th 2004 [6]. Since 1977, 85 political parties nominated candidates to, at least, one general election in the Basque Country electoral district. General election data have been considered because, in Spain, experts consider that general elections give a more realistic political/ideological landscape than local elections [7].

Now, let us classify the parties with respect to their relation with the political objective "independence". To do this, a survey is prepared to be answered from the party's election manifestos. The survey consists of the following questions (or ideological characteristics):

(1) Nationalist (Yes/No),
(2) Religious (Yes/No),
(3) Defense violence (Yes/No),
(4) Interventionist (Yes/No),
(5) Ecologist (Yes/No),
(6) Independence (Yes/No),
(7) Ideology (right wing or center/left wing/nationalist).

It should be mentioned that data of some political parties were not available, but these parties obtained a very small number of votes (less than 1%). Furthermore, some questions could not be answered for some parties due to the lack of details, or ambiguity, in the manifesto.

A non-parametric bivariant analysis [8, Chap. 9] is carried out in order to determine the ideological characteristics (questions of the survey) related with the defense of the use of the violence to get the independence. These characteristics were "independence", "nationalism" and "ideology", with associated p-values less than 0.01. In the multiple correspondence analysis [9, Chap. 10] three different profiles can be seen (see Fig. 1), nationalist parties related with independence and the use of violence, right-wing and center parties against independence and, in the middle of these profiles, left-wing parties with a non homogeneous and/or ambiguous position respect to independence and the use of violence.

These three profiles lead us to do a non-hierarchical cluster analysis with three groups of parties, whose definition is determined by the following characteristics:

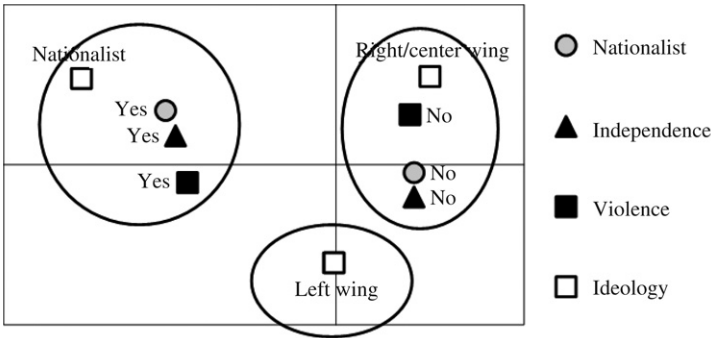

Fig. 1. Correspondence analysis shows three different profiles, nationalist parties, right-wing and center parties, and left-wing parties with ambiguous positions respect to independence and the use of violence.

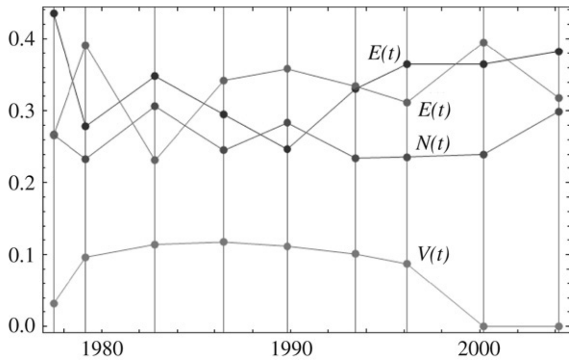

Fig. 2. This figure shows the percentage of votes of each subpopulation in each election. Vertical lines correspond to electoral days. Let us consider in our study data between 1979 and 1996, where the major part of time the Socialist Party (PSOE) was in the government, and the same policy against terrorism can be supposed.

- Group G_1: non-nationalist parties against independence and the use of the violence.
- Group G_2: nationalist parties agreeing with independence but disagreeing with the use of the violence.
- Group G_3: nationalist parties agreeing with independence and the use of the violence.

The determination of groups G_1, G_2 and G_3 allows us to classify 75 of the 85 parties. Therefore, the population of the Basque Country can be divided into four subpopulations

- $E(t)$, number of people who share the common ideological characteristics of parties in G_1 at time t,
- $N(t)$, number of people who share the common ideological characteristics of parties in G_2 at time t,
- $V(t)$, number of people who share the common ideological characteristics of parties in G_3 at time t, and
- $A(t)$, the rest of the people at time t. It includes people who do not share the ideological characteristics of groups G_1, G_2 and G_3 or people who abstain.

Fig. 2 shows the percentage of votes of each subpopulation in each election.

Two considerations should be mentioned here to understand the drastic changes in Fig. 2 at the beginning and at the end. The general elections in 1977 were the first celebrated after the dictator Franco died. Lots of parties presented candidates, the political situation was not clear and it is reflected in data. In 2000 and 2004, the "law of parties" forbade parties in group G_3 from nominating candidates, in fact, this law outlawed parties that do not condemn the violence. Notice that in 2000 and 2004 abstention increased, but this was because the votes for parties of group G_3 were considered void, increasing the subpopulation $A(t)$.

The above comments lead us to consider only election data since 1979 to 1996 where the major part of time the Socialist Party (PSOE) governed Spain and the same policy against terrorism can be supposed in order to fit the model we will develop in Section 3.

Table 1
Data corresponding to graphic in Fig. 2

Election date	$E(t)$	$N(t)$	$V(t)$	$A(t)$
Jun 15th, 1977	0.435392	0.266825	0.0316636	0.26612
Mar 1st, 1979	0.278466	0.233303	0.0967287	0.391502
Oct 28th, 1982	0.347978	0.306366	0.114331	0.231325
Jun 22nd, 1986	0.294959	0.245271	0.117587	0.342183
Dec 17th, 1989	0.246631	0.283516	0.111871	0.357982
Jun 6th, 1993	0.330461	0.234575	0.100969	0.333994
Mar 3rd, 1996	0.364871	0.236203	0.0872077	0.311718
Mar 12th, 2000	0.364974	0.239676	0	0.39535
Mar 14th, 2004	0.382756	0.299592	0	0.317652

Taking into account that, in Spain, only people older than 18 can vote and supposing that children and teenagers have the same ideology as their parents, let us assume that data in Table 1 is an ideological landscape of the whole population in Basque Country.

3. Type-epidemiological mathematical model

Consider the population of the Basque Country divided into four subpopulations determined in Section 2, that is, E, N, V and A.

Let us consider demographic data in the model. Then, let us assume that:

- The number of births $\Lambda(t)$ and the number of deaths $\Phi(t)$ are proportional to the number of individuals in each subpopulation.
- Terrorism does not increase substantially the number of deaths. In fact, ETA selects objectives and it is not usual they kill indiscriminately.
- The immigration $\Gamma(t)$ and emigration $\Sigma(t)$ in Basque Country are also included. It is considered that immigration and emigration only occurs in subpopulations E and A due to the terror pressure in proportions α_1 and α_2, respectively, to be determined.

In order to determine the rest of the transition terms, computation of the partial correlation coefficients have been used. This coefficient studies the linear relation between two variables under the influence of a third variable [8]. To carry out this study, let us take data of Table 1 corresponding to elections from March 1st 1979 to March 3rd 1996.

The partial correlation coefficient between subpopulations E and A under the influence of V is -0.8409 with a p-value of 0.009. It means that there is a linear inverse relation between E and A under V, that is, under V an increase of subpopulation E implies a decrease of subpopulation A and vice versa. Moreover, the linear correlation coefficient between E and A without the presence of V is not significant. Therefore the transition between E and A is modeled by the term

$$\beta_1 E(t) \frac{V(t)}{T(t)},$$

where $\beta_1 > 0$ indicates that the transition is due to the pressure of violent acts and $\beta_1 < 0$ indicates a strict law enforcement.

Analogously, a similar situation occurs between subpopulations A and V under the pressure of V. Then, the transition between subpopulations A and V is modeled by

$$k\beta_1 A(t) \frac{V(t)}{T(t)},$$

with $k > 0$.

On the other hand, the partial correlation coefficient between subpopulations N and A under the influence of E is -0.6292 with a p-value of 0.05 and there is a linear inverse relation between N and A under E. Also, the linear correlation coefficient between N and A without the presence of E is not significant. Therefore the transition between N and A is modeled by the term

$$\beta_2 N(t) \frac{E(t)}{T(t)}.$$

Then, the system of differential equations that models the evolution of ideologies in Basque Country under the pressure of the violence of ETA is given by

$$E'(t) = \Lambda(t) E(t) + \alpha_2 \Gamma(t) - \beta_1 E(t) \frac{V(t)}{T(t)} - \Phi(t) E(t) - \alpha_1 \Sigma(t), \tag{1}$$

$$N'(t) = \Lambda(t) N(t) - \beta_2 N(t) \frac{E(t)}{T(t)} - \Phi(t) N(t), \tag{2}$$

$$V'(t) = \Lambda(t) V(t) + k\beta_1 A(t) \frac{V(t)}{T(t)} - \Phi(t) V(t), \tag{3}$$

$$\begin{aligned} A'(t) = {} & \Lambda(t) A(t) + (1-\alpha_2) \Gamma(t) + \beta_1 E(t) \frac{V(t)}{T(t)} + \beta_2 N(t) \frac{E(t)}{T(t)} \\ & - k\beta_1 A(t) \frac{V(t)}{T(t)} - \Phi(t) A(t) - (1-\alpha_1) \Sigma(t), \end{aligned} \tag{4}$$

$$T(t) = E(t) + N(t) + V(t) + A(t). \tag{5}$$

The above system of differential equations can be represented by the diagram of Fig. 3.

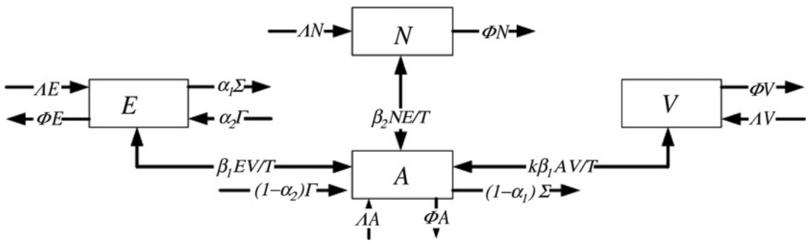

Fig. 3. Diagram corresponding to the model defined by the system of differential equations (1)–(5).

4. Scaling the model (1)–(5)

Data obtained in Section 2 is related to the percentages of population meanwhile model (1)–(5) is related to the number of individuals. It leads us to transform (by scaling) the model into the same units as data, because one of our objectives is to fit data with the model in next section.

Hence, following ideas developed in papers [10,11] about how to scale models where the population is varying in size, adding equations (1)–(4) one gets

$$T'(t) = [\Lambda(t) - \Phi(t)] T(t) + \Gamma(t) - \Sigma(t). \tag{6}$$

Dividing both members of (6) by $T(t)$ we have that

$$\frac{T'(t)}{T(t)} = \Lambda(t) - \Phi(t) + \frac{\Gamma(t) - \Sigma(t)}{T(t)}. \tag{7}$$

If we define the rates (depending on time)

$$e = \frac{E}{T}, \qquad n = \frac{N}{T}, \qquad v = \frac{V}{T}, \qquad a = \frac{A}{T}, \qquad \gamma = \frac{\Gamma}{T}, \qquad \sigma = \frac{\Sigma}{T}. \tag{8}$$

Eq. (7) can be transformed into

$$\frac{T'}{T} = \Lambda - \Phi + \gamma - \sigma. \tag{9}$$

On the other hand, let us compute the derivative of e, defined in (8). Using (9) we obtain that,

$$e' = \frac{E'T - ET'}{T^2} = \frac{E'}{T} - \frac{E}{T}\frac{T'}{T} = \frac{E'}{T} - e\left[\Lambda - \Phi + \gamma - \sigma\right]. \tag{10}$$

In an analogous way, we also have that,

$$n' = \frac{N'}{T} - n\left[\Lambda - \Phi + \gamma - \sigma\right],$$
$$v' = \frac{V'}{T} - v\left[\Lambda - \Phi + \gamma - \sigma\right],$$
$$a' = \frac{A'}{T} - a\left[\Lambda - \Phi + \gamma - \sigma\right].$$

Now, consider equation (1). If we divide it by T, we have

$$\frac{E'}{T} = \Lambda\frac{E}{T} + \alpha_2\frac{\Gamma}{T} - \beta_1\frac{E}{T}\frac{V}{T} - \Phi\frac{E}{T} - \alpha_1\frac{\Sigma}{T},$$

using (10) and substituting by the corresponding rates defined in (8) one gets

$$e' + e\left[\Lambda - \Phi + \gamma - \sigma\right] = \Lambda e + \alpha_2\gamma - \beta_1 ev - \Phi e - \alpha_1\sigma,$$

obtaining the scaled equation

$$e' = (\sigma - \gamma)\, e + \alpha_2\gamma - \beta_1 ev - \alpha_1\sigma. \tag{11}$$

The remainding equations can be scaled in the same way to obtain

$$n' = (\sigma - \gamma)\, n - \beta_2 ne, \tag{12}$$
$$v' = (\sigma - \gamma)\, v + k\beta_1 av, \tag{13}$$
$$a' = (\sigma - \gamma)\, a + (1 - \alpha_2)\,\gamma + \beta_1 ev + \beta_2 ne - k\beta_1 av - (1 - \alpha_1)\,\sigma. \tag{14}$$

Notice that the scaled system of differential equations (11)–(14) is also a non-autonomous system because the immigration (γ) and emigration (σ) rates depend on time.

5. Model fitting

Taking data in Table 1 corresponding to elections from March 1st 1979 to March 3rd 1996, let us fit data with the scaled model (11)–(14).

Moreover demographic data from [12], in particular annual population, immigration and emigration data in the interval 1979 to 1996 are considered. Hence, in order to compute the immigration and emigration rate functions $\gamma(t)$ and $\sigma(t)$, we divide each immigration and emigration datum by the corresponding population datum. Then, we use linear interpolation to construct both functions, γ and σ.

As initial condition of the model (11)–(14), it is considered

$$\begin{aligned} E(t_0) &= 0.278466, & N(t_0) &= 0.233303, \\ V(t_0) &= 0.0967287, & A(t_0) &= 0.391502, \end{aligned} \tag{15}$$

where t_0 corresponds to March 1st 1979 (see Table 1). In order to compute the best fitting, we carried out computations with *Mathematica* [13] and we implemented the function

$$\mathbb{F} : \mathbb{R}^5 \longrightarrow \mathbb{R}$$
$$(\beta_1, \beta_2, k, \alpha_1, \alpha_2) \longrightarrow \mathbb{F}(\beta_1, \beta_2, k, \alpha_1, \alpha_2)$$

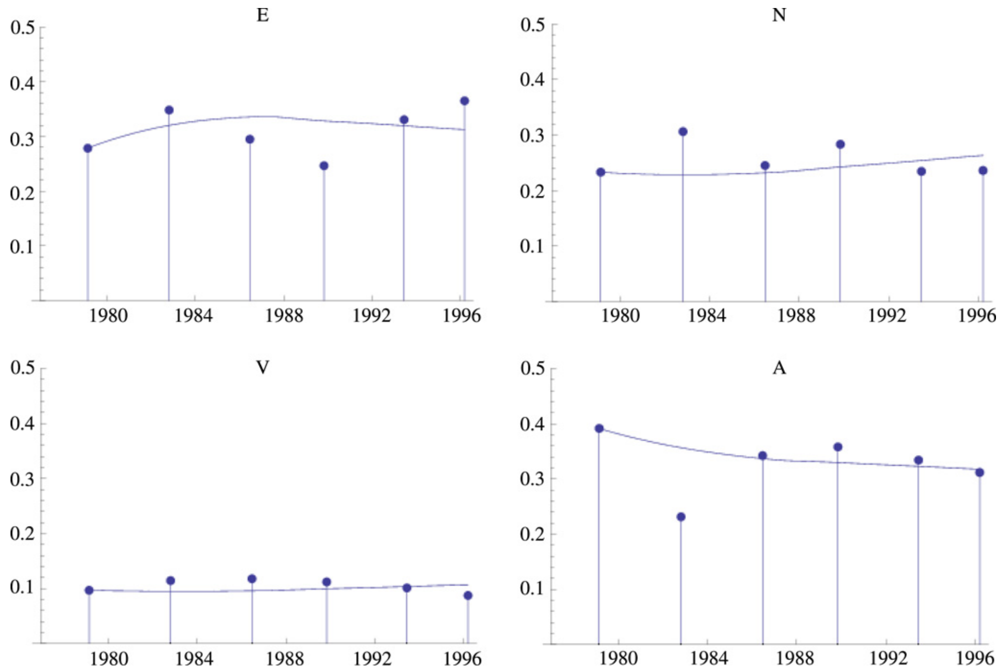

Fig. 4. Model fitting since March 1st 1979 to March 3rd 1996 for each subpopulation, E, N, V and A. Points are data of election days (Table 1) and the continuous lines the solution of the system of differential equations.

which variables are β_1, β_2, k, α_1 and α_2 such that:

(1) Solve numerically (*NDSolve[]*) the system of differential equations (11)–(14) with initial values (15),
(2) For $t =$ Oct 28th 1982, Jun 22nd 1986, Dec 17th 1989, Jun 6th 1993 and Mar 3rd 1996, corresponding to election days, evaluate the computed numerical solution for each subpopulation $E(t)$, $N(t)$, $V(t)$, $A(t)$.
(3) Compute the mean square error between the values obtained in Step 2 and the electoral data from Oct 28th 1982 to Mar 3rd 1996, (see Table 1).

Function $\mathbb{F}$ takes values in $\mathbb{R}^5$ (β_1, β_2, k, α_1 and α_2) and returns a positive real number. Hence, we can try to minimize this function using the Nelder–Mead algorithm [14,15], that does not need the computation of any derivative or gradient, which is impossible to know in this case.

In order to find a global minimum the feasible chosen domain is

$$D = [-1, 1] \times [-1, 1] \times [-1, 1] \times [0, 1] \times [0, 1] \subset \mathbb{R}^5,$$

and it is divided in 1125 disjoint subdomains of size $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ where, in each one, the Nelder–Mead algorithm is applied. We stored all the minima obtained and, among them, the values of β_1, β_2, k, α_1 and α_2, with restrictions $0 \leq \alpha_1, \alpha_2 \leq 1$ and $k > 0$, that minimize the function $\mathbb{F}$ are

$$\begin{aligned} \beta_1 &= 0.0606615, \\ \beta_2 &= -0.0339111, \\ k &= 0.460359, \\ \alpha_1 &= 0.869563, \\ \alpha_2 &= 0.997438, \end{aligned} \tag{16}$$

and the value of the function in the global minimum, that is, the mean square error, is 0.19756. The values of α_1 and α_2 indicate that immigration and emigration are concentrated in E subpopulation.

The graphical representation of the model fitting can be seen in Fig. 4.

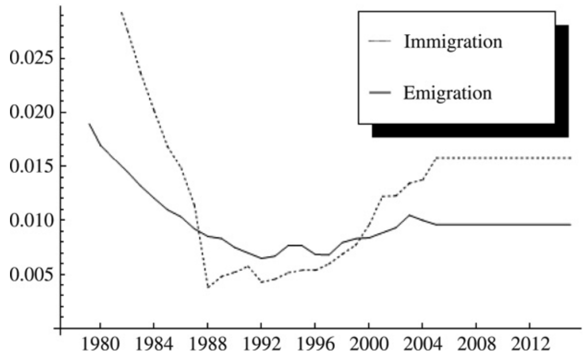

Fig. 5. Graphic of immigration and emigration rates from 1979 until 2015. Notice that from 2005 these rates are constant, equal to the ones in 2005.

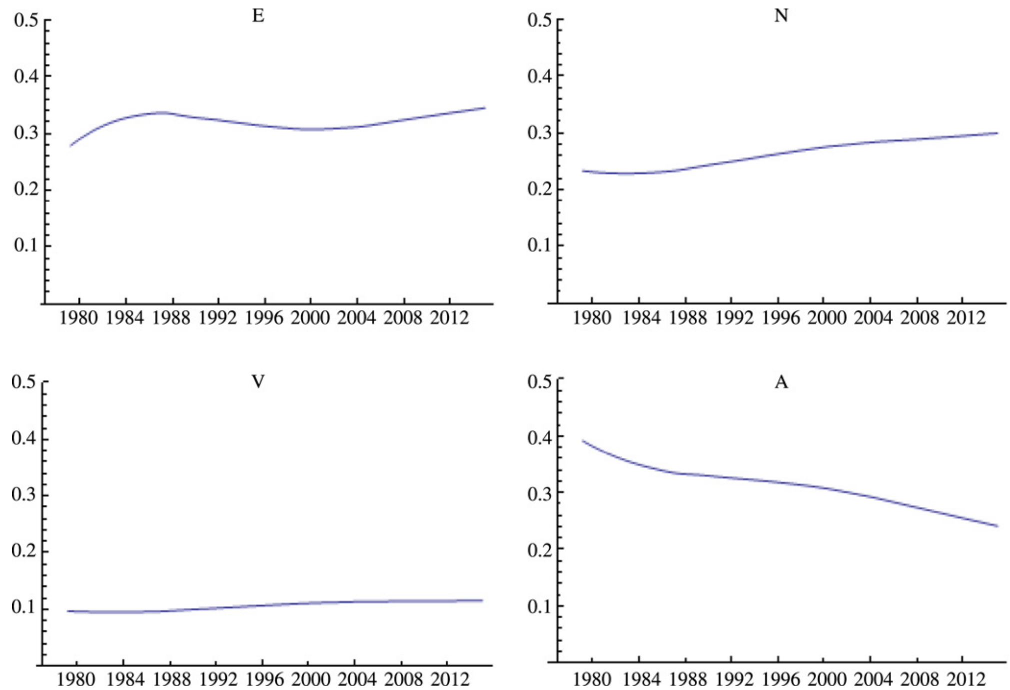

Fig. 6. Simulation from 1979 to 2015 with parameters of (16). There is a slight increasing of E, N and V at abstentionism expense.

6. Simulations

In this section let us present some simulations in order to predict future short-term ideological evolution. Simulations are done until year 2015 and we only change parameter β_1 because this represents the pressure of terrorism.

One question that emerges here is that immigration and emigration data are only available until 2005. Then, let us consider until 2015 a constant situation equal to year 2005 (see Fig. 5).

The first simulation presented here is the natural evolution with parameters given by (16) computed in the model fitting. As we can see in Fig. 6 subpopulations E, N and V are slight increasing at subpopulation A's expense.

The second simulation supposes an increase of law enforcement against terrorist groups. Therefore parameter β_1 is changed from 0.0606615 to -0.3. In this case there is an increase of subpopulation E, a slight increase in subpopulation N and a decrease of subpopulation V as can be seen in Fig. 7.

Conversely, if we consider an increase in terrorist activities, say from $\beta_1 = 0.0606615$ to $\beta_1 = 0.3$, subpopulation E decreases and V increases (see Fig. 8).

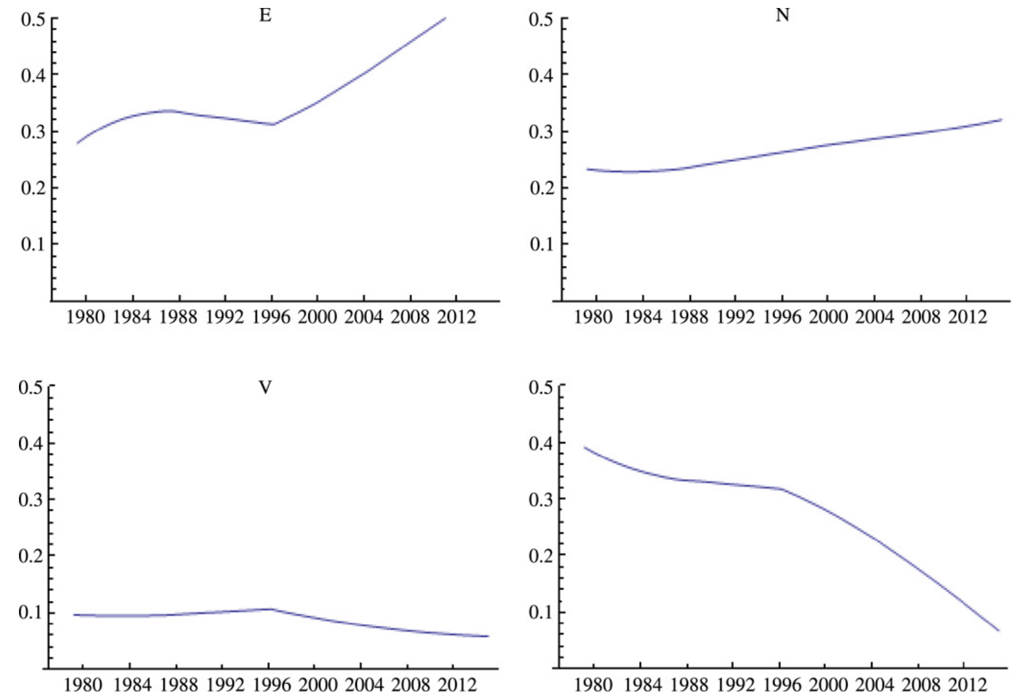

Fig. 7. In this graphic, parameter β_1 has been changed from 0.0606615 to −0.3. Then an increase of subpopulation E and a decrease of V has occurred.

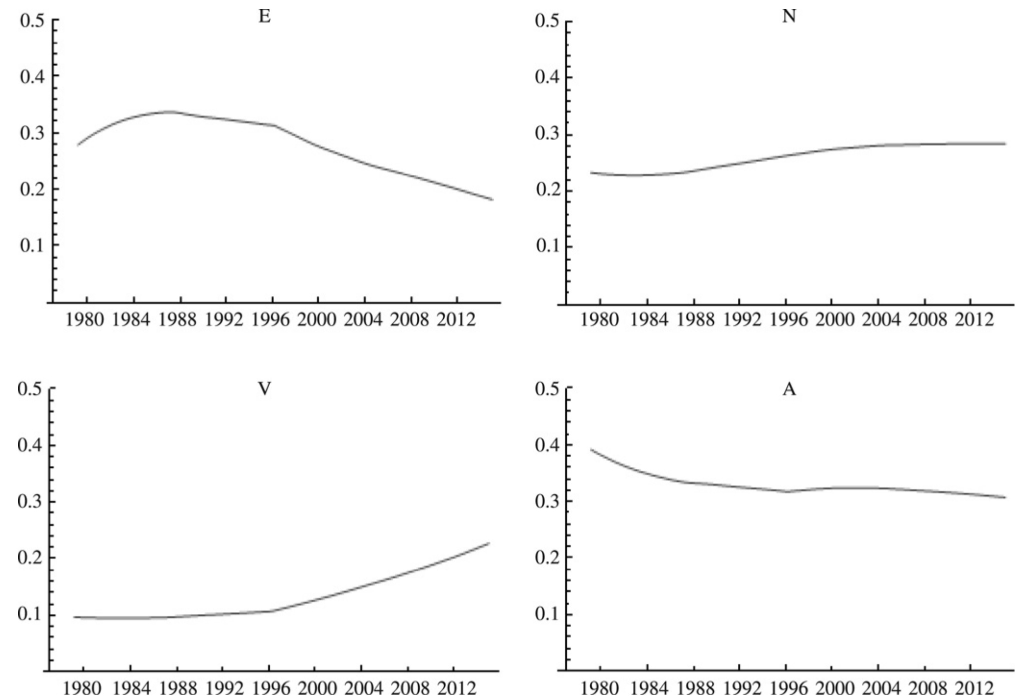

Fig. 8. An increasing in the terrorist activities, that is an increase of β_1 from 0.0606615 to 0.3, implies a decrease of subpopulation E and an increase of subpopulation V.

Our last simulation tries to explain what has happened in recent years. During the time President Aznar governed Spain, 1996–2004, it applied "law of parties" with a strong law enforcement against terrorists and its supporter groups.

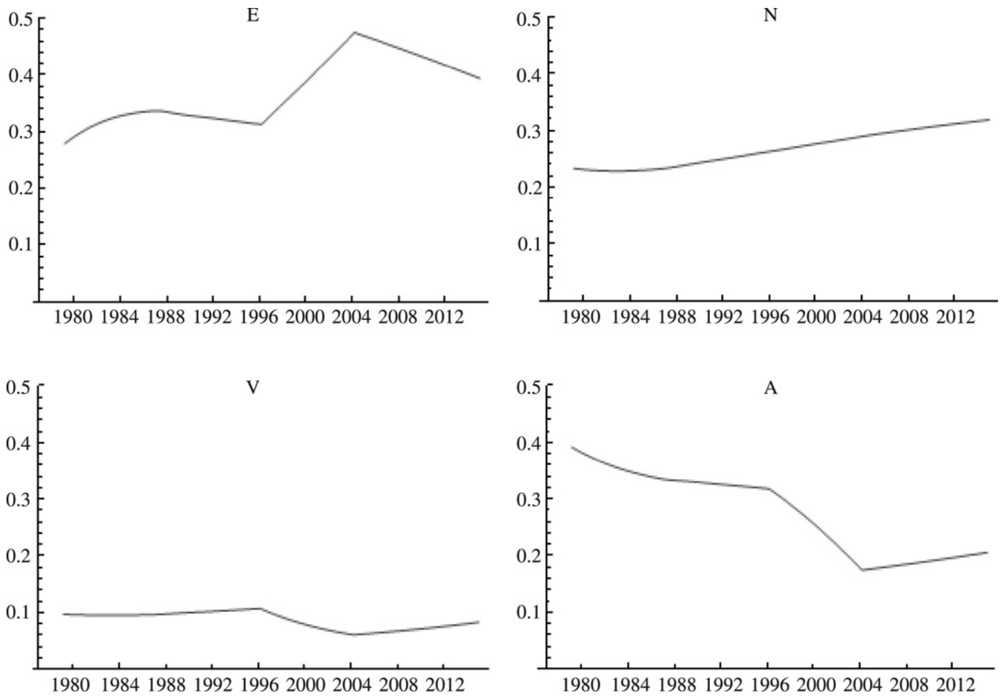

Fig. 9. Simulation from 1996 to 2015. During 1996–2004 President Aznar governed with strong law enforcement (say $\beta_1 = -0.6$). From 2004 President Zapatero gives some concessions to terrorist supporters (say $\beta_1 = 0.4$). Here we assume President Zapatero will govern, at least, until 2015.

Since 2004, President Zapatero has governed Spain and he has relaxed the law enforcement, and permitted parties in V to present some candidates to local elections (that means access to public budget to finance terrorist acts) and there have been reported meetings between spokesmen of Zapatero's government and the terrorist band ETA to negotiate political objectives in exchange for a cease-fire. In order to simulate this situation, during President Aznar time we consider $\beta_1 = -0.6$ and now with President Zapatero $\beta_1 = 0.4$. The results can be seen in Fig. 9.

Comparison between simulation of Fig. 9 and election data in Fig. 2 is not appropriate because "law of parties" banned ETA supporter groups, they could not present candidates, so the election data was not reliable, that is, did not represent the percentage of members of subpopulation V.

Notice that, with independence of what is the variation of β_1, subpopulation N remains stable or increasing.

7. Conclusions

In this paper we present a type-epidemiological mathematical model to analyze the ideological evolution of a society where extreme groups influence, with their terrorist acts, the population daily.

There are a few previous models of this type applied to study extreme behavior, for instance [1], but to our knowledge, this paper is the only study applied to a real situation, considering demographic and electoral data: The case of the Basque Country in Spain.

After model fitting, the model is used to obtain short-time predictions of the evolution of ideologies over time under different scenarios. Furthermore, an approach to recent and current times with Presidents Aznar and Zapatero has been done.

Our approach is a new step to understanding the transmission dynamics of extreme behavior and opens a way to apply type-epidemiological mathematical models to real situations.

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