The 2nd International Conference on Complexity Science & Information Engineering

Market Equilibrium Based on Renewable Energy Resources and Demand Response in Energy Engineering

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Abstract

Smart grid enables the integration of large-scale renewable energy resources (RERs) into the power system, but the subsequent intermittency and uncertainty can have an adverse impact on the networks’ reliability, safety and operation efficiency. Meanwhile, demand response helps greatly mitigate the negative impact associated with RERs. Hence, from the engineering’s perspective, the complexity and intelligence of the power system have been on an unprecedented level. In such a complex and intelligent power system, it is essential to investigate the impact of RERs and demand response on the market equilibrium in order to help market participants to make scientific decisions. In this paper, firstly, an overall model of major market participants together with the constraints of transmission and generation is established. Then, the energy market is analyzed with RERs’ uncertainties and demand response. Finally, a 4-bus network is utilized to validate theoretical results, indicating that as the uncertainties increase, power system’s operation costs and equilibrium shift will be enlarged; and the effect of demand response can narrow the equilibrium shift and reduce RERs’ integration costs.

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Keywords: renewable energy resources; energy engineering; market equilibrium; uncertainties; perturbation analysis

1. Introduction

With increasing concerns about global energy resources shortage, greenhouse gas effect and environmental pollution, smart grid has gained popularity globally for its unique advantages. Consequently, a large amount of smart technologies about renewable energy resources (RERs) generation and integration, storage technology, control signals to loads, energy efficiency and smart buildings have emerged. Admittedly, with the rapid development of RERs generation and integration technology, large-scale integration of RERs will bring about intermittency and randomness to the networks, which will certainly lead to integration costs. Meanwhile, demand response, which is aimed at guiding consumers’ behaviors according to the price signal, plays a significant role in mitigating such integration costs. Therefore, from the engineering’s perspective, the complexity and intelligence of the power system have been on an unprecedented level due to the comprehensive impact of RERs and demand response. So it is essential for us to investigate such impact on the market equilibrium to provide support for participants in the electricity market to make scientific decisions.

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Open access under CC BY-NC-ND license, doi:10.1016/j.sepro.2011.11.053
Literature review about the impact of RERs and demand response on the power system is presented here. Firstly, it is known that RERs respond actively to deal with energy and environment problems, but they have the disadvantage of intermittency and uncertainty due to their physical characteristics. One of the most serious problems that may emerge is limited dispatchability of intermittent generation [1]. In particular, for wind farms, problems such as the variability in resource availability, errors in forecasting resource availability and loads should also be taken into consideration [2]. The frequency modulation, peak regulation and economic planning and operation of the overall power system may be influenced by the integration of RERs as well [3]. Secondly, in recent years, there is a rapid development and self-improvement in demand response, which plays a significant role in reducing potential forecast error and redispach costs [4-6]. Load shape and operation efficiency will be improved using demand response [7]. It is assumed that the real-time pricing is implemented. When actual wind generation is less than forecast, consumer demand will decrease due to high costs associated with ancillary services used to compensate the generation shortage. Consequently, costs to meet consumer demand will fall. Similarly, when actual wind generation is more than forecast, since wind generation has zero marginal cost there is an increase in electricity demand [8, 9]. Nevertheless, none of the studies involved problems concerning the comprehensive impact of RERs and demand response on the market equilibrium point. Therefore, this paper concentrates on the market equilibrium analysis based on the integration of RERs and demand response.

The remainder of this paper is organized as follows: the general market equilibrium model based on agent behaviors is presented in Section 2. In Section 3, the market equilibrium in the absence of RERs and demand response is formulated and then perturbation analysis is given. Section 4 describes the suitability of the model using a case study and therefore the conclusion is obtained. Concluding remarks are shown in Section 5.

2. General market equilibrium model based on agent behaviors

Participants in the electricity market mainly consist of generating companies, consumer companies and independent system operator (ISO). In this paper, attention is merely paid to the corresponding agent behaviors of the three types of participants. Models of agent behaviors will be presented in this section, and then the market equilibrium is established. The key is to determine the generation of units and electricity demand of consumers, so that the market equilibrium can be obtained considering capacity constraints and the social welfare is maximized at the same time. It should be noted that the constraints that generating companies, consumer companies and ISO are subject to may conflict with each other, making the optimization problem more complex. For example, electricity price has links with both generating companies and consumer companies. As far as the consumers are concerned, low price means
low expenses so that they prefer low price. For generating companies, low price with previous costs leads to falling profits so that they are more willing to accept high price. It is obvious to see the conflict of constraints associated with the electricity price. In this case, ISO have to compensate generation companies who have expensive units used to ensure power balance. As a consequence, under the market equilibrium, the electricity price and amount, which are referred to the optimal local marginal price (LMP) and the optimal dispatch respectively, will be obtained. In addition, LMP is the corresponding shadow price. Models of three kinds of participants will be established in the following parts of this section.

2.1. Model of generating companies

It is assumed that the total number of generating units in the generating company is $T_g$. According to the power characteristics of generating units, the production of each unit $j$ is divided into $r$ power blocks, $j=1,2,...,T_g$, $r=1,2,...,T_g$. The power amount generated by unit $j$ in block $r$ is described as $A^r_j$, and the relevant linear operation cost is described as $C^r_j$. In addition, $p_n^j$ is the LMP of unit $j$ at node $n$. The objective of the generating company is to maximize its overall profit, so the optimization model is stated as:

$$\text{Max} \sum_{j \in g} \sum_{r=1}^{T_g} (p_n^j - C^r_j) A^r_j$$ (1)

$$\sum_{r=1}^{T_g} A^r_j \leq A^\text{max}_g : \xi_j; \forall j \in g, r=1,...,T_g$$ (2)

$$0 \leq A^r_j \leq A^\text{max}_g : \psi_j; \forall j \in g, r=1,...,T_g$$ (3)

Where, $\xi_j$ and $\psi_j$ denote the corresponding shadow prices. $A^r_j$ are decision variables. Attention should be paid to $p_n^j$, which are variables in the overall OPF problem, but remain constant in this optimization problem. Necessary and sufficient conditions for the optimization model are shown as:

$$(C^r_j + \xi_j + \psi_j - p_n^j) \times A^r_j = 0; \forall j \in g, r=1,...,T_g$$ (4)

$$(A^\text{max}_g - \sum_{r=1}^{T_g} A^r_j) \times \xi_j = 0; \forall j \in g$$ (5)

$$(A^r_j - A^\text{max}_g) \times \psi_j = 0; \forall j \in g, r=1,...,T_g$$ (6)

Dual variables $\xi_j$ and $\psi_j$ are used here, corresponding to the KKT conditions [12].

2.2. Model of consumer companies

It is assumed that the total number of electricity-consuming units in the consumer company is $T_d$. According to the power characteristics of these units, the demand of each unit $k$ is divided into $h$ blocks, $k=1,2,...,T_d$, $h=1,2,...,T_d$. The power amount demanded by unit $k$ in block $h$ is described as $A^h_k$, and the relevant linear utility function is denoted as $U^h_k$, which means the value created by the consumer company while consuming electricity. In addition, $p_n^k$ is the LMP of unit $k$ at node $n$. The objective of the consumer
company is to maximize its overall profit, and profit is described as the difference between $U^{kh}$ and $p^h_n$. Therefore, the optimization model is stated as follows:

$$\text{Max} \sum_{k=1}^{n} \sum_{h=1}^{r} (U^{kh}_d - p^k_n) A^{kh}_d$$

(7)

$$A^{kh}_d \leq A^{kh}_{d_{max}} : \tau_k, \forall k \in d_q; h = 1,\ldots,T_d$$

(8)

$$0 \leq A^{kh}_{d_{min}} \leq \sum_{h=1}^{T_d} A^{kh}_d : \eta_{kh}, \forall k \in d_q; h = 1,\ldots,T_d$$

(9)

Where, $\tau_k$ and $\eta_{kh}$ mean corresponding shadow prices. $A^{kh}_d$ are decision variables. Similarly, attention should be paid to $p^h_n$, which are variables in the overall OPF problem, but remain constant in this optimization problem. Necessary and sufficient conditions for the optimization model are shown as:

$$(p^k_n - U^{kh}_d + \tau_k - \eta_{kh}) \times A^{kh}_d = 0; \forall k \in d_q; h = 1,\ldots,T_d$$

(10)

$$\left( \sum_{h=1}^{T_d} A^{kh}_d - A^{kh}_{d_{min}} \right) \times \eta_{kh} = 0; \forall k \in d_q; h = 1,\ldots,T_d$$

(11)

$$(-A^{kh}_{d_{max}} + A^{kh}_d) \times \tau_k = 0; \forall k \in d_q; h = 1,\ldots,T_d$$

(12)

Dual variables $\tau_k$ and $\eta_{kh}$ are included here, corresponding to the KKT conditions [12].

2.3. Model of ISO

It is assumed that the electricity market in this paper is wholesale and operates as follows: Firstly, generating companies submit their bidding generation of the units and the corresponding price to the pool, and consumer companies offer their bidding electricity demand and the corresponding price at the same time. Then, the electricity market is cleared by ISO using an appropriate market-clearing method resulting in equilibrium price and production and consumption schedules. In the end, the market equilibrium is obtained and the corresponding equilibrium price is LMP. Actually, network constraints such as losses and line capacity limits may be included in the market-clearing method. There are two points to be considered here. The first is the determination of LMP. That is to say, the price at a certain node for the power generated by the generation company is its LMP, and similarly, the price at a certain node for the power consumed by the consumer company is its LMP. The second point is to determine which constraints to be selected. Network constraints mainly consist of transmission losses and line capacity limits, and power flow may be affected by technologies constraints. For example, power flow may encounter congestion when approaching its maximum limit. In addition, heat losses in power lines are obvious, but they are not included in the model for ease of exposition.

Standard market-clearing method is based on the social welfare which is maximized in this optimization model [13].

$$\text{Max} \left( \sum_{k=1}^{n} \sum_{h=1}^{r} U^{kh}_d A^{kh}_d - \sum_{j=1}^{T_j} \sum_{r=1}^{T_r} C_{gr}^{j} A^{pr}_g \right)$$

(13)
\[ \sum_{j=0}^{T_j} \sum_{r=1}^{T_r} A_{g}^{r} - \sum_{k=0}^{T_k} \sum_{r=1}^{T_r} A_{d}^{k} - \sum_{j=0}^{T_j} B_{xy} [\delta_x - \delta_y] = 0; \forall x \in N \] (14)

\[ B_{xy} [\delta_x - \delta_y] \leq A_{xy}^{\max}, \forall x \in N; \forall y \in \Omega \] (15)

Considering the objective function, the first term means the revenue due to surpluses of bids from generating companies, and the second term denotes the revenue due to surpluses of bids from consumer companies. Also, constraints (14) and (15) are based on power balance and capacity limits, respectively. And \( \varepsilon_x \) and \( \sigma_{xy} \) are associated Lagrange multipliers.

Actually, this optimization problem can be restated as the relationship between power blocks levels \( A_{g}^{r} \), demand blocks levels \( A_{d}^{k} \), the voltage angle \( \delta_x \), and dual variables \( \varepsilon_x \) and \( \sigma_{xy} \) such that

\[ (C_{gh}^{r} - p_{n}^{g}) \times A_{g}^{r} = 0; \forall j \in g; r = 1,...,T_g \] (16)

\[ (p_{k}^{h} - U_{d}^{k}) \times A_{d}^{k} = 0; \forall k \in d; h = 1,...,T_d \] (17)

\[ \left( \sum_{j=0}^{T_j} \sum_{r=1}^{T_r} A_{g}^{r} - \sum_{k=0}^{T_k} \sum_{r=1}^{T_r} A_{d}^{k} + \sum_{j=0}^{T_j} B_{xy} [\sigma_{xy} - \varepsilon_x] \right) \times \delta_x = 0; \forall x \in N \] (18)

\[ (-\sum_{j=0}^{T_j} \sum_{r=1}^{T_r} A_{g}^{r} + \sum_{k=0}^{T_k} \sum_{r=1}^{T_r} A_{d}^{k} + \sum_{m=0}^{\Omega} B_{xy} [\delta_x - \delta_y]) \times \varepsilon_x = 0; \forall x \in N; \varepsilon > 0 \] (19)

\[ \left( A_{xy}^{\max} - B_{xy} [\delta_x - \delta_y] \right) \times \sigma_{xy} = 0; \forall x \in N; \forall y \in \Omega \] (20)

\[ \delta_x \geq 0; \forall x \in N \] (21)

Where, \( A_{g}^{r} \), \( A_{d}^{k} \) and \( p_{n}^{g} \) are decision variables. In short, models of agent behaviors including generating companies, consumer companies and ISO have already been formulated. The key is to determine the equilibrium point which optimizes the three types of participants at the same time. The equilibrium point is subject to the following conditions: 1) maximum profit for each generating company; 2) maximum utility for each consumer company; 3) maximum net social welfare for ISO.

2.4. Introduce of perturbation parameters

Scholars at home and abroad have investigated the impact of RERs on power system. Research results indicate that the intermittency and uncertainty of RERs will lead to uncertainties of power generation and thus influence the market equilibrium. To measure the uncertainties effectively, \( A_{g}^{r} \) are introduced into the decision variable \( A_{g}^{r} \) in the model of generation companies. The procedure is as follows: 1) \( A_{g}^{r} \) are divided into \( A_{g}^{r} (j=1,...,n_T) \) and \( A_{g}^{r} (j=1,...,n_R) \), where \( n_T \) means traditional dispatchable generating units and \( n_R \) means renewable energy resources such as wind and solar energy, which are certainly non-dispatchable; 2) \( A_{g}^{r} \) are introduced into non-dispatchable generating companies, as

\[ \overline{A}^{\prime}_{gk} = A_{gk}^{r} (1 - \Delta_{gk}^{r}), 0 < \Delta_{gk}^{r} < 1 \] (22)

It should be noted that \( \Delta_{gk}^{r} \) may come from forecast errors, and are out of control and nonlinear.

Demand response can reduce integration costs of RERs by optimizing users’ electricity-consuming behaviours using price signal. Dispatchable load \( A_{d}^{k} \) are introduced when consumers’ units are divided into diapatchable and non-diapatchable ones by generating companies. It is assumed that \( A_{d}^{k} \) are affected by demand response obviously and are sensible to price changes. To measure the effect of demand
response effectively, control parameter $\gamma^{kh}_d$ are therefore introduced to reflect the response of users to price changes in the form of real-time pricing under demand response.

$$\overline{A}^{kh}_d = A^{kh}_d (1 - \gamma^{kh}_d), 0 < \gamma^{kh}_d < 1$$  \hspace{1cm} (23)

Where, $\overline{A}^{kh}_d$ denote electricity consumed by unit $k$ at block $h$ in the form of real-time pricing under demand response. Positive $\gamma^{kh}_d$ mean that electricity consumed in this block has decreased, while negative $\gamma^{kh}_d$ mean that electricity consumed in this block has increased. In this paper, attention is merely paid to the positive $\gamma^{kh}_d$, because there may be a decrease in power supply due to the integration of RERs. For example, positive $\gamma^{kh}_d$ will emerge if $\Delta^{jr}_g > 0$, which means that power generated by non-dispatchable units has fallen.

As the control parameter can effectively reflect the impact of RTP on electricity consumed by users, it is described as the curtailment factor.

Now, $A^{h'}_g$ are utilized to measure the uncertainties of RERs, and $\gamma^{kh}_d$ are employed to reflect the mitigation effect of demand response. Both $A^{h'}_g$ and $\gamma^{kh}_d$ are referred to the perturbation parameters, but $A^{h'}_g$ are unknown and $\gamma^{kh}_d$ are assumed to be controllable.

It should be noted that $\gamma^{kh}_d$ are used to measure the effect of RTP in the model. The inherent assumption here is that the non-dispatchable consumer company will suitably adjust its demand after observing the LMP, which is the solution of (10)-(12). Also, curtailment factor $\gamma^{kh}_d$ can reflect the effect of changes in users’ electricity-using behaviours. Hence, it is assumed that RTP is equal to LMP in this paper.

3. Market equilibrium analysis based on RERs and demand response

3.1. Market equilibrium model

It is clear that LMP in model of generating companies and consumer companies are input variables, and in the optimization model of ISO, they are dual variables. Therefore, there is a tight link between the three optimization models.

As each of these three sets of optimization problems are linear programming problems, Karush-Kuhn-Tucker(KKT) optimality conditions is employed to explain the optimal solutions. The optimality conditions of the three optimization problems bring about a Mixed Linear Complementarity Problem(MLCP), which corresponds to the market equilibrium. As a result, the market equilibrium is obtained when the necessary and sufficient conditions (4)-(6), (10)-(12) and (16)-(21) are satisfied simultaneously. This optimization problem can be simply stated as the solution of the following MLCP: A vector $x^* \in \mathbb{R}^n$ is required to meet the following constraints:

$$x^T(Mx + q) = 0, x \geq 0, Mx + q \geq 0$$ \hspace{1cm} (24)

Where, $x \in \mathbb{R}^{n \times 1}$ is the variable vector, square metric $M \in \mathbb{R}^{n \times n}$ and $q \in \mathbb{R}^{n \times 1}$ includes operating costs, the bidding of generating companies and consumer companies, maximum and minimum power generated by generating companies, maximum and minimum power consumed by consumer companies and maximum thermal limit of transmission lines. The solution $x^*$ of MLCP is the market equilibrium point, which is stated as follows:

$$x^* = \text{MLCP}(M, q)$$ \hspace{1cm} (25)
**Definition 1**: A matrix $M$ is called a P-matrix if its all principal minors are positive.

Two theorems that discuss the MLCP are given here [14, 15].

**Theorem 1**: In the MLCP defined in (24), $M$ is a P-matrix if and only if the $MLCP(M, q)$ has a unique solution for any $q \in \mathbb{R}^n$, moreover, if $M$ is a P-matrix then there is a neighborhood $N$ of $M$, such that all matrices in $N$ are P-matrix. Therefore, we can define a solution function $x(M, q) : M \times \mathbb{R}^n \rightarrow \mathbb{R}^n$, where $x(M, q)$ is the solution of $MLCP(M, q)$ and $R^n_+ = \{ x \in \mathbb{R}^n | x \geq 0 \}$.

**Theorem 2**: Matrix $A$ is a P-matrix if and only if $(I - D + DA)$ is nonsingular for any diagonal matrix $D = \text{diag}(d), 0 \leq d_i \leq 1, i = 1, 2, \ldots, n$. Furthermore, if $x(A, q)$ and $x(B, p)$ are the solution of the corresponding $MLCP(A, q)$ and $MLCP(B, p)$ respectively, then:

$$\beta(A) = \max_{d \in [0, 1]} \left\| (I - D + DA)^{-1} D \right\|$$

Dependence of $x(M, q)$ on the market parameters is described in *Theorem 2*. It is assumed that all components for $x$ in (24) have been suitably normalized so as to satisfy the condition $\|x^*\| \leq 1$.

### 3.2. Perturbation analysis

The solution of $MLCP(M, q)$ is the market equilibrium point in the absence of RERs and demand response. It is known that renewable energy resources are used widely to deal with energy crisis and environmental pollution. For example, wind and solar energy are commonly utilized to generate power. Nevertheless, the intermittency and randomness of such resources have caused uncertainties in power generation, which also has a big impact on the market equilibrium. Fortunately, demand response can reduce the integration costs of RERs. As is stated above, $\Delta_{w/g}$ denote the uncertainties caused by wind forecast errors, and $\lambda_{kh}$ denote the effect of demand response. The $M$ and $q$ will change accordingly in the presence of perturbation parameters, and $M + \Delta M$ and $q + \Delta q$ are used to describe the change. Hence, the corresponding optimization problem and the corresponding equilibrium point are stated as follows:

$$x^T((M + \Delta M)x + (q + \Delta q)) = 0, x \geq 0, (M + \Delta M)x + (q + \Delta q) \geq 0$$

$$x^*_\lambda = MLCP(M + \Delta M, q + \Delta q)$$

**Definition 2**: Define non-dimensional perturbation parameters $\phi_M$ and $\phi_q$, constant $\lambda$ and $N$ are defined respectively as:

$$\|\Delta M\| \leq \phi_M \|M\|$$

$$\|\Delta q\| \leq \phi_q \|q\|$$

$$\lambda = \varepsilon_M \beta(M)\|M\|$$

$$N : \{ \Delta M, \|\beta(M)\| \|\Delta M\| \leq \lambda \}$$
The key is to find the relation between the two optimal solution $x^*$ and $x^*_\Delta$, in the following theorem.  

**Theorem 3**: If the nominal market has a unique solution $x^*$ and $\lambda < 1$, then the perturbed market has a unique solution $x^*_\Delta$ and satisfies the following inequality:

$$
\|x^* - x^*_\Delta\| \leq \frac{2\varepsilon}{1 - \lambda} \beta(M)
$$

(34)

It is necessary to prove Theorem 3. Two steps are to be taken: First, prove that if the nominal market has a unique solution $x^*$ and $\lambda < 1$, then the perturbed market has a unique solution $x^*_\Delta$. Second, prove the inequality stated in (34).

(1) **Unique solution in perturbed market**

If the nominal market in (24) has a unique solution, according to Theorem 1, the matrix $M$ is $P$-matrix. Then, according to Theorem 2, this implies that $(I - D + DM)$ is nonsingular for any diagonal matrix. The following equality is obtained:

$$(I - D + D(M + \Delta M)) = (I - D + DM)(I + M_0\Delta M)$$

(35)

Where,

$$M_0 = (I - D + DM)^{-1} D$$

(36)

Considering the definition of $\beta(M)$, the following inequality is satisfied:

$$\|M_0\Delta M\| \leq \beta(M)\|M\| \leq \lambda; \forall \Delta M \in N$$

(37)

**Theorem 3** indicates that $I + M_0\Delta M$ is nonsingular for any $\Delta M \in N$ due to $\lambda < 1$. Hence, $(I - D + D(M + \Delta M))$ is nonsingular from (35) and then the perturbed market in (28) has a unique solution for any $\Delta M \in N$ when $\lambda < 1$ is met.

(2) **The inequality**

According to (26) in the Theorem 2, the following inequality is met:

$$\|x^* - x^*_\Delta\| \leq \beta(M + \Delta M)\|\Delta M x^* + \Delta q\|$$

(38)

$\beta(M + \Delta M)$ can be restated as

$$(I - D + D(M + \Delta M))^{-1} D = (I + M_0(\Delta M))^{-1} (I - D + DM)^{-1} D$$

(39)

And

$$\|I + M_0(\Delta M))^{-1}\| \leq \frac{1}{1 - \beta(M)\|M\|} \leq \frac{1}{1 - \lambda}$$

(40)
The following inequality is obtained after taking norms on both sides of (39):

\[ \beta(M + \Delta M) \leq \frac{1}{1 - \lambda} \beta(M) \]  

(41)

From (38), there is an inequality stated as follows:

\[ \|x^* - x^*_M\| \leq \frac{1}{1 - \lambda} \beta(M) \|\Delta M + \Delta q\| \]  

(42)

After considering the definition of \( \Delta M \) and \( \Delta q \):

\[ \|x^* - x^*_M\| \leq \frac{2\varepsilon}{1 - \lambda} \beta(M) \]  

(43)

Where, \( \varepsilon = \max \{ \varepsilon_M \| M \|, \varepsilon_q \| q \| \} \), which is the desired bound.

To be simple, \( \mu \) is defined as

\[ \mu = \frac{2\varepsilon}{1 - \lambda} \beta(M) \]  

(44)

It can be seen from Theorem 3 that \( \mu \) is the maximum equilibrium shift caused by the uncertainties of integration of RERs into the power system.

4. Case study

4.1. Initial data

This paper focuses on the market equilibrium based on RERs and demand response. Wind power is integrated into the power networks for its wide application. A 4-bus network is utilized to evaluate the equilibrium of the electricity market with wind power and demand response.

The 4-bus network is presented in Figure 1. Among the 4 buses, there are two traditional generating units which are dispatchable in bus 1 and a wind generator which is non-dispatchable in bus 2. The intermittency and randomness of wind generation in bus 2 can therefore be compensated by power generated in bus 1, ensuring the reliability of power supply. There are power consumptions at nodes 3 and 4. The maximum and minimum power supply and demand of these market participants are indicated in Table 1, and the minimum value of those is 0. To be simple, it is assumed that every demand and generator uses one block to bid on the market.

\[ g_1 \quad 1 \quad g_2 \]
\[ 2 \quad 3 \quad 4 \]
\[ d_1 \quad d_2 \]

Fig. 1 the 4-bus power network
The bids of generators and consumers are also indicated in Table 1. It can be seen that the bids of the amount of power generated by \( g_1 \) and \( g_2 \) is 20MW, while the price offered by \( g_1 \) is 50$/MWh, which is much higher than 6$/MWh given by the wind generator in bus 2. Obviously, wind power generation has the advantage of low costs, thus promoting the rapid development of wind power. Bids of electricity demand of \( d_1 \) and \( d_2 \) are 16MW and 10MW respectively, with the prices of 6.4$/MWh and 8.5$/MWh respectively.

Table 1 Maximum power, minimum power and bids of generators and consumers

<table>
<thead>
<tr>
<th></th>
<th>( g_1 )</th>
<th>( g_2 )</th>
<th>( d_1 )</th>
<th>( d_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum power</td>
<td></td>
<td></td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>MW</td>
<td>20</td>
<td>26</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>Minimum power</td>
<td></td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>MW</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bidding amount</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(MW)</td>
<td>20</td>
<td>20</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>Price ($/MWh)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>6</td>
<td>6.4</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Transmission lines data from the generating companies to consumer companies are shown in Table 2. Susceptance \( B_{xy} \) and the line capacity limits \( A_{xy}^{max} \) are included. The line parameters are per unit with three-phase base of 230 kV and 10 MVA. It can be seen from Table 2 that capacity limits from wind generators to consumers are relatively higher due to the uncertainties of wind power generation. For example, when there is larger amount of wind power generation than expected, larger capacity is necessary to enable the electricity to be transported to the consumers. Hence, there will be an increase in the operation costs of the overall power networks. It is important to enlarge the capacity to ensure the reliability of the networks.

Table 2 Transmission lines data

<table>
<thead>
<tr>
<th>Bus x-bus y</th>
<th>X (pu)</th>
<th>Capacity limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>0.0505</td>
<td>100</td>
</tr>
<tr>
<td>1-4</td>
<td>0.0374</td>
<td>100</td>
</tr>
<tr>
<td>2-3</td>
<td>0.0374</td>
<td>150</td>
</tr>
<tr>
<td>2-4</td>
<td>0.0637</td>
<td>200</td>
</tr>
</tbody>
</table>

4.2. Data analysis under market equilibrium

The generation power of generating companies and consumer companies and price under market equilibrium are obtained in the absence of the effects of uncertainties of RERs and demand response. The results are given in Table 3.

Table 3 Results of the nominal market equilibrium

<table>
<thead>
<tr>
<th>Bus x</th>
<th>Generation power(MW)</th>
<th>Demand power(MW)</th>
<th>Price($/MWh)</th>
<th>sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-</td>
<td>0.7000</td>
<td>26.00</td>
</tr>
<tr>
<td>2</td>
<td>26.00</td>
<td>-</td>
<td>0.7000</td>
<td>26.00</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>16.00</td>
<td>0.7000</td>
<td>26.00</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>10.00</td>
<td>0.7000</td>
<td>26.00</td>
</tr>
<tr>
<td>sum</td>
<td>26.00</td>
<td>26.00</td>
<td>0.7000</td>
<td>26.00</td>
</tr>
</tbody>
</table>

It can be seen from Table 3 that the price in the equilibrium market is 0.75$/MWh. The amount of power offered by \( g_1 \) is zero, while \( g_2 \) provides 26MW to meet the electricity amount demanded by both \( d_1 \) and \( d_2 \). It is obvious that the efficiency and equity are maximized when the market equilibrium is satisfied.

Transmission lines data under market equilibrium are indicated in Table 4.
Table 4 Transmission lines data under market equilibrium

<table>
<thead>
<tr>
<th>Bus x-bus y</th>
<th>1-3</th>
<th>1-4</th>
<th>2-3</th>
<th>2-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P(MW)</td>
<td>3.432</td>
<td>-3.432</td>
<td>12.556</td>
<td>13.428</td>
</tr>
<tr>
<td>Q(MVAr)</td>
<td>0.0319</td>
<td>0.0236</td>
<td>0.0695</td>
<td>0.0632</td>
</tr>
<tr>
<td>Loss</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P(MW)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q(MVAr)</td>
<td>0</td>
<td>0.0146</td>
<td>0.1027</td>
<td>0.0738</td>
</tr>
</tbody>
</table>

4.3. Perturbation analysis

In this 4-bus network, the uncertainties are introduced in the wind generator in bus 2, which are denoted as $\Delta g^i$. It is assumed that electricity demand of consumers is sensible to price changes. The effect of demand response in the form of RTP is denoted as $\gamma_d^i$.

(1) Uncertainties of RERs

First, the uncertainties incorporating wind generation under fixed loads is to be measured. $\Delta g^i$ is used as the measure of uncertainties of wind generators, and the corresponding operation costs $F_g$ are defined as

$$F_g = C_g^1 A_g^1 + C_g^2 A_g^2$$

(45)

Results of operation costs and equilibrium shift for a range of $\Delta g^i$ are obtained by simulation. Table 5 shows the results. It can be seen that as the $\Delta g^i$ increases, the operation costs $F_g$ and equilibrium shift $\mu$ will have an upward trend.

Table 5 Operation costs and equilibrium shift

<table>
<thead>
<tr>
<th>$\Delta g^i$</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_g$</td>
<td>11.197</td>
<td>10.463</td>
<td>10.725</td>
<td>11.208</td>
<td>11.679</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.0515</td>
<td>0.0632</td>
<td>1.4196</td>
<td>1.5304</td>
<td>1.6817</td>
</tr>
</tbody>
</table>

(2) Effect of demand response

It is time to numerically evaluate the effect of the second perturbation parameter $\gamma_d^i$ that is introduced in this paper. Table 6 indicates the equilibrium shift for a range of $\gamma_d^i$ with different wind forecast errors. It is observed that when $\Delta g^i$ is at a fixed level, equilibrium shift will decrease with increasing curtailment factor $\gamma_d^i$, which means that demand response can mitigate the equilibrium shift, and thus reduce the cost of uncertainty in the presence of a wind forecast error.

Table 6 Equilibrium shift under different wind forecast errors and curtailment factor

<table>
<thead>
<tr>
<th>$\Delta g^i$</th>
<th>$\gamma_d^i=0$</th>
<th>$\gamma_d^i=0.03$</th>
<th>$\gamma_d^i=0.06$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0677</td>
<td>0.0434</td>
<td>0.0262</td>
</tr>
<tr>
<td>0.03</td>
<td>1.4398</td>
<td>0.0601</td>
<td>0.0559</td>
</tr>
<tr>
<td>0.04</td>
<td>1.5362</td>
<td>1.4357</td>
<td>0.0763</td>
</tr>
</tbody>
</table>

5. Conclusion

This paper concentrates on the market equilibrium analysis based on models of agent behaviors including generating companies, consumer companies and ISO. Perturbation analysis is then implemented in consideration of the uncertainties of RERs and effect of demand response. Simulation results of the 4-bus network indicate that demand response can effectively reduce the integration costs of RERs, which helps participates in the electricity market to make scientific decisions to maximize their own benefits.
Acknowledgements

The author would like to thank financial support given to this work by the Energy Foundation under contract number Grant 70671041. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

References


