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Constraints on the CKM angle γ from $B \rightarrow K^{*\pm}\pi^\mp$

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Abstract

We present constraints on the CKM parameter $\gamma = \arg V_{ub}^*$ formed within the framework of SU(3) symmetry and based on charmless hadronic B decays to $K^{*\pm}\pi^\mp$ and other pseudoscalar-vector final states. For strong phases of $\mathcal{O}(10^\circ)$, our analysis weakly favors $\cos \gamma < 0$. We also estimate that a determination of γ with an experimental uncertainty of less than 10° can be attained with an order-of-magnitude improvement in the precision of the experimental inputs, but SU(3) symmetry breaking could introduce corrections approaching the size of the current experimental uncertainties.

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In the Standard Model, the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix [1] gives rise to CP-violating phenomena through its single complex phase. This phase can be probed experimentally by measuring decay rates and CP asymmetries for charmless hadronic B decays that receive contributions from amplitudes with differing weak phases. In the flavor SU(3) decomposition of the amplitudes in pseudoscalar-vector (PV) final states [2], the $b \rightarrow u\bar{u}s$ transition $B \rightarrow K^{*\pm}\pi^\mp$ is dominated by two amplitudes, color-allowed tree and gluonic penguin, which interfere with a weak phase $\pi - \gamma$, where $\gamma = \arg V_{ub}^*$, and with an unknown strong phase δ . We extract $\cos \gamma$ in two ways, employing Monte Carlo simulation to propagate experimental uncertainties and ratios of meson decay constants to account for SU(3) symmetry breaking. The first uses only $B \rightarrow K^{*\pm}\pi^\mp$, and the second adds information from $B \rightarrow \phi(\bar{K})$. In both cases, the magnitudes of the penguin and tree amplitudes must be known, and we estimate these from CKM unitarity and measured branching fractions for other $B \rightarrow PV$ decays.

An alternative method of constraining γ [3] makes use of observables pertaining to $b \rightarrow c$ transitions and to mixing in the neutral B and K systems, with a resultant 95% confidence level (C.L.) allowed interval for γ of $[38^\circ, 80^\circ]$. In contrast, the analysis presented in this Letter uses rare, charmless $b \rightarrow u, d, s$ transitions without reference to mixing-induced CP violation. A discrepancy between the constraints on γ from charmless hadronic B decays and those from B and K mixing might arise from new physics contributions to either B and K mixing or the $b \rightarrow s$ or $b \rightarrow d$ penguins.

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Global analyses of charmless hadronic B decays in the framework of QCD-improved factorization [4,5] find a value for γ of approximately 80° . However, these fits predict smaller branching fractions for $B \rightarrow K^{*\pm}\pi^\mp$ and $(\bar{K}^*)^0\pi^\pm$ than are observed experimentally, and removing these modes from the above analyses improves the fit quality. It has been suggested [6] that the $B \rightarrow K^*\pi$ modes may receive dynamical enhancements not accounted for in Refs. [4] and [5]. Our analysis focuses on $B \rightarrow K^{*\pm}\pi^\mp$ with input from a modest number of other $B \rightarrow PV$ decays, thus providing a complement to the global fits.

Following the notation in Ref. [2] for SU(3) invariant amplitudes, we denote color-allowed tree amplitudes by t and gluonic penguins by p . Amplitudes for $|\Delta S| = 1$ transitions are primed, while those for $\Delta S = 0$ transitions are unprimed. A subscript P or V indicates whether the spectator quark hadronizes into the pseudoscalar or vector meson, respectively. Since $B \rightarrow K^{*+}K^\mp$ is dominated by penguin annihilation and W -exchange contributions or rescattering effects, and these decays have not been observed experimentally, we neglect such amplitudes in our analysis. The transition amplitude for $B \rightarrow K^{*\pm}\pi^\mp$ is $A(K^{*\pm}\pi^\mp) = -(p'_P + t'_P)$. The amplitudes t' and p' carry the CKM matrix elements $V_{ub}^*V_{us}$ and $V_{tb}^*V_{ts}$ with weak phases γ and π , respectively. The amplitudes for the two charge states are given by

$$A(K^{*+}\pi^-) = |p'_P| - |t'_P|e^{i\gamma}e^{i\delta}, \quad (1)$$

$$A(K^{*-}\pi^+) = |p'_P| - |t'_P|e^{-i\gamma}e^{i\delta}, \quad (2)$$

and we can express the CP-averaged amplitude as

$$\frac{1}{2}\left[|A(K^{*+}\pi^-)|^2 + |A(K^{*-}\pi^+)|^2\right] = |p'_P|^2 + |t'_P|^2 - 2|p'_P||t'_P|\cos\gamma\cos\delta. \quad (3)$$

We identify squared amplitudes, $|A|^2 = A^*A$, with branching fractions, \mathcal{B} , and we absorb all numerical factors, like G_F , m_B , phase space integrals, decay constants, form factors, and CKM matrix elements, into the definitions of the amplitudes. Most of the B branching fraction measurements in the literature are calculated assuming equal production of charged and neutral mesons. We correct these branching fractions by the ratio of B^+B^- to $B^0\bar{B}^0$ production rates, f_{+-}/f_{00} , as well as by the ratio of charged to neutral lifetimes, τ_+/τ_0 . Because the constraints on γ are constructed from ratios of branching fractions, we scale only the neutral B branching fractions by the product $\mathcal{F} \equiv f_{+-}/f_{00} \cdot \tau_+/\tau_0$, which is measured directly in Refs. [7] and [8]. Thus, $\cos\gamma\cos\delta$ can be expressed in terms of the CP-averaged branching fraction $\mathcal{B}(K^{*\pm}\pi^\mp)$:

$$\cos\gamma\cos\delta = \frac{|p'_P|^2 + |t'_P|^2 - \mathcal{B}(K^{*\pm}\pi^\mp)\mathcal{F}}{2|p'_P||t'_P|}. \quad (4)$$

For a given value of δ , γ is determined to a twofold ambiguity. The rate difference between $\bar{B}^0 \rightarrow K^{*-}\pi^+$ and $B^0 \rightarrow K^{*+}\pi^-$, which is proportional to $\sin\gamma\sin\delta$, provides an additional observable that allows us to disentangle γ and δ :

$$\cos(\gamma + \delta) = \frac{|p'_P|^2 + |t'_P|^2 - \mathcal{B}(K^{*+}\pi^-)\mathcal{F}}{2|p'_P||t'_P|}, \quad (5)$$

$$\cos(\gamma - \delta) = \frac{|p'_P|^2 + |t'_P|^2 - \mathcal{B}(K^{*-}\pi^+)\mathcal{F}}{2|p'_P||t'_P|}, \quad (6)$$

which leads to

$$\gamma = \frac{1}{2}\left[\cos^{-1}\frac{|p'_P|^2 + |t'_P|^2 - \mathcal{B}(K^{*+}\pi^-)\mathcal{F}}{2|p'_P||t'_P|} + \cos^{-1}\frac{|p'_P|^2 + |t'_P|^2 - \mathcal{B}(K^{*-}\pi^+)\mathcal{F}}{2|p'_P||t'_P|}\right], \quad (7)$$

$$\delta = \frac{1}{2}\left[\cos^{-1}\frac{|p'_P|^2 + |t'_P|^2 - \mathcal{B}(K^{*+}\pi^-)\mathcal{F}}{2|p'_P||t'_P|} - \cos^{-1}\frac{|p'_P|^2 + |t'_P|^2 - \mathcal{B}(K^{*-}\pi^+)\mathcal{F}}{2|p'_P||t'_P|}\right]. \quad (8)$$

Table 1

Input parameters used to constrain γ . Branching fractions and partial rate differences are given in units of 10^{-6} . Except for the last two entries, branching fractions are averaged over charge conjugate states

| Parameter | References | | | Value |
|--|---|-----------------------------------|---|--|
| \mathcal{F} | [7,8] | | | 1.11 ± 0.07 |
| f_{K^*}/f_ρ | [20] | | | 1.04 ± 0.02 |
| $ V_{us} $ | [21] | | | 0.2196 ± 0.0026 |
| $ V_{ud} $ | [21] | | | 0.9734 ± 0.0008 |
| $ V_{ub} $ | [21] | | | $(3.66 \pm 0.51) \times 10^{-3}$ |
| $ V_{cb} $ | [21] | | | $(4.07 \pm 0.10) \times 10^{-2}$ |
| $\mathcal{A}_{\text{CP}}(K^{*\pm}\pi^\mp)$ | [22] | | | $0.26^{+0.33+0.10}_{-0.34-0.08}$ |
| A | [11] | | | $-0.18 \pm 0.08 \pm 0.03$ |
| C | [11] | | | $0.36 \pm 0.18 \pm 0.04$ |
| ΔC | [11] | | | $0.28^{+0.18}_{-0.19} \pm 0.04$ |
| ρ_{AC} | [12] | | | -0.080 |
| $\rho_{A\Delta C}$ | [12] | | | -0.059 |
| $\rho_{C\Delta C}$ | [12] | | | 0.176 |
| | CLEO | BABAR | Belle | |
| $\mathcal{B}(\rho^\pm\pi^\mp)$ | $27.6^{+8.4+4.2}_{-7.4}$ [23] | $22.6 \pm 1.8 \pm 2.2$ [11] | $20.8^{+6.0+2.8}_{-6.3-3.1}$ [24] | 22.8 ± 2.5 |
| $\mathcal{B}(\bar{K}^*0\pi^\pm)$ | $7.6^{+3.5}_{-3.0} \pm 1.6$ [23] | $15.5 \pm 1.8^{+1.5}_{-3.2}$ [25] | $19.3^{+4.2+4.1}_{-3.9-7.1}$ [26] | 12.4 ± 2.5 |
| $\mathcal{B}(K_{S^0}^{*\pm}\pi^\mp)$ | $\left. \vphantom{\mathcal{B}(K_{S^0}^{*\pm}\pi^\mp)} \right\} 16^{+6}_{-5} \pm 2.0$ [27] | | $20.3^{+7.5}_{-6.6} \pm 4.4$ [28] | $\left. \vphantom{\mathcal{B}(K_{S^0}^{*\pm}\pi^\mp)} \right\} 16.4^{+4.2}_{-4.0}$ |
| $\mathcal{B}(K_{K^0}^{*\pm}\pi^\mp)$ | | | $13.0^{+3.9+2.0+6.9}_{-3.6-1.8-6.1}$ [29] | |
| $\mathcal{B}(\phi K^\pm)$ | $5.5^{+2.1}_{-1.8} \pm 0.6$ [30] | $10.0^{+0.9}_{-0.8} \pm 0.5$ [31] | $10.7 \pm 1.0^{+0.9}_{-1.6}$ [32] | 9.6 ± 0.8 |
| $\mathcal{B}(\phi\bar{K}^0)$ | $5.4^{+3.7}_{-2.7} \pm 0.7$ [30] | $7.6^{+1.3}_{-1.2} \pm 0.5$ [31] | $10.0^{+1.9+0.9}_{-1.7-1.3}$ [32] | 8.1 ± 1.1 |
| $\mathcal{B}(\rho^\pm\pi^\mp)_\rho$ | | [11,12,23,24] | | 13.9 ± 2.7 |
| $\Delta(\rho^-\pi^+)$ | | [11,12,23,24] | | -2.9 ± 4.6 |
| $\mathcal{B}(K^{*+}\pi^-)$ | | [11,12,20,22–24,27–29] | | $14.4^{+4.4}_{-4.0}$ |
| $\mathcal{B}(K^{*-}\pi^+)$ | | [11,12,20,22–24,27–29] | | $18.7^{+4.8}_{-4.6}$ |

These expressions for γ and δ are subject to a fourfold ambiguity: $\{\gamma, \delta\} \rightarrow \{\delta, \gamma\}$, $\{-\gamma, -\delta\}$, or $\{-\delta, -\gamma\}$.

The charge-separated branching fractions $\mathcal{B}(K^{*+}\pi^-)$ and $\mathcal{B}(K^{*-}\pi^+)$ appearing in Eqs. (5)–(8) can be determined directly from $\mathcal{B}(K^{*\pm}\pi^\mp)$ and the CP asymmetry $\mathcal{A}_{\text{CP}}(K^{*\pm}\pi^\mp)$. In addition, SU(3) symmetry relates the rate difference $\Delta(K^{*-}\pi^+) \equiv \mathcal{B}(K^{*-}\pi^+) - \mathcal{B}(K^{*+}\pi^-)$ to the corresponding $\Delta S = 0$ quantity, $\Delta(\rho^-\pi^+) \equiv \mathcal{B}(\bar{B}^0 \rightarrow \rho^-\pi^+) - \mathcal{B}(B^0 \rightarrow \rho^+\pi^-)$ [9,10]:

$$\Delta(K^{*-}\pi^+) = - \left[\frac{f_{K^*} F_1^{B \rightarrow \pi}(m_{K^*}^2)}{f_\rho F_1^{B \rightarrow \pi}(m_\rho^2)} \right]^2 \Delta(\rho^-\pi^+). \quad (9)$$

To attain greater precision on $\mathcal{B}(K^{*+}\pi^-)$ and $\mathcal{B}(K^{*-}\pi^+)$, we combine information on $\Delta(\rho^-\pi^+)$ with the measurements of $\mathcal{B}(K^{*\pm}\pi^\mp)$ and $\mathcal{A}_{\text{CP}}(K^{*\pm}\pi^\mp)$ listed in Table 1. These inputs are given relative weights that minimize the uncertainties on $\mathcal{B}(K^{*+}\pi^-)$ and $\mathcal{B}(K^{*-}\pi^+)$, and we account for the correlation between CLEO's $\mathcal{A}_{\text{CP}}(K^{*\pm}\pi^\mp)$ and $\mathcal{B}(K^{*\pm}\pi^\mp)$ measurements, which are made with the same dataset and technique.

The BABAR analysis of $B \rightarrow \pi^+\pi^-\pi^0$ [11], which determines the CP asymmetry and dilution parameters A , C , and ΔC defined in Ref. [11], allows us to evaluate $\Delta(\rho^-\pi^+) = -(A + C + A\Delta C) \cdot \mathcal{B}(\rho^\pm\pi^\mp)$. We propagate the uncertainties on these parameters with their correlations [12] to obtain $\Delta(\rho^-\pi^+) = -(2.9 \pm 4.6) \times 10^{-6}$. Thus, taking the form factor ratio in Eq. (9) to be unity, we find $\mathcal{B}(K^{*+}\pi^-) = (14.4^{+4.4}_{-4.0}) \times 10^{-6}$ and $\mathcal{B}(K^{*-}\pi^+) =$

$(18.7^{+4.8}_{-4.6}) \times 10^{-6}$ with a correlation coefficient of 0.53. The correlation coefficients between $\Delta(\rho^- \pi^+)$ and these two branching fractions are 0.45 for $\mathcal{B}(K^{*+} \pi^-)$ and -0.50 for $\mathcal{B}(K^{*-} \pi^+)$.

A second method of estimating γ uses $B \rightarrow K^{*\pm} \pi^\mp$ and $B \rightarrow \phi K^\pm$. The possibility of constraining γ from these decays was first noticed by Gronau and Rosner [13], and the concrete formulation of this method was subsequently put forth by Gronau [14,15]. The SU(3) decomposition of the $B \rightarrow \phi K^\pm$ amplitude is $A(\phi K^\pm) = p'_P - \frac{1}{3} P'_{EW}$ to $\mathcal{O}(\lambda)$, where P'_{EW} denotes the electroweak penguin contribution. The weak phase of P'_{EW} is the same as that of p'_P , and its strong phase is expected to be the same as in t'_P because of the similarity of their flavor topologies [14,15]. Thus, the ratio of the CP-averaged branching fractions for $B \rightarrow K^{*\pm} \pi^\mp$ and $B \rightarrow \phi K^\pm$ provides a measure of γ up to a twofold ambiguity:

$$\cos \gamma = \frac{1}{2r \cos \delta} \left[1 + r^2 - R \left(1 - 2 \cos \delta \left| \frac{P'_{EW}}{3p'_P} \right| + \left| \frac{P'_{EW}}{3p'_P} \right|^2 \right) \right], \quad (10)$$

where $r \equiv |t'_P/p'_P|$, and

$$R \equiv \frac{|A(K^{*+} \pi^-)|^2 + |A(K^{*-} \pi^+)|^2}{|A(\phi K^+)|^2 + |A(\phi K^-)|^2}. \quad (11)$$

Both $B \rightarrow \phi K^\pm$ and $B \rightarrow \phi(\bar{K}^0)$ receive the same SU(3) amplitude contributions [2], so we can improve the statistical precision of Eq. (11) by combining both channels:

$$R = \frac{\mathcal{B}(K^{*\pm} \pi^\mp) \mathcal{F}}{[\sigma_0^2 \mathcal{B}(\phi K^\pm) + \sigma_+^2 \mathcal{B}(\phi \bar{K}^0) \mathcal{F}] / (\sigma_+^2 + \sigma_0^2)}, \quad (12)$$

where σ_+ and σ_0 refer to the uncertainties on $\mathcal{B}(\phi K^\pm)$ and $\mathcal{B}(\phi \bar{K}^0) \mathcal{F}$, respectively. To determine γ with this method, the size of δ must be known. It is believed, based on perturbative [16] and statistical [17] calculations, that $0^\circ < |\delta| < 90^\circ$. In the simulation, we fix $|P'_{EW}|$ to be $\frac{1}{2}|p'_P|$, as given by factorization calculations [13,18,19], and we evaluate the dependence of our results on $|P'_{EW}/p'_P|$ and δ .

In both of the above methods of constraining γ (involving Eqs. (4)–(8) and Eq. (10)), numerical values of $|t'_P|$ and $|p'_P|$ are given by other $B \rightarrow PV$ branching fractions [20]. The penguin amplitude is simply

$$|p'_P| = \sqrt{\mathcal{B}(\bar{K}^0 \pi^\pm)}. \quad (13)$$

The tree amplitude is taken from the $\Delta S = 0$ transition $B \rightarrow \rho^\pm \pi^\mp$ and related to the $|\Delta S| = 1$ amplitude through SU(3)-breaking factors:

$$|t'_P| = \left| \frac{V_{us}}{V_{ud}} \right| \frac{f_{K^*}}{f_\rho} |t_P|. \quad (14)$$

The experimentally measured $\mathcal{B}(\rho^\pm \pi^\mp)$ represents a sum over $B^0 \rightarrow \rho^\pm \pi^\mp$ and $\bar{B}^0 \rightarrow \rho^\pm \pi^\mp$ decays:

$$\mathcal{B}(\rho^\pm \pi^\mp) = \frac{1}{\mathcal{F}} (|t_P + p_P|^2 + |t_V + p_V|^2). \quad (15)$$

We isolate $|t_P + p_P|$ with the BABAR $B \rightarrow \pi^+ \pi^- \pi^0$ analysis [11], which provides

$$\mathcal{B}(\rho^\pm \pi^\mp)_P \equiv \frac{1}{2} [\mathcal{B}(B^0 \rightarrow \rho^+ \pi^-) + \mathcal{B}(\bar{B}^0 \rightarrow \rho^- \pi^+)] = \frac{1}{\mathcal{F}} |t_P + p_P|^2 \quad (16)$$

$$= \frac{1}{2} \mathcal{B}(\rho^\pm \pi^\mp) (1 + AC + \Delta C). \quad (17)$$

Based on the experimental inputs in Table 1, we find $\mathcal{B}(\rho^\pm \pi^\mp)_P = (13.9 \pm 2.7) \times 10^{-6}$ and a correlation coefficient between $\mathcal{B}(\rho^\pm \pi^\mp)_P$ and $\Delta(\rho^- \pi^+)$ of 0.05.

Extracting $|t_P|$ from $\mathcal{B}(\rho^\pm\pi^\mp)_P$ requires estimates of the magnitude and phase of p_P . Its magnitude is obtained from the analogous $|\Delta S| = 1$ amplitude:

$$|p_P| = \left| \frac{V_{td}}{V_{ts}} \right| \frac{f_\rho}{f_{K^*}} |p'_P|. \quad (18)$$

In the SU(3) limit, p_P and t_P have the same relative strong phase as that between p'_P and t'_P . Their relative weak phase, however, is $\gamma + \beta$, where γ is unknown, a priori. Therefore, we must solve for $\cos \gamma$ and $|t'_P|$ simultaneously.

Using CKM unitarity, the parameters $|V_{td}/V_{ts}|$ and β can be eliminated in favor of $|V_{ub}/V_{cb}|$ and γ via the relations

$$\left| \frac{V_{td}}{V_{ts}} \right|^2 = |V_{us}|^2 - 2|V_{us}| \left| \frac{V_{ub}}{V_{cb}} \right| \cos \gamma + \left| \frac{V_{ub}}{V_{cb}} \right|^2, \quad (19)$$

$$\sin \beta = \left| \frac{V_{ts}}{V_{td}} \right| \left| \frac{V_{ub}}{V_{cb}} \right| \sin \gamma, \quad (20)$$

$$\cos \beta = \left| \frac{V_{ts}}{V_{td}} \right| \left(|V_{us}| - \left| \frac{V_{ub}}{V_{cb}} \right| \cos \gamma \right). \quad (21)$$

By making these substitutions, we remove our dependence on $\sin 2\beta$ measurements involving $b \rightarrow c$ transitions and $B^0\text{--}\bar{B}^0$ mixing, and we remain sensitive to new physics which may affect these processes and charmless $b \rightarrow u, d, s$ transitions differently.

From the above unitarity relations and the CP-averaged branching fraction

$$\mathcal{B}(\rho^\pm\pi^\mp)_P = |p_P|^2 + |t_P|^2 + 2|p_P||t_P| \cos(\gamma + \beta) \cos \delta, \quad (22)$$

we find the following expression for $|t'_P|$:

$$|t'_P| = \left| \frac{V_{us}}{V_{ud}} \right| |p'_P| y \left\{ 1 \pm \sqrt{1 - \frac{1}{y^2} \left(|V_{us}|^2 - 2|V_{us}| \left| \frac{V_{ub}}{V_{cb}} \right| \cos \gamma + \left| \frac{V_{ub}}{V_{cb}} \right|^2 \right) + \frac{f_{K^*}^2 \mathcal{B}(\rho^\pm\pi^\mp)_P \mathcal{F}}{f_\rho^2 |p'_P|^2 y^2}} \right\}, \quad (23)$$

where

$$y \equiv \left(\left| \frac{V_{ub}}{V_{cb}} \right| - |V_{us}| \cos \gamma \right) \cos \delta. \quad (24)$$

Using Eq. (23) to calculate $|t'_P|$ from $B \rightarrow K^{*\pm}\pi^\mp$, $(\bar{K}^*)^0\pi^\pm$, and $\rho^\pm\pi^\mp$ depends on a choice of δ as well as knowledge of γ , and an iterative solution is required. The fixed strong phase appearing in Eq. (24) is distinct from the strong phase in the simulated quantities $\cos \gamma \cos \delta$ (Eq. (4)) and $\cos(\gamma \pm \delta)$ (Eqs. (5) and (6)). To distinguish these two strong phases, we denote the one entering Eq. (24) by $\delta_{t'_P}$. Below, we verify that the simulated values of $\cos \gamma \cos \delta$ and $\cos(\gamma \pm \delta)$ are insensitive to the choice of $\delta_{t'_P}$. In the second method of constraining $\cos \gamma$, we simulate Eq. (10) with $\delta_{t'_P} = \delta$.

Experimental measurements of the following quantities are given as input to the simulation: \mathcal{F} , f_{K^*}/f_ρ , $|V_{us}|$, $|V_{ud}|$, $|V_{ub}|$, $|V_{cb}|$, $\mathcal{A}_{\text{CP}}(K^{*\pm}\pi^\mp)$, the $B \rightarrow \rho^\pm\pi^\mp$ parameters A , C , and ΔC , and the CP-averaged branching fractions for $B \rightarrow \rho^\pm\pi^\mp$, $(\bar{K}^*)^0\pi^\pm$, $K^{*\pm}\pi^\mp$, ϕK^\pm , and $\phi(\bar{K}^*)^0$. These parameters are simulated with Gaussian or bifurcated Gaussian (different widths above and below the peak) distributions, and their values are summarized in Table 1. The input that contributes the largest uncertainty to γ is the $B \rightarrow K^{*\pm}\pi^\mp$ branching fraction.

For the five branching fractions, we combine all publicly presented measurements, with statistical and systematic errors added in quadrature. Where possible, the contribution from f_{+-}/f_{00} to the systematic error has been removed, since it is included coherently in the simulation. We neglect all other correlations among the systematic errors.

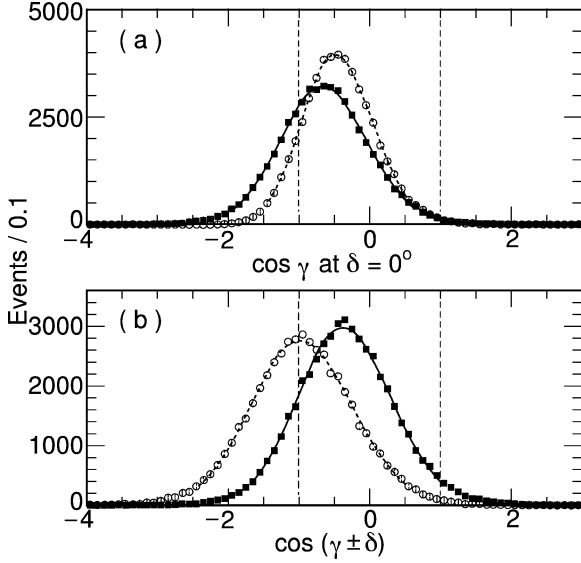


Fig. 1. Simulated distributions of $\cos \gamma \cos \delta$ from Eq. (4) [(a), solid squares], $\cos(\gamma + \delta)$ from Eq. (5) [(b), solid squares], and $\cos(\gamma - \delta)$ from Eq. (6) [(b), open circles] using $B \rightarrow K^{*\pm}\pi^\mp$, as well as $\cos \gamma$ from Eq. (10) [(a), open circles] using $B \rightarrow K^{*\pm}\pi^\mp$ and $B \rightarrow \phi(\bar{K})$ with $|P'_{EW}/p'_P| = 0.5$, all with $\delta_{t'_P} = 0^\circ$. Overlaid on the histograms are the fits to bifurcated Gaussians. The dashed lines demarcate the physical region.

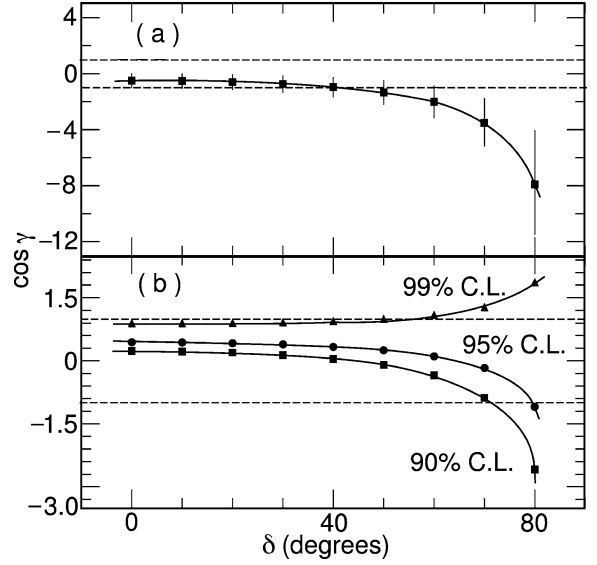


Fig. 2. Peak values (a) of and upper limits (b) on $\cos \gamma$ from Eq. (10) as a function of $\delta = \delta_{t'_P}$, using $B \rightarrow K^{*\pm}\pi^\mp$ and $B \rightarrow \phi(\bar{K})$ with $|P'_{EW}/p'_P| = 0.5$. The asymmetric errors on the peak values give the bifurcated Gaussian widths of the simulated distributions. The dashed lines demarcate the physical region.

Fig. 1 shows the simulated distribution of $\cos \gamma \cos \delta$ from Eq. (4), with $\delta_{t'_P} = 0^\circ$. Fitting this distribution to a bifurcated Gaussian yields the measurement $\cos \gamma \cos \delta = -0.68^{+0.63}_{-0.59}$, which suggests constructive interference between t'_P and p'_P . The 90%, 95%, and 99% C.L. upper limits on $\cos \gamma \cos \delta|_{\delta_{t'_P}=0^\circ}$ are 0.16, 0.42, and 0.94, respectively. Based on the smallness of direct CP asymmetries in $B \rightarrow K\pi$, one can infer a strong phase between tree and penguin amplitudes in these decays of $(8 \pm 10)^\circ$ [33]. If the strong phases in $B \rightarrow PV$ decays are as small as in two-pseudoscalar (PP) final states, then our analysis weakly favors $\cos \gamma < 0$. The variation of $\cos \gamma \cos \delta$ with $\cos \delta_{t'_P}$ is roughly linear, with a slope of $d \cos \gamma \cos \delta / d \cos \delta_{t'_P} = 0.11$. Also shown in Fig. 1 is the distribution of $\cos \gamma$ from Eq. (10), with $\delta = \delta_{t'_P} = 0^\circ$ and $|P'_{EW}/p'_P| = 0.5$. Here, we obtain $\cos \gamma|_{\delta=0^\circ} = -0.50^{+0.53}_{-0.47}$ and 90%, 95%, and 99% C.L. upper limits on $\cos \gamma|_{\delta=0^\circ}$ of 0.23, 0.44, and 0.89.

From Fig. 1, we also find $\cos(\gamma + \delta)$ and $\cos(\gamma - \delta)$ from Eqs. (5) and (6) to be $-0.39^{+0.69}_{-0.63}$ and $-0.99^{+0.74}_{-0.69}$, respectively, with a correlation coefficient of 0.61. Considering only the 47% of trials where both quantities acquire physical values, the distributions of the weak and strong phases imply $\gamma = (113^{+20}_{-30})^\circ$ and $\delta = (-13 \pm 17)^\circ$, with a correlation coefficient of 7×10^{-5} . Because of the fourfold ambiguity of the γ/δ system, we fix $\delta_{t'_P}$ to 0° rather than equating it to the simulated value of δ . The variations of $\cos(\gamma + \delta)$ and $\cos(\gamma - \delta)$ with $\cos \delta_{t'_P}$ are given by $d \cos(\gamma + \delta) / d \cos \delta_{t'_P} = 0.06$ and $d \cos(\gamma - \delta) / d \cos \delta_{t'_P} = 0.13$. The values of γ and δ both change by less than 2° between $\delta_{t'_P} = 0^\circ$ and $\delta_{t'_P} = 80^\circ$.

Fig. 2 shows the dependence of $\cos \gamma$ from Eq. (10) on $\delta = \delta_{t'_P}$, with $|P'_{EW}/p'_P| = 0.5$. The peak values are plotted with asymmetric error bars representing the widths of the bifurcated Gaussian distributions. By demanding that $\cos \gamma$ peak in the physical region, one can infer that $|\delta| < 41^\circ$. The variation of $\cos \gamma$ with $|P'_{EW}/p'_P|$ is linear, with a slope $d \cos \gamma / d |P'_{EW}/p'_P| = 0.28 - 1.51 \cos \delta_{t'_P}$. Incorporating the $B \rightarrow \phi(\bar{K})$ decays in the measurement

of γ results in greater precision than using $B \rightarrow K^{*\pm}\pi^\mp$ alone, but the theoretical uncertainties incurred are also larger.

Using the simulation of Eq. (4), we also determine the ratio $r = 0.30_{-0.05}^{+0.07}$ at $\delta_{t'_p} = 0^\circ$ with a $\delta_{t'_p}$ dependence given by $r = 0.25 + 0.09 \cos \delta_{t'_p} - 0.04 \cos^2 \delta_{t'_p}$. The inverse ratio for $\Delta S = 0$ decays, $|p_P/t_P| = \frac{1}{r} |V_{us}/V_{ud}| |V_{td}/V_{ts}|$, is found to be $0.43 - 0.49 \cos \delta_{t'_p} + 0.26 \cos^2 \delta_{t'_p}$, which takes the value $0.20_{-0.02}^{+0.03}$ at $\delta_{t'_p} = 0^\circ$.

The widths of the generated distributions presented above are dominated by experimental uncertainties on the input branching fractions, $\mathcal{A}_{\text{CP}}(K^{*\pm}\pi^\mp)$, A , C , and ΔC . We study the improvement in the resolutions of $\cos \gamma \cos \delta$, $\cos(\gamma \pm \delta)$, and $\cos \gamma|_{\delta=0^\circ}$, collectively denoted by $\hat{\sigma}_{\cos \gamma}$, as these measurement uncertainties are reduced while maintaining the central values at their current positions, with $\delta_{t'_p} = 0^\circ$ and $|P'_{\text{EW}}/p'_P| = 0.5$. It is found that $\hat{\sigma}_{\cos \gamma}$ scales with the size of the experimental uncertainties until the latter reach 10% of their current values, where the resolution of γ is $\mathcal{O}(10^\circ)$. At this point, $\hat{\sigma}_{\cos \gamma}$ begins to be dominated by the uncertainty on \mathcal{F} , and only by lowering $\sigma_{\mathcal{F}}$ can $\hat{\sigma}_{\cos \gamma}$ be reduced any further.

We have modeled SU(3) symmetry breaking effects in ratios of $\Delta S = 0$ to $|\Delta S| = 1$ amplitudes with the purely real ratio of decay constants f_{K^*}/f_ρ . Repeating the simulation without SU(3) breaking (i.e., with $f_{K^*}/f_\rho = 1$) results in changes to $\cos \gamma \cos \delta$, $\cos(\gamma \pm \delta)$, and $\cos \gamma|_{\delta=0^\circ}$ of 0.05 or smaller. Recent studies based on QCD-improved factorization [10,34] have suggested that SU(3) breaking could be as large as 30% and that the amplitude ratios may possess a small complex phase. To probe the impact of such effects, we reinterpret f_{K^*}/f_ρ as a phenomenological parameter and scale it by $\pm 30\%$ of the value given in Table 1, neglecting any possible complex phases. We find shifts of $_{-0.32}^{+0.21}$ in $\cos \gamma \cos \delta$, $_{-0.45}^{+0.32}$ in $\cos(\gamma + \delta)$, $_{-0.18}^{+0.12}$ in $\cos(\gamma - \delta)$, and $_{-0.30}^{+0.19}$ in $\cos \gamma|_{\delta=0^\circ}$. Thus, in this conservative estimate, SU(3) breaking effects are roughly 15–70% of the current experimental uncertainties. To obtain meaningful constraints on γ , future experimental advances must be accompanied by an improved understanding of SU(3) breaking.

In conclusion, we have formed constraints on γ as a function of δ and $|P'_{\text{EW}}/p'_P|$ using branching fractions of and CP asymmetries in $B \rightarrow PV$ decays. At present, experimental uncertainties overwhelm the theoretical uncertainties arising from the model dependence of $|V_{ub}|$ and $|V_{cb}|$, but they are the same order of magnitude as the uncertainties in SU(3) symmetry breaking. For strong phases of $\mathcal{O}(10^\circ)$ or smaller, our analysis favors $\cos \gamma < 0$, which agrees with indications from $B \rightarrow PP$ decays [33,35,36]. However, the current experimental precision does not yet permit a stringent comparison with fits reliant upon B and K mixing.

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