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6D Higgsless Standard Model

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Abstract

We present a six-dimensional Higgsless Standard Model with a realistic gauge sector. The model uses only the Standard Model gauge group $SU(2)_L \times U(1)_Y$ with the gauge bosons propagating in flat extra dimensions compactified on a rectangle. The electroweak symmetry is broken by boundary conditions, and the correct splitting between the W and Z gauge boson masses can be arranged by suitable choice of the compactification scales. The higher Kaluza–Klein excitations of the gauge bosons decouple from the effective low-energy theory due to dominant brane kinetic terms. The model has the following two key features compared to five-dimensional models. The dimensional couplings in the bulk Lagrangian, responsible for electroweak symmetry breaking using mixed boundary conditions, are of order the electroweak scale. Moreover, with respect to "oblique" corrections, the agreement with the precision electroweak parameters is improved compared to five-dimensional warped or flat space models. We also argue that the calculability of Higgsless models can be ameliorated in more than five dimensions.

1. Introduction

The Standard Model (SM) of electroweak interactions [1], based on the gauge symmetry group $SU(2)_L \times U(1)_Y$, provides a highly successful description of electroweak precision tests (EWPT) [2,3]. One fundamental ingredient of the SM is the Higgs mechanism [4], which accomplishes electroweak symmetry breaking (EWSB) and at high energies unitarizes massive $W^{\pm}$ and Z scattering through the presence of the scalar Higgs doublet [5]. However, no fundamental scalar particle has been observed yet in Nature, and as long as there is no direct evidence for the existence of the Higgs boson, the actual mechanism of EWSB remains a mystery. In case the Higgs boson will also not be found at the Tevatron or the LHC, it will therefore be necessary to consider alternative ways to achieve EWSB without a Higgs.

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It is well known, that in extra dimensions gauge symmetries can also be broken by boundary conditions (BCs) on a compact space [6]. Here, a geometric "Higgs" mechanism ensures tree-level unitarity of longitudinal gauge boson scattering through a tower of Kaluza–Klein (KK) [7] excitations [8]. The SM in $(\text{TeV})^{-1}$-sized extra dimensions with gauge symmetry breaking by BCs, in connection with the problem of breaking supersymmetry in string theory, was first considered in Ref. [9]. In theories using only usual orbifold BCs [10] for gauge symmetry breaking, however, it is generally difficult to reduce the rank of a gauge group, as it would be required for realistic EWSB. Rank reduction, on the other hand, is easily achieved in the recently proposed new type of Higgsless models for EWSB [11–15], which employ mixed (neither Dirichlet nor Neumann) BCs.[1] The mixed BCs, when consistent with the variation of a gauge invariant action, correspond to a soft breaking of the gauge symmetry, since they can be ultraviolet completed by a boundary Higgs field.

The original model for Higgsless EWSB [11] is an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory compactified on an interval $[0, \pi R]$ in five-dimensional (5D) flat space. At one end of the interval, $SU(2)_R \times U(1)_{B-L}$ is broken to $U(1)_Y$. At the other end, $SU(2)_L \times SU(2)_R$ is broken to the diagonal subgroup $SU(2)_D$, thereby leaving only $U(1)_Q$ of electromagnetism unbroken in the effective four-dimensional (4D) theory. Although this model exhibited some similarities with the SM, the ρ parameter deviated from unity by $\sim 10\%$ and the lowest KK excitations of the $W^\pm$ and Z were too light (~ 240 GeV) to be in agreement with experiment. These problems have later been resolved by considering the setup in the truncated anti-de Sitter (AdS) space of the Randall–Sundrum model [17]. Here, the generators broken on the Planck brane can be associated via the AdS/CFT correspondence [18] in the 4D dual [19] theory with a global custodial $SU(2)$ symmetry [20], while the electroweak symmetry has been broken by the presence of the TeV brane alone [12]. As a consequence, in the strongly coupled 4D theory, violation of custodial isospin remains (even after inclusion of radiative corrections) only of order $\sim 1\%$, while the higher KK resonances of the gauge bosons would decouple below ~ 1 TeV [12,13]. In this framework, it is possible to generate realistic quark and lepton masses with viable couplings to $W^\pm$ and Z, when the fermions propagate in the bulk [13,14]. Based on the same gauge group, similar effects can be realized in 5D flat space [15], when 4D brane kinetic terms [21–23] dominate the contribution from the bulk. In fact, brane kinetic terms seem also to be required in Higgsless warped space models [24], to evade disagreement with EWPT due to tree-level "oblique" corrections [25–27].

In 5D Higgsless models, a ρ parameter close to unity is achieved at the expense of enlarging the SM gauge group by an additional gauge group $SU(2)_R$, which introduces a gauged custodial symmetry in the bulk. Inspired by dimensional deconstruction [28,29], one can consider the $SU(2)_L \times SU(2)_R$ subgroup of the model as belonging to a chain of 5D gauge theories with product group structure $SU(2)_1 \times SU(2)_2 \times \cdots \times SU(2)_N \supset SU(2)_L \times SU(2)_R$, which is broken down to $SU(2)_D$ by BCs (for a discussion of Higgsless EWSB in deconstruction see Ref. [30]). From the deconstruction point of view, such a product group may be reduced to a single six-dimensional (6D) parent gauge group $SU(2)_L$, while keeping essential features of the corresponding 5D theory. Hence, it should be possible to obtain consistent 6D Higgsless models of EWSB, which are based only on the SM gauge group $SU(2)_L \times U(1)_Y$ and allow the ρ parameter to be set equal to unity. There is yet another advantage of going beyond five dimensions. In more than five dimensions, the physical space can be reduced (e.g., by orbifold BCs) to a domain smaller than the periodicity of the wavefunctions. As a result, the S, T, and U parameters [25] would become suppressed by higher powers of the loop expansion parameter of the theory, thereby potentially improving the calculability of Higgsless models.

In this Letter, we consider a Higgsless model for EWSB in six dimensions, which is based only on the SM gauge group $SU(2)_L \times U(1)_Y$, where the gauge bosons propagate in the bulk. The model is formulated in flat space with the two extra dimensions compactified on a rectangle and EWSB is achieved by imposing consistent BCs. The higher KK resonances of $W^\pm$ and Z decouple below ~ 1 TeV through the presence of a dominant 4D brane induced gauge kinetic term. The ρ parameter is arbitrary and can be set exactly to one by an appropriate

[1] For GUT breaking with mixed BCs see Ref. [16].

choice of the bulk gauge couplings and compactification scales. Unlike in the 5D theory, the mass scale of the lightest gauge bosons W and Z is solely set by the dimensionful bulk couplings, which (upon compactification via mixed BCs) are responsible for EWSB. We calculate the tree-level oblique corrections to the S, T, and U parameters and find that they are in better agreement with data than in proposed 5D warped and flat Higgsless models. Non-oblique corrections, however, can generally lead to a tension between the bottom quark mass and the $Z \to b\bar{b}$ coupling, which could be modified at the level of current experimental uncertainties. By considering the scattering of a scalar propagating in S^1/Z_2 and $S^1/(Z_2 \times Z_2')$ extra dimensions, we estimate the raising of the strong coupling scale, which could improve the calculability of Higgsless models formulated on these manifolds.

The Letter is organized as follows. In Section 2, we introduce the 6D model on a rectangle and discuss the symmetry breaking by BCs. In Section 3, we determine the wavefunctions in the presence of the brane terms, vacuum polarizations and KK spectra of the gauge bosons. We compare the oblique corrections to EWPT in Section 4. Non-oblique corrections of the SM couplings due to the generation of heavy fermion masses are then discussed in Section 5. Next, in Section 6, we estimate the strong coupling scale on different orbifold extra dimensions and outline potential implications for an improved calculability of Higgsless models. Finally, in Section 7, we present our summary and conclusions.

2. The model

Let us consider a 6D $SU(2)_L \times U(1)_Y$ gauge theory in a flat space–time background, where the two extra spatial dimensions are compactified on a rectangle.[2] The coordinates in the 6D space are written as $z_M = (x_\mu, y_m)$, where the 6D Lorentz indices are denoted by capital Roman letters $M = 0, 1, 2, 3, 5, 6$, while the usual 4D Lorentz indices are symbolized by Greek letters $\mu = 0, 1, 2, 3$, and the coordinates y_m $(m = 1, 2)$ describe the fifth and sixth dimension.[3] The physical space is thus defined by $0 \leqslant y_1 \leqslant \pi R_1$ and $0 \leqslant y_2 \leqslant \pi R_2$, where R_1 and R_2 are the compactification radii of a torus T^2, which is obtained by identifying the points of the two-dimensional plane R^2 under the actions $T_5 : (y_1, y_2) \to (y_1 + 2\pi R_1, y_2)$ and $T_6 : (y_1, y_2) \to (y_1, y_2 + 2\pi R_2)$. We denote the $SU(2)_L$ and $U(1)_Y$ gauge bosons in the bulk, respectively, by $A^a_M(z_M)$ $(a = 1, 2, 3$ is the gauge index) and $B_M(z_M)$. The action of the gauge fields in our model is given by

$$\mathcal{S} = \int d^4x \int_0^{\pi R_1} dy_1 \int_0^{\pi R_2} dy_2 \left(\mathcal{L}_6 + \delta(y_1)\delta(y_2)\mathcal{L}_0\right), \tag{1}$$

where $\mathcal{L}_6$ is a 6D bulk gauge kinetic term and $\mathcal{L}_0$ is a 4D brane gauge kinetic term localized at $(y_1, y_2) = (0, 0)$, which read, respectively,

$$\mathcal{L}_6 = -\frac{M_L^2}{4} F^a_{MN} F^{MNa} - \frac{M_Y^2}{4} B_{MN} B^{MN}, \qquad \mathcal{L}_0 = -\frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu a} - \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu}, \tag{2}$$

with field strengths $F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + f^{abc} A^b_M A^c_N$ (f^{abc} is the structure constant) and $B_{MN} = \partial_M B_N - \partial_N B_M$. In Eq. (2), the quantities M_L and M_Y have mass dimension $+1$, while g and g' are dimensionless. Since the boundaries of the manifold break translational invariance and are "singled out" with respect to the points in the interior of the rectangle, brane terms like $\mathcal{L}_0$ can be produced by quantum loop effects [21,22] or arise from classical singularities in the limit of vanishing brane thickness [23].

Unlike in five dimensions (for a discussion of the $\xi \to \infty$ limit in generalized 5D R_ξ gauges see, e.g., Ref. [32] and also Ref. [11]), we cannot go to a unitary gauge where all fields $A^a_{5,6}$ $(a = 1, 2, 3)$ and $B_{5,6}$ are identically

[2] Chiral compactification on a square has recently been considered in Ref. [31].

[3] For the metric we choose a signature $(+, -, -, -, -, -)$.

set to zero. Instead, there will remain after dimensional reduction one combination of physical scalar fields in the spectrum.[4] To make these scalars sufficiently heavier than the Lee–Quigg–Thacker bound of ≈ 2 TeV, we can assume, e.g., a seventh dimension compactified on $\mathcal{S}^1/Z_2$ with compactification radius $R_3 \lesssim R_1, R_2$. By setting $A^a_{5,6,7} = B_{5,6,7} = 0$ (A^a_7 and B_7 are the seventh components of the gauge fields) on all boundaries of this manifold, the associated scalars can acquire for compactification scales $R_1^{-1}, R_2^{-1} \simeq 1$–$2$ TeV, masses well above 2 TeV. Therefore, at low energies $\lesssim 2$–3 TeV, we have a model without any *light* scalars and will, in what follows, neglect the heavy scalar degrees of freedom.

Since the Lagrangian in Eq. (2) does not contain any explicit gauge symmetry breaking, we can obtain consistent new BCs on the boundaries by requiring the variation of the action to be zero. Variation of the action in Eq. (2) yields after partial integration

$$\begin{aligned}
\delta S = {} & \int d^4x \int_{y_1=0}^{\pi R_1} dy_1 \int_{y_2=0}^{\pi R_2} dy_2 \left[M_L^2\left(\partial_M F^{aM\mu} - f^{abc}F^{bM\mu}A_M^c\right)\delta A_\mu^a + M_Y^2 \partial_M B^{M\mu}\delta B_\mu\right] \\
& + \int d^4x \int_{y_2=0}^{\pi R_2} dy_2 \left[M_L^2 F^a_{5\mu}\delta A^{a\mu} + M_Y^2 B_{5\mu}\delta B^\mu\right]_{y_1=0}^{\pi R_1} \\
& + \int d^4x \int_{y_1=0}^{\pi R_1} dy_1 \left[M_L^2 F^a_{6\mu}\delta A^{a\mu} + M_Y^2 B_{6\mu}\delta B^\mu\right]_{y_2=0}^{\pi R_2} \\
& + \int d^4x \left[\frac{1}{g^2}\left(\partial_\mu F^{a\mu\nu} - f^{abc}F^{b\mu\nu}A_\mu^c\right)\delta A_\nu^c + \frac{1}{g'^2}\partial_\mu B^{\mu\nu}\delta B_\nu\right]_{(y_1,y_2)=(0,0)} = 0,
\end{aligned} \tag{3}$$

where we have (as usual) assumed that the gauge fields and their derivatives go to zero for $x_\mu \to \infty$. The bulk terms in the first line in Eq. (3), lead to the familiar bulk equations of motion. Moreover, since the minimization of the action requires the boundary terms to vanish as well, we obtain from the second and third line in Eq. (3) a set of consistent BCs for the bulk fields.

We break the electroweak symmetry $SU(2)_L \times U(1)_Y \to U(1)_Q$ by imposing on two of the boundaries following BCs:

$$\text{at } y_1 = \pi R_1: \quad A^1_\mu = 0, \quad A^2_\mu = 0, \tag{4a}$$

$$\text{at } y_2 = \pi R_2: \quad \partial_{y_2}\left(M_L^2 A^3_\mu + M_Y^2 B_\mu\right) = 0, \quad A^3_\mu - B_\mu = 0. \tag{4b}$$

The Dirichlet BCs in Eq. (4a) break $SU(2)_L \to U(1)_{I_3}$, where $U(1)_{I_3}$ is the $U(1)$ subgroup associated with the third component of weak isospin I_3. The BCs in Eq. (4b) break $U(1)_{I_3} \times U(1)_Y \to U(1)_Q$, leaving only $U(1)_Q$ unbroken on the entire rectangle (see Fig. 1). Note, in Eq. (4b), that the first BC involving the derivative with respect to y_2 actually follows from the second BC $\delta A^3_\mu = \delta B_\mu$ by minimization of the action. The gauge groups $U(1)_{I_3}$ and $U(1)_{I_3} \times U(1)_Y$ remain unbroken at the boundaries $y_1 = 0$ and $y_2 = 0$, respectively. Locally, at the fixed point $(y_1, y_2) = (0, 0)$, $SU(2)_L \times U(1)_Y$ is unbroken. We can restrict ourselves, for simplicity, to the solutions which are relevant to EWSB, by imposing on the other two boundaries the following Dirichlet BCs:

$$\text{at } y_1 = 0: \quad A^{1,2}_\mu(z_M) = \bar{A}^{1,2}_\mu(x_\mu), \tag{5a}$$

$$\text{at } y_2 = 0: \quad A^3_\mu(z_M) = \bar{A}^3_\mu(x_\mu), \quad B_\mu(z_M) = \bar{B}_\mu(x_\mu), \tag{5b}$$

[4] We thank H. Murayama and M. Serone for pointing out this fact.

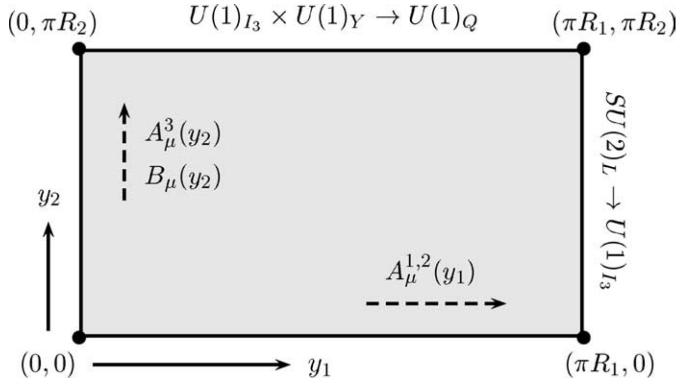

Fig. 1. Symmetry breaking of $SU(2)_L \times U(1)_Y$ on the rectangle. At one boundary $y_1 = \pi R_1$, $SU(2)_L$ is broken to $U(1)_{I_3}$ while on the boundary $y_2 = \pi R_2$ the subgroup $U(1)_{I_3} \times U(1)_Y$ is broken to $U(1)_Q$, which leaves only $U(1)_Q$ unbroken on the entire rectangle. Locally, at the fixed point $(0,0)$, $SU(2)_L \times U(1)_Y$ remains unbroken. The dashed arrows indicate the propagation of the lowest resonances of the gauge bosons.

where the bar indicates a boundary field. The Dirichlet BCs in Eqs. (5a), (5b) require $A^{1,2}_\mu$ to be independent of y_2, while A^3_μ and B_μ become independent of y_1, such that we can generally write $A^{1,2}_\mu = A^{1,2}(x_\mu, y_1)$, $A^3_\mu = A^3_\mu(x_\mu, y_2)$, and $B_\mu = B_\mu(x_\mu, y_2)$. For the transverse[5] components of the gauge fields the bulk equations of motion then take the forms

$$\big(p^2 + \partial^2_{y_1}\big) A^{1,2}_\mu(x_\mu, y_1) = 0, \qquad \big(p^2 + \partial^2_{y_2}\big) A^3_\mu(x_\mu, y_2) = 0, \qquad \big(p^2 + \partial^2_{y_2}\big) B_\mu(x_\mu, y_2) = 0, \tag{6}$$

where $p^2 = p_\mu p^\mu$ and $p_\mu = i\partial_\mu$ is the momentum in the uncompactified 4D space. Since we assume all the gauge couplings to be small, we will, in what follows, treat A^a_μ approximately as a "free" field (i.e., without self interaction) and drop all cubic and quartic terms in A^a_μ.

We assume that the fermions, in the first approximation, are localized on the brane at $(y_1, y_2) = (0,0)$, away from the walls of electroweak symmetry breaking. This choice will avoid any unwanted non-oblique corrections to the electroweak precision parameters.

3. Effective theory

The total effective 4D Lagrangian in the compactified theory $\mathcal{L}_{\text{total}}$ can be written as $\mathcal{L}_{\text{total}} = \mathcal{L}_0 + \mathcal{L}_{\text{eff}}$, where $\mathcal{L}_{\text{eff}} = \int_0^{\pi R_1} dy_1 \int_0^{\pi R_2} dy_2\, \mathcal{L}_6$ denotes the contribution from the bulk, which follows from integrating out the extra dimensions. After partial integration along the y_1 and y_2 directions, we obtain for $\mathcal{L}_{\text{eff}}$ the non-vanishing boundary term

$$\mathcal{L}_{\text{eff}} = -M_L^2 \pi R_2 \big[\bar{A}^1_\mu \partial_{y_1} A^{1\mu} + \bar{A}^2_\mu \partial_{y_1} A^{2\mu}\big]_{y_1=0} - \pi R_1 \big[M_L^2 \bar{A}^3_\mu \partial_{y_2} A^{3\mu} + M_Y^2 \bar{B}_\mu \partial_{y_2} B^\mu\big]_{y_2=0}, \tag{7}$$

where we have applied the bulk equations of motion and eliminated the terms from the boundaries at $y_1 = \pi R_1$ and $y_2 = \pi R_2$ by virtue of the BCs in Eqs. (4). Notice, that in arriving at Eq. (7) we have redefined the bulk gauge fields as $A_\mu \to A'_\mu \equiv A_\mu/\sqrt{2}$ to canonically normalize the kinetic energy terms of the KK modes. In order to determine $\mathcal{L}_{\text{total}}$ explicitly, we first solve the equations of motion in Eq. (6) and insert the solutions into the expression for

[5] Note that $\partial_M F^{aM\mu} = p^2 P_{\mu\nu}(p) A^{a\mu} + (\partial^2_{y_1} + \partial^2_{y_2}) A^a_\nu = 0$, where $P_{\mu\nu}(p) = g_{\mu\nu} - p_\mu p_\nu / p^2$ is the operator projecting onto transverse states.

$\mathcal{L}_{\mathrm{eff}}$ in Eq. (7). The most general solutions for Eqs. (6) can be written as

$$A_\mu^{1,2}(x_\mu, y_1) = \bar{A}_\mu^{1,2}(x_\mu)\cos(py_1) + b_\mu^{1,2}(x_\mu)\sin(py_1), \tag{8a}$$

$$A_\mu^{3}(x_\mu, y_2) = \bar{A}_\mu^{3}(x_\mu)\cos(py_2) + b_\mu^{3}(x_\mu)\sin(py_2), \tag{8b}$$

$$B_\mu(x_\mu, y_2) = \bar{B}_\mu(x_\mu)\cos(py_2) + b_\mu^{Y}(x_\mu)\sin(py_2), \tag{8c}$$

where $p = \sqrt{p_\mu p^\mu}$ and we have already applied the BCs in Eqs. (5). The coefficients $b_\mu^a(x_\mu)$ and $b_\mu^Y(x_\mu)$ are then determined from the BCs in Eqs. (4). For $b_\mu^{1,2}(x_\mu)$, e.g., we find from the BCs in Eq. (4a) that $b_\mu^{1,2}(x_\mu) = -\bar{A}_\mu^{1,2}(x_\mu)\cot(p\pi R_1)$ and hence one obtains

$$A_\mu^{1,2}(x_\mu, y_1) = \bar{A}_\mu^{1,2}(x_\mu)\bigl[\cos(py_1) - \cot(p\pi R_1)\sin(py_1)\bigr]. \tag{9a}$$

In a similar way, one arrives after some calculation at the solutions

$$\begin{aligned} A_\mu^{3}(x_\mu, y_2) &= \bar{A}_\mu^{3}(x_\mu)\left[\cos(py_2) + \frac{M_L^2\tan(p\pi R_2) - M_Y^2\cot(p\pi R_2)}{M_L^2 + M_Y^2}\sin(py_2)\right] \\ &\quad + \bar{B}_\mu(x_\mu)\frac{M_Y^2\tan(p\pi R_2) + M_Y^2\cot(p\pi R_2)}{M_L^2 + M_Y^2}\sin(py_2), \end{aligned} \tag{9b}$$

$$\begin{aligned} B_\mu(x_\mu, y_2) &= \bar{A}_\mu^{3}(x_\mu)\frac{M_L^2\tan(p\pi R_2) + M_L^2\cot(p\pi R_2)}{M_L^2 + M_Y^2}\sin(py_2) \\ &\quad + \bar{B}_\mu(x_\mu)\left[\cos(py_2) + \frac{M_Y^2\tan(p\pi R_2) - M_L^2\cot(p\pi R_2)}{M_L^2 + M_Y^2}\sin(py_2)\right]. \end{aligned} \tag{9c}$$

Inserting the wavefunctions in Eqs. (9) into the effective Lagrangian in Eq. (7), we can rewrite $\mathcal{L}_{\mathrm{eff}}$ as

$$\mathcal{L}_{\mathrm{eff}} = \bar{A}_\mu^a \Sigma_{aa}(p^2)\bar{A}^{a\mu} + \bar{A}_\mu^3 \Sigma_{3B}(p^2)\bar{B}^\mu + \bar{B}_\mu \Sigma_{BB}(p^2)\bar{B}^\mu, \tag{10}$$

where $(aa) = (11), (22)$, and (33) and the momentum-dependent coefficients Σ are given by

$$\begin{aligned} \Sigma_{11}\bigl(p^2\bigr) &= \Sigma_{22}\bigl(p^2\bigr) = \pi R_2 M_L^2 p\cot(p\pi R_1), \\ \Sigma_{33}\bigl(p^2\bigr) &= -\pi R_1 M_L^2 p\frac{M_L^2\tan(p\pi R_2) - M_Y^2\cot(p\pi R_2)}{M_L^2 + M_Y^2}, \\ \Sigma_{3B}\bigl(p^2\bigr) &= -2\pi R_1 M_L^2 M_Y^2 p\frac{\tan(p\pi R_2) + \cot(p\pi R_2)}{M_L^2 + M_Y^2}, \\ \Sigma_{BB}\bigl(p^2\bigr) &= -\pi R_1 M_Y^2 p\frac{M_Y^2\tan(p\pi R_2) - M_L^2\cot(p\pi R_2)}{M_L^2 + M_Y^2}. \end{aligned} \tag{11}$$

The Σ's can be viewed as the electroweak vacuum polarization amplitudes which summarize in the low energy theory the effect of the symmetry breaking sector. The presence of these terms leads at tree level to oblique corrections (as opposed to vertex corrections and box diagrams) of the gauge boson propagators and affects electroweak precision measurements [25,26]. Since $\mathcal{L}_{\mathrm{eff}}$ in Eq. (7) generates effective mass terms for the gauge bosons in the 4D theory,[6] the KK masses of the $W^\pm$ bosons are found from the zeros of the inverse propagator as given by the solutions of the equation

$$\Sigma_{11}\bigl(p^2\bigr) - \frac{p^2}{2g^2} = 0. \tag{12}$$

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[6] For an effective field theory approach to oblique corrections see, e.g., Ref. [27].

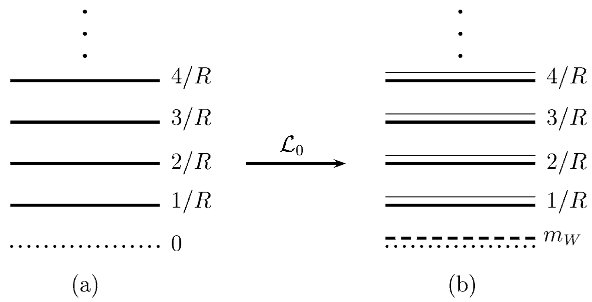

Fig. 2. Effect of the brane kinetic terms $\mathcal{L}_0$ on the KK spectrum of the gauge bosons (for example, of $W^\pm$). Solid lines represent massive excitations, the bottom dotted lines would correspond to the zero modes which have been removed by the BCs. Without the brane terms (a), the lowest KK excitations are of order $1/R \simeq 1$ TeV. After switching on the dominant brane kinetic terms (b), the zero modes are approximately "restored" with a small mass $m_W \ll 1/R$ (dashed line), while the higher KK-levels receive small corrections to their masses (thin solid lines) and decouple below ~ 1 TeV.

To determine the KK masses of the gauge bosons, we will from now on assume that the brane terms $\mathcal{L}_0$ dominate the bulk kinetic terms, i.e., we take $1/g^2, 1/g'^2 \gg (M_{L,Y}\pi)^2 R_1 R_2$. As a result, we find for the $W^\pm$'s the mass spectrum

$$m_n = \frac{n}{R_1}\left(1 + \frac{2g^2 M_L^2 R_1 R_2}{n^2} + \cdots\right), \quad n = 1, 2, \ldots,$$
$$m_0^2 = \frac{2g^2 M_L^2 R_2}{R_1} + \mathcal{O}\big(g^4 M_L^4 R_2^2\big) = m_W^2, \tag{13}$$

where we identify the lightest state with mass m_0 with the $W^\pm$. Observe in Eqs. (13), that the inclusion of the brane kinetic terms $\mathcal{L}_0$ for $1/R_1, 1/R_2 \gtrsim \mathcal{O}(\text{ TeV})$ leads to a decoupling of the higher KK-modes with masses m_n $(n > 0)$ from the electroweak scale, leaving only the $W^\pm$ states with a small mass m_0 in the low-energy theory (see Fig. 2). Note that a similar effect has been found for warped models in Ref. [33].

The calculation of the mass of the Z boson goes along the same lines as for $W^\pm$, but requires, due to the mixing of $\bar{A}^3_\mu$ with $\bar{B}_\mu$ in Eq. (10), the diagonalization of the kinetic matrix

$$M_{\text{kin}} = \begin{pmatrix} \Sigma_{33}(p^2) - \frac{p^2}{2g^2} & \frac{1}{2}\Sigma_{3B}(p^2) \\ \frac{1}{2}\Sigma_{3B}(p^2) & \Sigma_{BB}(p^2) - \frac{p^2}{2g'^2} \end{pmatrix}, \tag{14}$$

which has the eigenvalues

$$\lambda_\pm\big(p^2\big) = \frac{1}{2}\left(\Sigma_{33}\big(p^2\big) - \frac{p^2}{2g^2} + \Sigma_{BB}\big(p^2\big) - \frac{p^2}{2g'^2}\right) \pm \frac{1}{2}\sqrt{\left(\Sigma_{33}(p^2) - \frac{p^2}{2g^2} - \Sigma_{BB} + \frac{p^2}{2g'^2}\right)^2 + \Sigma_{3B}^2\big(p^2\big)}, \tag{15}$$

where the KK towers of the γ and Z are given by the solutions of the equations $\lambda_-(p^2) = 0$ (for γ) and $\lambda_+(p^2) = 0$ (for Z), respectively. By taking in Eq. (15) the limit $p^2 \to 0$, it is easily seen that $\lambda_-(p^2) = 0$ has a solution with $p^2 = 0$, which we identify with the massless γ of the SM, corresponding to the unbroken gauge group $U(1)_Q$. The lowest excitation in the tower of solutions to $\lambda_+(p^2) = 0$ has a mass-squared

$$m_Z^2 = \frac{2(g^2 + g'^2)M_L^2 M_Y^2 R_1}{(M_L^2 + M_Y^2)R_2} + \mathcal{O}\big(g^4 M_L^4 R_2^2\big), \tag{16}$$

which we identify with the Z of the SM. All other KK modes of the γ and Z have masses of order $\gtrsim 1/R_2$ and thus decouple for $1/R_1, 1/R_2 \gtrsim \mathcal{O}(\text{ TeV})$, leaving only a massless γ and a Z with mass m_Z in the low-energy theory.

4. Relation to EWPT

One important constraint on any model for EWSB results from the measurement of the ρ parameter, which is experimentally known to satisfy the relation $\rho = 1$ to better than 1% [2]. In our model, we find from Eqs. (13) and (16) a fit of the natural zeroth-order SM relation for the ρ parameter in terms of

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{g^2}{g^2 + g'^2} \frac{M_L^2 + M_Y^2}{M_Y^2} \left(\frac{R_2}{R_1}\right)^2 \frac{1}{\cos^2\theta_W} = 1, \tag{17}$$

where $\theta_W \approx 28.8°$ is the Weinberg angle of the SM. For definiteness, we will choose in the following the 4D brane couplings g and g' to satisfy the usual SM relation $g^2/(g^2 + g'^2) = \cos^2\theta_W \approx 0.77$. Defining $\rho = 1 + \Delta\rho$, we then obtain from Eq. (17) that $\Delta\rho = 0$ if the bulk kinetic couplings and compactification radii satisfy the relation

$$\left(M_L^2 + M_Y^2\right)/M_Y^2 = R_1^2/R_2^2. \tag{18}$$

Although we can thus set $\Delta\rho = 0$ by appropriately dialing the gauge couplings and the size of the extra dimensions, we observe in Eq. (10) that $\mathcal{L}_{\mathrm{eff}}$ introduces a manifest breaking of custodial symmetry (which transforms the three gauge bosons A_μ^a among themselves) and will thus contribute to EWPT via oblique corrections to the SM parameters.[7]

To estimate the effect of the oblique corrections in our model let us consider in the 4D effective theory a general vacuum polarization tensor $\Pi_{AB}^{\mu\nu}(p^2)$ between two gauge fields A and B which can (for canonically normalized fields) be expanded as [27]

$$i\Pi_{\mu\nu}^{AB}(p^2) = i g_A g_B \left[\Pi_{AB}^{(0)} + p^2 \Pi_{AB}^{(1)}\right] g_{\mu\nu} + p_\mu p_\nu \text{ terms}, \tag{19}$$

where g_A and g_B are the couplings corresponding to the gauge fields A and B, respectively. After going in $\mathcal{L}_{\mathrm{eff}}$ back to canonical normalization by redefining $A_\mu^a \to A'_\mu \equiv A_\mu^a/g$ and $B_\mu \to B'_\mu \equiv B_\mu/g'$, we identify $\Sigma_{aa}(p^2) \simeq \frac{1}{2}[\Pi_{aa}^{(0)} + p^2\Pi_{aa}^{(1)}]$, for $(aa) = (11), (22), (33), (BB)$, while $\Sigma_{3B}(p^2) \simeq \Pi_{3B}^{(0)} + p^2\Pi_{3B}^{(1)}$. From Eqs. (11) we then obtain the polarization amplitudes

$$\begin{aligned}
&\Pi_{11}^{(0)} = \Pi_{22}^{(0)} = 2M_L^2 \frac{R_2}{R_1}, && \Pi_{11}^{(1)} = \Pi_{22}^{(1)} = -2\frac{\pi^2 M_L^2}{3} R_1 R_2, \\
&\Pi_{33}^{(0)} = 2\frac{M_L^2 M_Y^2}{M_L^2 + M_Y^2} \frac{R_1}{R_2}, && \Pi_{33}^{(1)} = -2\frac{\pi^2 M_L^2 R_1 R_2}{M_L^2 + M_Y^2} \left(M_L^2 + \frac{1}{3} M_Y^2\right), \\
&\Pi_{3B}^{(0)} = -2\frac{M_L^2 M_Y^2}{M_L^2 + M_Y^2} \frac{R_1}{R_2}, && \Pi_{3B}^{(1)} = -\frac{4}{3} \frac{\pi^2 M_L^2 M_Y^2}{M_L^2 + M_Y^2} R_1 R_2.
\end{aligned} \tag{20}$$

A wide range of effects from new physics on EWPT can be parameterized in the ϵ_1, ϵ_2, and ϵ_3 framework [26], which is related to the S, T, and U formalism of Ref. [25] by $\epsilon_1 = \alpha T$, $\epsilon_2 = -(\alpha U/4)\sin^2\theta_W$, and $\epsilon_3 = (\alpha S/4)\sin^2\theta_W$. The experimental bounds on the relative shifts with respect to the SM expectations are roughly of the order $\epsilon_1, \epsilon_2, \epsilon_3 \lesssim 3 \times 10^{-3}$ [34]. From Eqs. (20) we then obtain for these parameters explicitly

$$\epsilon_1 = g^2 \frac{\Pi_{11}^{(0)} - \Pi_{33}^{(0)}}{m_W^2} = -2g^2 \frac{M_L^2}{m_W^2} \frac{R_1}{R_2} \left[\frac{M_Y^2}{M_L^2 + M_Y^2} - \left(\frac{R_2}{R_1}\right)^2\right], \tag{21a}$$

[7] Note, however, that in the limit $p^2 \to 0$, we have $\Sigma_{11} = \Sigma_{33}$, which restores custodial symmetry.

$$\epsilon_2 = g^2\big(\Pi_{33}^{(1)} - \Pi_{11}^{(1)}\big) = -g^2\frac{4\pi^2}{3}\frac{M_L^4}{M_L^2 + M_Y^2}R_1R_2, \tag{21b}$$

$$\epsilon_3 = -g^2\Pi_{3B}^{(1)} = g^2\frac{4\pi^2}{3}\frac{M_L^2M_Y^2}{M_L^2 + M_Y^2}R_1R_2, \tag{21c}$$

where we have used in the last equation that $-\epsilon_3/(gg') = \Pi_{3\gamma}^{(1)}/\sin^2\theta_W - \Pi_{33}^{(1)} = \cot\theta_W\,\Pi_{3B}^{(1)}$ [26]. Note in Eq. (21a), that for our choice of parameters we have $\epsilon_1 = \Delta\rho = 0$. The quantities $|\epsilon_2|$ and $|\epsilon_3|$, on the other hand, are bounded from below by the requirement of having sufficiently many KK modes below the strong coupling (or cutoff) scale of the theory. Using "naive dimensional analysis" (NDA) [35,36], one obtains for the strong coupling scale Λ of a D-dimensional gauge theory [37] roughly $\Lambda^{D-4} \simeq (4\pi)^{D/2}\Gamma(D/2)/g_D^2$, where g_D is the bulk gauge coupling. In our 6D model, we would therefore have $\Lambda \simeq \sqrt{2}(4\pi)^{3/2}M_{L,Y}$ which leads for $M_{L,Y} \simeq 10^2$ GeV to a cutoff $\Lambda \simeq 6$ TeV. Assuming for simplicity $M_L = M_Y$, it follows from Eq. (18) that $R_2 = R_1/\sqrt{2}$, and using Eqs. (21b) and (21c) we obtain

$$\epsilon_3 \simeq \frac{g^2}{96\sqrt{2}\pi}(\Lambda R_2)^2 \simeq 2.3\times 10^{-3}(g\Lambda R_2)^2, \tag{22}$$

while $\epsilon_2 \simeq \epsilon_3$. It is instructive to compare the value for ϵ_3 in our 6D setup as given by Eq. (22) with the corresponding result of the 5D model in Ref. [15]. We find that by going from 5D to 6D, the strong coupling scale of the theory is lowered from ~ 10 TeV down to ~ 6 TeV. Despite the lowering of the cutoff scale, however, the parameter ϵ_3 is in the 6D model by $\sim 15\%$ smaller than the corresponding 5D value.[8] This is due to the fact that in the 6D model the bulk gauge kinetic couplings satisfy $M_L = M_Y \simeq 100$ GeV, while they take in 5D the values $M_L \simeq M_Y \simeq 10$ GeV, which is one order of magnitude below the electroweak scale. From Eq. (22) we then conclude that one can take for the inverse loop expansion parameter $\Lambda R_2 \simeq 1/g \approx 1.6$ in agreement with EWPT. Like in the 5D case, however, the 6D model seems not to admit a loop expansion parameter in the regime $\Lambda R_2 \gg 1$ as required for the model to be calculable.

5. Non-oblique corrections and fermion masses

In the previous discussion, we have assumed that the fermions are (approximately) localized at $(y_1, y_2) = (0, 0)$. This would make the fermions exactly massless, since they have no access to the EWSB at $y_1 = \pi R_1$ and $y_2 = \pi R_2$. In this limiting case, the effects on the electroweak precision parameters $(\epsilon_1, \epsilon_2, \epsilon_3/S, T, U)$ come from the oblique corrections due to the vector self energies as given by Eq. (10). A more realistic case will be to extend the fermion wave functions to the bulk, i.e., to the walls of EWSB, where fermion mass operators of the form $C\bar{\Psi}_L\Psi_R$ (C is some appropriate mass parameter) can be written. Thus, although the fermion wave functions will be dominantly localized at $(0, 0)$, the profile of the wavefunctions in the bulk will be such that it will have small contributions from the symmetry breaking walls, giving rise to fermion masses. The hierarchy of fermion masses would then be accommodated by some suitable choice of the parameters C [20].

To make the incorporation of heavy fermions in our model explicit, let us introduce the 6D chiral quark fields $\mathcal{Q}_i$, $\mathcal{U}_i$, and $\mathcal{D}_i$ ($i = 1, 2, 3$ is the generation index), where $\mathcal{Q}_i$ are the isodoublet quarks, while $\mathcal{U}_i$ and $\mathcal{D}_i$ denote the isosinglet up and down quarks, respectively. For the cancellation of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge and gravitational anomalies we assume that $\mathcal{Q}_i$ have positive and $\mathcal{U}_i, \mathcal{D}_i$ have negative $SO(1, 5)$ chiralities [38]. Next,

[8] Notice that in Ref. [15], the strong coupling scale is defined by $1/\Lambda = 1/\Lambda_L + 1/\Lambda_R$, while we assume for $M_L = M_Y$ that $\Lambda = \Lambda_L = \Lambda_Y$.

we consider the action of the top quark fields with zero bulk mass, which is given by

$$\begin{aligned}\mathcal{S}_{\text{fermion}} = & \int dx^4 \int_0^{\pi R_1} dy_1 \int_0^{\pi R_2} dy_2\, i\big(\bar{\mathcal{Q}}_3 \Gamma^M D_M \mathcal{Q}_3 + \bar{\mathcal{U}}_3 \Gamma^M D_M \mathcal{U}_3\big) \\ & + \int dx^4 \int_0^{\pi R_1} dy_1 \int_0^{\pi R_2} dy_2\, K\delta(y_1)\delta(y_2) i\big[\bar{\mathcal{Q}}_3 \Gamma^\mu D_\mu \mathcal{Q}_3 + \bar{\mathcal{U}}_3 \Gamma^\mu D_\mu \mathcal{U}_3\big] \\ & + \int dx^4 \int_0^{\pi R_1} dy_1 \int_0^{\pi R_2} dy_2\, C\delta(y_1 - \pi R_1)\delta(y_2 - \pi R_2)\bar{\mathcal{Q}}_{3L}\mathcal{U}_{3R} + \text{h.c.},\end{aligned} \tag{23}$$

where we have added in the second line 4D brane kinetic terms with a (common) gauge kinetic parameter $K = [m]^{-2}$ at $(y_1, y_2) = (0, 0)$ and in the third line we included a boundary mass term with coefficient $C = [m]^{-1}$, which mixes $\mathcal{Q}_{3L}$ and $\mathcal{U}_{3R}$ at $(y_1, y_2) = (\pi R_1, \pi R_2)$. Note, that the addition of the boundary mass term in the last line of Eq. (23) is consistent with gauge invariance, since $U(1)_Q$ the only gauge group surviving at $(y_1, y_2) = (\pi R_1, \pi R_2)$. Consider now first the limit of a vanishing brane kinetic term $K \to 0$. Like in the 5D case [14], appropriate Dirichlet and Neumann BCs for $\mathcal{Q}_{3L,R}$ and $\mathcal{U}_{3L,R}$ would give, in the KK tower corresponding to the top quark, a lowest mass eigenstate, which is a Dirac fermion with mass m_t of the order $m_t \sim C/R^2$, where we have defined the length scale $R \sim R_1 \sim R_2$. Next, by analogy with the generation of the $W^\pm$ and Z masses, switching on a dominant brane kinetic term $K/R^2 \gg 1$, ensures an approximate localization of $\mathcal{Q}_{3L}$ and $\mathcal{U}_{3R}$ at $(y_1, y_2) = (0, 0)$ and leads to $m_t \sim C/K$ [15]. Now, the typical values of non-oblique corrections to the SM gauge couplings coming from the bulk are[9] $\sim CR/K \sim m_t/(1/R)$ and keeping these contributions under control, the compactification scale $1/R$ must be sufficiently large. Like in 5D models, this generally introduces a possible tension between the 3rd generation quark masses and the coupling of the Z to the bottom quark. Replacing in the above discussion $\mathcal{U}_{3L,R}$ with $\mathcal{D}_{3L,R}$ and m_t by the bottom quark mass $m_b(m_Z) \approx 3$ GeV, we thus estimate for $1/R \sim 1$ TeV a shift of the SM $Z \to \bar{b}_L b_L$ coupling by roughly $\sim 0.3\%$, which is of the order of current experimental uncertainties.[10] Similarly, we predict in our model the coupling of the Z to the top quark to deviate by $\sim 10\%$ from the SM value, which can be checked in the electroweak production of single top in the Tevatron Run 2. It can also be tested in the $t\bar{t}$ pair production in a possible future linear collider.

6. Improving the calculability

To improve the calculability of the model, it seems necessary to raise (for given $1/g_D^2$) the strong coupling scale Λ, which would allow the appearance of more KK modes below the cutoff. In fact, it has recently been argued that the compactification of a 5D gauge theory on an orbifold S^1/Z_2 gives a cutoff which is by a factor of 2 larger than the NDA estimate obtained for an uncompactified space [34]. Let us now demonstrate this effect explicitly by repeating the NDA calculation of Ref. [35] on an orbifold following the methods of Refs. [22] and [39]. For this purpose, consider a 5D scalar field $\phi(x_\mu, y)$ (where we have defined $y = y_1$), propagating in an S^1/Z_2 orbifold extra dimension. The radius of the 5th dimension is R and periodicity implies $y + 2\pi R \sim y$. As a consequence, the momentum in the fifth dimension is quantized as $p_5 = n/R$ for integer n. Under the Z_2 action $y \to -y$ the scalar transforms as $\phi(x_\mu, y) = \pm\phi(x_\mu, -y)$, where the $+$ $(-)$ sign corresponds to ϕ being even (odd) under Z_2.

[9] The factor C becomes obvious when treating the brane fields in Eq. (23) as 4D fields, in which case $C = [m]^{+1}$ and $K = [m]^0$.

[10] The LEP/SLC fit of $\Gamma_b/\Gamma_{\text{had}}$ in Z decay requires the shift of the $Z \to \bar{b}_L b_L$ coupling to be $\lesssim 0.3\%$ [3].

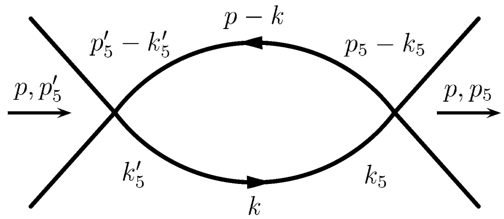

Fig. 3. One-loop diagram for ϕ–ϕ scattering on S^1/Z_2. The total incoming momentum is (p, p_5') and the total outgoing momentum is (p, p_5). Generally, it is possible that $|p_5'| \neq |p_5|$, since the orbifold fixed points break 5D translational invariance.

The scalar propagator on this space is given by [22,39]

$$D(p, p_5, p_5') = \frac{i}{2}\left\{\frac{\delta_{p_5,p_5'} \pm \delta_{-p_5,p_5'}}{p^2 - p_5^2}\right\}, \tag{24}$$

where the additional factor $1/2$ takes into account that the physical space is only half of the periodicity. Consider now the one-loop ϕ–ϕ scattering diagram in Fig. 3. The total incoming momentum is (p, p_5') and the total outgoing momentum is (p, p_5), which can in general be different, since 5D translation invariance is broken by the orbifold boundaries. Locally, however, momentum is conserved at the vertices. The diagram then reads

$$i\Sigma = \frac{1}{4}\frac{\lambda^2}{2}\frac{1}{2\pi R}\sum_{k_5,k_5'}\int \frac{d^4k}{(2\pi)^4}\left\{\frac{\delta_{k_5,k_5'} \pm \delta_{-k_5,k_5'}}{k^2 - k_5^2}\right\}\left\{\frac{\delta_{(p_5-k_5),(p_5'-k_5')} \pm \delta_{-(p_5-k_5),(p_5'-k_5')}}{(p-k)^2 - (p_5-k_5)^2}\right\}, \tag{25}$$

where λ is the quartic coupling and the additional factor $1/4$ results from working on S^1/Z_2. After summing over k_5', the integrand can be written as

$$F(k_5) = \frac{1}{(k^2 - k_5^2)[(p-k)^2 - (p_5-k_5)^2]}\{\delta_{p_5 p_5'} + \delta_{p_5,-p_5'} \pm \delta_{2k_5,(p_5+p_5')} \pm \delta_{2k_5,(p_5-p_5')}\}. \tag{26}$$

In Eq. (26), the first two terms in the bracket conserve $|p_5'|$ and contribute to the bulk kinetic terms of the scalar. The last two terms, on the other hand, violate $|p_5'|$ conservation and thus lead to a renormalization of the brane couplings [22]. Note that these brane terms lead in Eq. (25) to a logarithmic divergence. Applying, on the other hand, to the bulk terms the Poisson resummation identity

$$\frac{1}{2\pi R}\sum_{m=-\infty}^{\infty} F(m/R) = \sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{-2\pi i k R n}F(k), \tag{27}$$

we obtain a sum of momentum space integrals, where the "local" $n = 0$ term diverges linearly like in 5D uncompactified space. This term contributes a linear divergence to the diagram such that the scattering amplitude becomes under order one rescalings of the random renormalization point for the external momenta of the order

$$i\Sigma \to \frac{\lambda^2}{4}\int\frac{d^5k}{(2\pi)^5}\left[k^2(p-k)^2\right]^{-1} \simeq \frac{\lambda^2}{2}\frac{\Lambda}{(4\pi)^{5/2}\Gamma(5/2)}, \tag{28}$$

where Λ is an ultraviolet cutoff. On S^1/Z_2, we thus indeed obtain for the strong coupling scale $\Lambda \simeq 48\pi^3\lambda^{-2}$, which is two times larger than the NDA value obtained in 5D uncompactified space. This is also in agreement with the definition of Λ for a 5D gauge theory on an interval given in Ref. [34].

Similarly, when the 5th dimension is compactified on $S^1/(Z_2 \times Z_2')$ [40], we expect a raising of Λ by a factor of 4 with respect to the uncompactified case. Let us briefly estimate how far this could improve the calculability of our 6D model. To this end, we assume, besides the two extra dimensions compactified on the rectangle, two additional extra dimensions with radii R_3 and R_4, each of which has been compactified on $S^1/(Z_2 \times Z_2')$. We assume that the gauge bosons are even under the actions of the $Z_2 \times Z_2'$ groups. Moreover, we take for the bulk kinetic coefficients in eight dimensions $M_L^4 = M_Y^4$ and set $R_3 = R_4 = R_2 = R_1/\sqrt{2}$. From the expression analogous to Eq. (21c), we then obtain the estimate $\epsilon_3 \simeq g^2(\pi M_L R_2)^4/3\sqrt{2}$, where the relative factor $(\pi R_2/2)^2$, arises from integrating over the physical space on each circle, which is only 1/4 of the circumference. With respect to the NDA value $\Lambda^4 \simeq (4\pi)^4\Gamma(4)M_L^4$ in uncompactified space, the cutoff gets now modified as $\Lambda^4 \to 16\Lambda^4$, implying that

$$\epsilon_3 \simeq \frac{g^2}{192\sqrt{2}}\left(\frac{\Lambda R_2}{4}\right)^4 \simeq 1.3 \times 10^{-3}\left(\frac{\Lambda R_2}{4}\right)^4. \tag{29}$$

In agreement with EWPT, the loop expansion parameter could therefore assume here a value $(\Lambda R_2)^{-1} \simeq 0.25$, corresponding to the appearance of 4 KK modes per extra dimension below the cutoff. Taking also a possible additional raising of Λ by a factor of $\sqrt{2}$ due to the reduced physical space on the rectangle into account, one could have $(\Lambda R_2)^{-1} \simeq 0.2$ with 5 KK modes per extra dimension below the cutoff. In conclusion, this demonstrates that by going beyond five dimensions, the calculability of Higgsless models could be improved by factors related to the geometry.

7. Summary and conclusions

In this Letter, we have considered a 6D Higgsless model for EWSB based only on the SM gauge group $SU(2)_L \times U(1)_Y$. The model is formulated in flat space with the two extra dimensions compactified on a rectangle of size $\sim (\text{TeV})^{-2}$. EWSB is achieved by imposing (in the unitary gauge) consistent BCs on the edges of the rectangle. The higher KK resonances of $W^\pm$ and Z decouple below ~ 1 TeV through the presence of a dominant 4D brane induced gauge kinetic term at the point where $SU(2)_L \times U(1)_Y$ remains unbroken. The ρ parameter is arbitrary and can be set exactly to unity by appropriately choosing the bulk gauge couplings and compactification scales. As a consequence of integrating out *two* extra dimensions, the mass scale of the gauge bosons is essentially independent of the compactification scales and thus set by the bulk gauge kinetic parameters M_L and M_Y alone, which are of the order of the electroweak scale. The resulting gauge couplings in the effective 4D theory arise essentially from the brane couplings, slightly modified (at the level of one percent) by the bulk interaction. Thus, the main role played by the bulk interactions is to break the electroweak gauge symmetry. We calculate the tree-level oblique corrections to the S, T, and U parameters and find them to be consistent with current data. Non-oblique corrections to the SM gauge couplings, however, can generally modify the coupling of the Z to the bottom quark at the level of current experimental uncertainties. By considering at one-loop the ϕ^4 interaction of a scalar ϕ propagating on S^1/Z_2 and $S^1/(Z_2 \times Z_2')$, we estimate the shift of the strong coupling scale for models formulated on these manifolds. We thus conclude that a stronger suppression of the tree-level oblique corrections could be obtained in the presence of one or two extra dimensions (in addition to the ones compactified on the rectangle), each of which has been compactified on $S^1/(Z_2 \times Z_2')$, thereby improving the calculability of the model.

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