Results in Physics 2 (2012) 216–220

Contents lists available at SciVerse ScienceDirect

Results in Physics

journal homepage: www.journals.elsevier.com/results-in-physics

Tinkerbell beams in a non-linear ring resonator

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ARTICLE INFO

Article history:
Received 27 August 2012
Accepted 13 November 2012
Available online 17 November 2012

Keywords:
Ring resonator
Tinkerbell map
Non-linear resonator

ABSTRACT

In this article the dynamical behaviour of a beam that behaves according to a Tinkerbell map (i.e. Tinkerbell beam), within a nonlinear ring phase-conjugated resonator is modelled. The ABCD matrix of an optical device able to generate a two dimensional Tinkerbell map is found in terms of the map parameters, the state variables and the resonator parameters. For the first time to our knowledge the conditions in order to obtain the dynamics of a beam behaving according to a Tinkerbell map are found within an optical resonator. Finally some of the main technical problems to build a resonator intracavity element able to produce Tinkerbell beams are presented.

1. Introduction

In this work the dynamical behaviour of a beam that spatially behaves according to a Tinkerbell map (i.e. Tinkerbell beam), within a nonlinear ring phase-conjugated resonator is modelled. It is shown that the behaviour of a beam within a ring optical resonator may be well described by a Tinkerbell map and the necessary conditions for its occurrence are discussed. The state of the beam, as well as the iterative map, is determined by its previous state. In particular, it stands out that the introduction of a specific intracavity element within a ring phase-conjugated resonator may produce beams described by a Tinkerbell map, which we call from now on "Tinkerbell beams". The idea of describing chaotic optical resonators through the introduction of map generating elements from a mathematical viewpoint was already explored elsewhere [1,2]. The approach to the Tinkerbell map is taken into account due to its experimental viability, the final mathematical description for the Tinkerbell beam generator makes us think that it can be physically constructed.

One of the major research areas for nonlinear optics and quantum electronics is optical phase conjugation (OPC). OPC may be achieved by two coherent optical beams propagating in opposite directions with a reversed wave front and identical transverse amplitude distributions. A pair of phase-conjugated beams possesses the unique feature that the aberration influence imposed on the forward beam as it goes through an inhomogeneous or disturbing medium is removed as the backward beam returns through the same medium. No matter how these beams are obtained, it is important for the generation of the backward phase-conjugated beam, the formation of the induced holographic grating and the subsequent wave-front restoration via a backward reading beam. OPC research growth shows a renovated momentum because of OPC-associated techniques usefulness in many different application areas: such as high-brightness laser oscillator/amplifier systems, cavity-less lasing devices, laser target-aiming systems, aberration correction for coherent-light transmission and reflection through disturbing media, long distance optical fibre communications with ultra-high bit-rate, optical phase locking and coupling systems, and novel optical data storage and processing systems [3].

One way to improve the output power performance of a phase conjugated laser oscillator is to use intracavity nonlinear elements [4,5]. Often, intracavity cells are utilised to compensate optical aberrations both in the resonator and due to thermal effects in the active medium, and to obtain a near diffraction limited output [6], they can also eliminate the need for a conventional Q-switch since its intensity-dependent reflectivity typically produces nanosecond pulse trains of diffraction limited quality.

OPC also help in the production of short holograms that do not exhibit in-depth diffraction deformation of the fine speckle pattern of the recording fields [7,8]. For instance, phase conjugation by Stimulated Brillouin Scattering (SBS) can help in the power scaling of fibres and solid-state lasers [9–11].

Several theoretical models have been proposed to describe OPCs in resonators; usually in SBS conjugated lasers the linear phase conjugated beam is obtained from the SBS reflection on one of the cavity mirrors [12]; however ring-phase conjugated resonators can also be used [13]. While the typical theoretical model of an OPC laser in transient operation [14] works taking into account the spatial and temporal dynamics of the input $E_P(z,t)$ and the Stokes $E_S(z,t)$ fields as well as the acoustic-wave amplitude $Q(z,t)$ in the SBS cell, it is known that spatial mode analysis of a laser may be carried out using transfer matrices, also called *ABCD*

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http://dx.doi.org/10.1016/j.rinp.2012.11.001

matrices. Matrices are a simple way to obtain the final position and angle of a ray that propagates through a complex optical system, such as a self-adaptive laser resonator with its oscillator made of a plane output coupler and an infinite nonlinear FWM medium in self-intersecting loop geometry [15]. In this article a phase conjugate ideal mirror is used as an element of a ring resonator able to present beam dynamics described by a Tinkerbell map. We believe that this kind of laser (spatially driven and controlled) can be very useful as a model instrument with several fine adjustable parameters. These laser systems can be used to model complex macroscopic systems (e.g. weather, oceanic currents and brain activity) as well as in cryptography and secure communication systems.

This paper is organised as follows. Section 2 reviews the ABCD matrix optic elements that are the starting point of this work, Section 3 presents some basic features of the nonlinear Tinkerbell map, Section 4 shows the main characteristics of the map generation matrix and the Tinkerbell Beams. Section 5 presents the general case for the Tinkerbell beams in a nonlinear ring phase-conjugated resonator, Section 6 discusses some of the main technical problems in order to build an intracavity element to generate the Tinkerbell beams discussed here, and finally Section 7 presents the conclusions.

2. *ABCD* matrix optics

A ray beam is described by two parameters, its high and its angle to an optical axis. When a ray travels through an optical element, it usually changes its direction. Mathematically the optical element can be represented by a 2×2 matrix, called the ABCD matrix. Any sequence of linear transformations follows the simple rules of matrix multiplication and can therefore be used when the ray travels through a whole optical system. To do so one has to assume cylindrical symmetry around the optical axis, and to define at a given position z both the perpendicular distance $y(z)$ of any ray to the optical axis and its angle with the same axis $\theta(z)$. The paraxial ray's transformation can therefore be described by the following equation [16]

$$\begin{pmatrix} y_{n+1} \\ \theta_{n+1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_n \\ \theta_n \end{pmatrix}. \tag{1}$$

All optical passive elements used to reflect, propagate and transmit light are represented by constant *ABCD* matrices whose determinant Det[*ABCD*] = n_b/n_a, where n_b and n_a are the refraction indexes before and after the optical element described by the matrix. Since typically n_b and n_a are the same, it holds that Det[*ABCD*] = 1. However, for active or nonlinear optical elements of the *ABCD* matrix, elements can be described by functions of various parameters.

It is important to note that since in the process of phase conjugation the incoming ray retraces exactly its incident path [9], the ideal phase conjugated *ABCD* matrix is

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2}$$

whose determinant is not 1 but -1. As already mentioned, typically the phase conjugation can be achieved by FWM or by a stimulated scattering process such as Brillouin. Upon reflection on a stimulated SBS phase conjugated mirror, the reflected wave has its frequency ω downshifted to $\omega - \delta = \omega(1 - \delta/\omega)$ where δ is the characteristic Brillouin downshift frequency of the mirror material (typically $\delta/\omega \ll 1$). Taking the downshifting frequency into account the ABCD matrix reads

$$\begin{pmatrix} 1 - \frac{\delta}{\omega} & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

The ideal matrix (2) must also be modified considering that in phase conjugation by SBS a light intensity threshold must be reached to obtain an exponential amplification of the scattered light; to model a real SBS phase conjugated mirror one has to include a Gaussian aperture of radius a at intensity $1/e^2$ placed before the ideal phase conjugated mirror, to ensure that only the fraction of a Gaussian incident beam with intensity above threshold is phase-conjugated reflected as a Gaussian beam. The specific matrix for this aperture is

$$\begin{pmatrix} 1 & 0 \\ -\frac{i\lambda}{\pi a^2} & 1 \end{pmatrix}, \tag{4}$$

where the aperture a is given as a function of the radial distribution of the intensity of the incident light beam. The ABCD matrix elements of a phase conjugated mirror depend on several parameters such as the incident light intensity, the wavelength and the Brillouin downshifting frequency [17]. For our next calculations we will use the ideal matrix given by Eq. (2).

3. Tinkerbell map

From an extensive list of dynamic maps [18], Duffing, Hénon and Tinkerbell are two dimensional nonlinear maps that have proven to be very useful in physical applications [18,19]. Since these maps are described by recurrence relations they can be written as a matrix system such as the one described by Eq. (1), that is

$$y_{n+1} = Ay_n + B\theta_n, \tag{5-a}$$
$$\theta_{n+1} = Cy_n + D\theta_n. \tag{5-b}$$

Since the Tinkerbell map [20] is a discrete-time dynamical system given by the equations

$$y_{n+1} = y_n^2 - \theta_n^2 + \alpha y_n + \beta\theta_n, \tag{6-a}$$
$$\theta_{n+1} = 2y_n\theta_n + \gamma y_n + \delta\theta_n, \tag{6-b}$$

where y_n and θ_n are the scalar state variables and α, β, γ, and δ are the map parameters, in order to write the Tinkerbell map as a matrix system such as Eq. (1) the following values for the coefficients A, B, C and D must hold

$$A(y_n, \alpha) = y_n + \alpha, \tag{7}$$
$$B(\theta_n, \beta) = -\theta_n + \beta, \tag{8}$$
$$C(\theta_n, \gamma) = 2\theta_n + \gamma, \tag{9}$$
$$D(\delta) = \delta. \tag{10}$$

To do so, the matrix coefficients have to depend on the previous state variables y_n and θ_n, so they are not constants anymore. They also depend on the Tinkerbell map parameters α, β, γ, and δ. With these considerations the *ABCD* Tinkerbell matrix system becomes

$$\begin{pmatrix} y_{n+1} \\ \theta_{n+1} \end{pmatrix} = \begin{pmatrix} y_n + \alpha & -\theta_n + \beta \\ 2\theta_n + \gamma & \delta \end{pmatrix} \begin{pmatrix} y_n \\ \theta_n \end{pmatrix}. \tag{11}$$

4. Ring phase-conjugated resonator with Tinkerbell dynamics

This section presents an optical resonator that produces beams following the Tinkerbell map dynamics; these beams will be called "Tinkerbell beams". Fig. 1 shows a ring phase-conjugated resonator consisting of two ideal mirrors, an ideal phase conjugated mirror and a yet unknown optical element described by an *ABCD* matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. The phase conjugated mirror (*PCM*) is separated by a distance l from each plane mirror M, while the Tinkerbell device is located between the two mirrors at a distance $l/2$ from each other. To calculate the full resonator matrix one has to take into

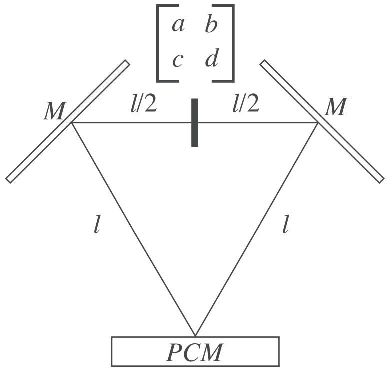

Fig. 1. Ring phase conjugated laser resonator with a chaos generating element.

account the identity matrix: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ for the plane mirrors M, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ for the ideal phase conjugated mirror PCM, $\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix}$ for an l and $l/2$ translations, and the unknown Tinkerbell map generating device matrix represented by $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Therefore, the total transformation ABCD matrix for a complete round trip is calculated as

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}. \tag{12}$$

The resulting matrix is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} a + \frac{3cl}{2} & b + \frac{3l}{4}(2a + 3cl + 2d) \\ -c & -(d + \frac{3cl}{2}) \end{pmatrix}. \tag{13}$$

If one does want a particular map to be reproduced by a ray in the optical resonator, each round trip described by (y_n, θ_n), has to be considered as an iteration of the selected map. In order to obtain Tinkerbell beams, Eq. (11) must be equated to Eq. (13), that is

$$a + \frac{3cl}{2} = \alpha + y_n, \tag{14}$$

$$b + \frac{3l}{4}(2a + 3cl + 2d) = \beta - \theta_n, \tag{15}$$

$$c = -\gamma - 2\theta_n, \tag{16}$$

$$d + \frac{3cl}{2} = -\delta. \tag{17}$$

Eqs. (14)–(17) define a system for the matrix elements a, b, c, d, that guarantees a Tinkerbell map behaviour for a given ray (y_n, θ_n) or a Tinkerbell beam i.e. the generation of a beam that behaves on (y_n, θ_n) according to a Tinkerbell map. These elements can be written in terms of the map parameters (α, β, γ and δ), the resonator's main parameter l and the ray state variables y_n and θ_n as

$$a = \alpha + \frac{3}{2}\gamma l + 3l\theta_n + y_n, \tag{18}$$

$$b = \frac{1}{4}(4\beta - 6\alpha l + 6\delta l - 9\gamma l^2 - 4\theta_n - 18l^2\theta_n - 6ly_n), \tag{19}$$

$$c = -2\theta_n - \gamma, \tag{20}$$

$$d = -\delta + \frac{3}{2}l(\gamma + 2\theta_n). \tag{21}$$

The introduction of the above values for the $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ matrix in Eq. (13) enables us to obtain Eq. (11). For any transfer matrix, elements A and D describe the lateral magnification while C depicts the focal length, whereas the device's optical thickness is given by $B = L/n$, where L is its length and n its refractive index. From Eqs. (18)–(21) it must be noted that the upper elements (a and b) of the device matrix depend on both state variables (y_n and θ_n) while the lower elements (c and d) only on the state variable θ_n. The study of the stability and chaos of the Tinkerbell map in terms of its parameters is a well-known topic [21,22]. In particular the behaviour of element b is very important from a practical point of view; Fig. 2 shows a computer calculation for the first 100 round trips of matrix element b of the Tinkerbell map generating device for a resonator of unitary length ($l = 1$) and map parameters $\alpha = 0$, $\beta = -0.6$, $\gamma = 0$ and $\delta = -1$, these parameters were found using brute force calculations and they were selected due to the matrix-element b behaviour (i.e. we were looking for the behaviour able to be achievable in experiments). As can be seen, the optical length of the map generating device (i.e. element b) varies on each round trip in a periodic form, this would require that the physical length of the device, its refractive index -or a combination of both- change in time. The actual design of a physical Tinkerbell map generating device for a unitary ring resonator must satisfy Eqs. (18)–(21), to do so its elements (a, b, c and d) must vary accordingly. The obtained matrix can be regarded as the matrix of a dynamic general lens.

5. Tinkerbell beams: general case

To obtain the Eqs. (18)–(21) b, the thickness of the Tinkerbell device, has to be very small (close to zero), so the translations before and after the device can be over the same distance $l/2$. In the previous numeric simulation b takes values up to 0.2, so the general case where the map generating element b is not assumed to be small must be studied. Eq. (12) then becomes

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{l-b}{2} \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & \frac{l-b}{2} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}, \tag{22}$$

where $0 < b < l$. Therefore the round trip total transformation matrix is now written as

$$\begin{pmatrix} a - \frac{c}{2}(b - 3l) & \frac{1}{4}[b^2c - 2b(-2 + a + 3cl + d) + 3l(2a + 3cl + 2d)] \\ -c & \frac{1}{2}(bc - 3cl - 2d) \end{pmatrix}. \tag{23}$$

From Eqs. (11) and (23) we obtain the following system of equations for the matrix elements a, b, c and d

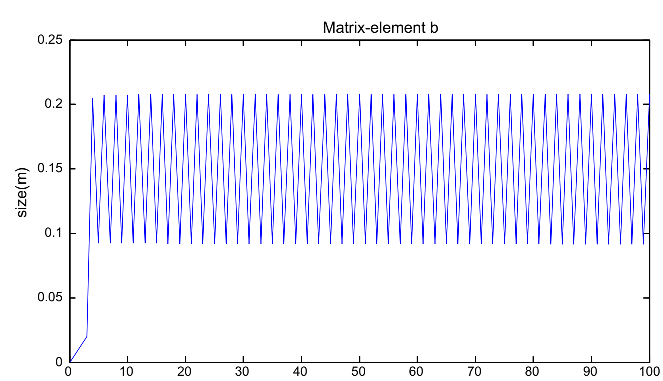

Fig. 2. Computer calculation of the magnitude of matrix element b of the Tinkerbell map generating device for a resonator with $l = 1$ and Tinkerbell parameters $\alpha = 0$, $\beta = -0.6$, $\gamma = 0$ and $\delta = -1$ for the first 100 round trips.

$$a - \frac{c}{2}(b - 3l) = \alpha + y_n, \tag{24}$$

$$\frac{1}{4}(b^2c - 2b(-2 + a + 3cl + d) + 3l(2a + 3cl + 2d)) = \beta - \theta_n, \tag{25}$$

$$-c = \gamma + 2\theta_n, \tag{26}$$

$$\frac{bc - 3cl - 2d}{2} = \delta. \tag{27}$$

The solution to this new system is written as

$$a = \alpha + \frac{3}{2}\gamma l + 3l\theta_n + y_n + \frac{1}{2\gamma + 4\theta_n} \times \begin{pmatrix} \gamma(2 - \alpha + \delta - 3\gamma l - 12l\theta_n - y_n) \\ +\theta_n(4 - 2\alpha + 2\delta - 12l\theta_n - 2y_n) \\ -(-\frac{\gamma}{2} - \theta_n)\sqrt{P^2 - Q} \end{pmatrix}, \tag{28}$$

$$b = \frac{1}{\gamma + 2\theta_n}\left(-2 + \alpha - \delta + 3\gamma l + 6l\theta_n + y_n + \frac{\sqrt{P^2 - Q}}{2}\right), \tag{29}$$

$$c = -\gamma - 2\theta_n, \tag{30}$$

$$d = \delta + \frac{3}{2}\gamma l + 3l\theta_n + \frac{1}{2\gamma + 4\theta_n} \times \begin{pmatrix} \gamma(2 - \alpha + \delta - 3\gamma l - 12l\theta_n - y_n) \\ +\theta_n(4 - 2\alpha + 2\delta - 12l\theta_n - 2y_n) \\ -(-\frac{\gamma}{2} - \theta_n)\sqrt{P^2 - Q} \end{pmatrix}, \tag{31}$$

where

$$P = 4 - 2\alpha + 2\delta - 6\gamma l - 12l\theta_n - 2y_n \quad \text{and}$$
$$Q = (4\gamma + 8\theta_n)(-4\beta + 6\gamma l - 6\delta l + 9\gamma l^2 + 4\theta_n + 18l^2\theta_n + 6ly_n).$$

It should be noted that if one takes into account the thickness of the map generating element, the equation complexity is substantially increased. Now only c has a simple relation with θ_n and γ, on the other hand a, b and d are dependent on both state variables, on all Tinkerbell parameters, as well as on the resonator length. When the calculation is performed for this new matrix with the following map parameters: $\alpha = 0.4$, $\beta = -0.4$, $\gamma = -0.3$ and $\delta = 0.225$, Fig. 3 is obtained. The behaviour observed in Fig. 3 for the matrix-element b can be obtained for several different parameters' combinations, as well as other dynamical regimes with a lack of relevance to our work. One can note that after a few iterations the device's optical thickness is small and constant, this should make easier the physical implementation of this device.

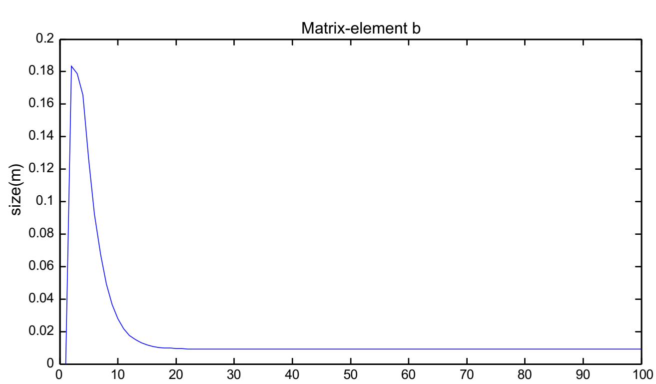

Fig. 3. Computer calculation of the magnitude of matrix element b of the Tinkerbell map generating device for a resonator with $l = 1$ and Tinkerbell parameters $\alpha = 0.4$, $\beta = -0.4$, $\gamma = -0.3$ and $\delta = 0.225$ for the first 100 round trips.

6. A practical Tinkerbell map generating device

As we have seen, in order to generate an *ABCD* matrix system such as (11) it is essential to introduce an intra-cavity element which will be responsible for taking into account the hysteresis and non-linearity of the dynamic system. The intra-cavity map generating device is described by a 2×2 matrix, where its elements are given by Eqs. (18)–(21). The equations describing the intracavity element are

$$y_{output} = ay_{input} + b\theta_{input}, \tag{32-a}$$

$$\theta_{output} = cy_{input} + e\theta_{input}. \tag{32-b}$$

The practical implementation of an intra-cavity element is technically a complex task due to the fact that the actual intra-cavity matrix is a dynamic one, its value depends not only on the map constants but also on the previous round-trip y_n and θ_n values. In particular it is required for the intra-cavity element a system able to detect and measure the position and angle of incidence of the input beam parameters, i.e. y_{input} and θ_{input}, this information should be optically or electronically processed (according to Eqs. (18)–(21)) in order to produce and generate the required output beam with new parameters i.e. y_{output} and θ_{output}. A general intra-cavity element does not exists yet.

The measurement of the impinging angle of a light beam can be implemented by several techniques, such as the use of collimators or interferometers. However, when the spatial coordinates are also of interest, as in this case, there is not a straightforward solution. A possible solution is the use of a matrix of photosensors mounted on a PZT-driven stage. As shown in Fig. 4, a projected spot results from the projection of the beam onto the plane of the photosensors. The angle can be obtained by measuring the spatial coordinates of the spot for two different positions. To obtain measurement speeds on the order of milliseconds it is necessary that the PZT stage be driven at relatively high speeds, e.g. the M-663 stage from Physik Instruments can reach displacement speeds of up to 400 mm/s (travel range of 19 mm with 100-nm resolution). A matrix of photosensor such as that offered by Centronic, i.e. 12×12 elements, each element of 1.4×1.4 mm, can be used as a first approximation. In this case, by considering the travel range of the stage, the maximum transverse displacement of the beam spot, at the sensor plane, would be 3 pix, where one pixel corresponds to one sensor element; for this computation it was assumed that ray angles are

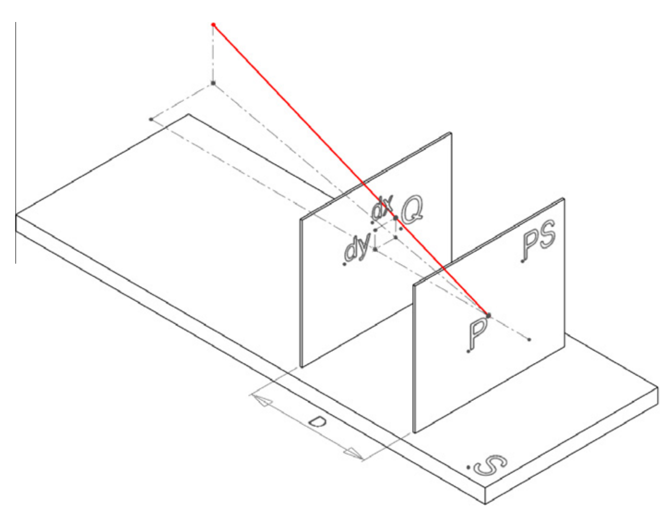

Fig. 4. A photosensor array (PS) is translated by a PZT stage (S) a distance D. This produces a displacement of the beam spot from P to Q, dx and dy. The incidence angles of the ray are given by $\theta_x = \tan^{-1}(dx/D)$ and $\theta_y = \tan^{-1}(dx/D)$. The light beam is indicated by the red line.

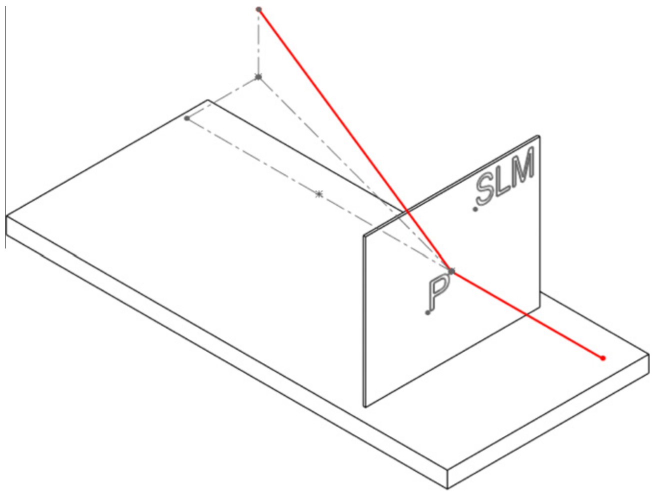

Fig. 5. Beam steering by a SLM. The beam impinges on the SLM from the right.

less than 15°. This arrangement would yield measurements with low accuracy. To increase the accuracy of the measurement, the separation between neighbouring elements should be decreased. This can be achieved by using a camera sensor, where the pitch may be as small as 4 μm at the maximum angle, the distance between the two positions of the beam spot can be as large as hundreds of pixels. However, in this case the complexity of the arrangement is increased.

On the other hand beam steering may be done by non-mechanical array devices, which provide high-speed pointing, see Fig. 5. Among these types of devices we can mention those based on liquid-crystal displays (spatial light modulators, SLM) and those on microelectromechanical systems (MEMS) [23]. In the former, the phase of each element of the matrix is changed by application of a low-voltage signal. In the devices based on MEMS, each element of the array consists of a micromirror, which generates tilt to steer the beam. Steering time is on the order of milliseconds.

7. Conclusions

This article shows how Tinkerbell beams can be produced if a particular device is introduced in a ring optical phase-conjugated resonator. The difference equations of the Tinkerbell map are explicitly introduced in an *ABCD* transfer matrix to control the beam behaviour. The matrix elements a, b, c and d of a map generating device are found in terms of the map parameters (α, β, γ and δ), the state variables (y_n and θ_n) and the resonator length. The mathematical characteristics of an optical device inside an optical resonator capable to produce Tinkerbell beams are found. In the general case a device with fixed size was obtained, opening the possibility of continuance of this work; that is the actual building of an optical device with these a, b, c and d matrix elements according to the description given and the experimental observation of Tinkerbell beams. Finally the discussion of some of the main technical problems in order to build an intracavity device able to produce Tinkerbell beams is also presented.

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