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Foreword

Special issue on automated deduction: Decidability, complexity, tractability

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Decidability and complexity issues are extremely important in automated deduction. Although general logical formalisms (predicate logic, set theory, number theory) are undecidable or not even recursively enumerable, it is often the case that in applications only special fragments need to be considered, which are decidable and sometimes even have low complexity. In the field of automated deduction, considerable effort was dedicated to the task of *identifying decidable problems* and studying their complexity, and in giving *uniform decision procedures* obtained with general-purpose methods such as resolution, tableau calculi or sequent calculi, and to study situations in which these methods can be tuned to provide algorithms with optimal complexity. In recent years, automated reasoning techniques found a large number of practical applications ranging from knowledge representation (reasoning in large databases and ontologies, reasoning in non-classical logics) to program verification and verification of reactive, real time or hybrid systems. The need of *efficient decision procedures* for application areas in which the complexity and size of formulae is really large – e.g. reasoning in large ontologies and problems arising from verification – was an important stimulus for the investigation of decidable (and even tractable) logical theories and their complexity. In most applications logical theories do not appear alone: the problems to be solved are of a heterogeneous nature, and need to be modeled using combinations of theories. Studying the limits of decidability and the

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complexity of reasoning in combinations of logical theories is a very active field of research, with numerous applications. The topics of papers collected in this special issue give an impression of some current fields of research in automated reasoning and of the state-of-the-art research in the area of decidability, complexity, and tractability in automated deduction. The papers in this special issue can be grouped into four categories: (i) Decidable fragments of first-order logic, (ii) (Context) unification and applications, (iii) Reasoning in large databases and ontologies, and (iv) Reasoning in combinations of theories.

In the paper *Decidable Fragments of Many-Sorted Logic*, Aharon Abadi, Alexander Rabinovich and Mooly Sagiv, study a decidable fragment of many-sorted first-order logic that allows the expression of a large variety of real world specifications. The motivation for this study is that using types often reduces the complexity of checking satisfiability/validity of formulae with quantifiers, when these quantify over different types. The main result in this paper is a characterization of a decidable fragment of many-sorted first-order logics important in applications.

The paper *Context Unification with One Context Variable*, written by Adrià Gascón, Guillem Godoy, Manfred Schmidt-Schauß and Ashish Tiwari, addresses the problem of context unification, which is a generalization of standard term unification. This problem is especially interesting because it is useful in interprocedural program analysis. The authors analyze the special case of context unification where the use of, at most, one context variable is allowed, and show that it is in NP. The consequences of this result are investigated, and particular cases where one-context unification is polynomial are presented.

The paper *Deciding inseparability and conservative extensions for the description logic \mathcal{EL}* , written by Carsten Lutz and Frank Wolter, addresses the problem of deciding whether two ontologies are inseparable w.r.t. a signature Σ , i.e., whether they have the same consequences formulated in Σ . The authors focus on the lightweight description logic \mathcal{EL} as an ontology language and consider query languages based on (i) subsumption queries, (ii) instance queries over ABoxes, (iii) conjunctive queries over ABoxes, and (iv) second-order logic. It is proved that for query languages (i) to (iii), inseparability is ExpTime-complete and that case (iv) is undecidable.

The next two papers address the problem of reasoning in combinations of theories.

The paper *Theory decision by decomposition*, written by Maria Paola Bonacina and Mnacho Echenim, addresses the problem of giving decision procedures for satisfiability modulo theories (SMT) of arbitrary quantifier-free formulae. The authors propose an approach that decomposes the formula in such a way that its definitional part, including the theory, can be compiled by a rewrite-based first-order theorem prover, and the residual problem can be decided by an SMT-solver. Thus, the complementary strengths of first-order provers and SMT-solvers can be exploited. The method is illustrated by giving decision procedures for the theories of records, integer offsets and arrays, and for combinations including such theories.

The paper *Combination of Convex Theories: Modularity, Deduction Completeness, and Explanation* written by Duc-Khanh Tran, Christophe Ringeissen, Silvio Ranise and H el ene Kirchner, presents a framework for designing cooperation schemas between decision procedures while maintaining modularity of their interfaces. This is used for specifying and proving the correctness of the combination schemas by Nelson-Oppen and Shostak. The authors then introduce the concept of “deduction complete satisfiability procedures” and provide a schema to modularly combine them. Finally, the problem of modularly constructing explanations by re-using available proof-producing procedures for the component theories is addressed.

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