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Erratum

Erratum to "Error bounds for spline-based quadrature methods for strongly singular integrals" [J. Comput. Appl. Math. 89 (1998) 257]☆

Kai Diethelm *

Institut für Angewandte Mathematik, Technische Universität Braunschweig, Pockelsstr. 14, 38106 Braunschweig, Germany

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The purpose of this note is to point out a mistake in the author's paper [2].
Indeed, Theorems 1 and 2 should have the following correct form.

Theorem 1. *Assume* $p-1<s\leqslant d+1$. *Let* $f^{(s)}$ *be bounded. Let* (J_n) *be*

- *a sequence of nodal spline approximation operators of degree* d *with* $x_j=a+(b-a)j/(dn)$ $(j=1,2,\ldots,dn)$, *or*
- *a sequence of not-a-knot spline interpolation operators of odd degree* $d\geqslant p$ *with uniform partitions*, *or*
- *a sequence of quasi-interpolatory spline approximation operators of degree* $d\geqslant p$ *with uniform partitions as described above.*

Denote by $H_{p,n}[f]:=H_p[J_n[f]]$ *the quadrature rule for* H_p *based on* J_n. *Then*, *for every* $\delta>0$

$$|H_p[f](t)-H_{p,n}[f](t)|=\begin{cases}\mathrm{O}(n^{p-1-s}\ln n) & \text{if } p\in\mathbb{N},\\ \mathrm{O}(n^{p-1-s}) & \text{if } p\notin\mathbb{N},\end{cases}$$

uniformly for all $t\in[a+\delta,b-\delta]$. *In the case* $p\in\mathbb{N}$, *the same result holds if we replace* H_p by H_p^*.

Theorem 2. *Assume* $p-1<s$. *Let* $J_n:\mathscr{A}^s[a,b]\to\mathscr{A}^s[a,b]$ *satisfy*

$$\sup\{\|(J_n[f]-f)^{(k)}\|_\infty\colon f\in C^s[a,b],\|f^{(s)}\|_\infty\leqslant 1\}=n^{k-s}\varepsilon_{k,n}, \tag{1}$$

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* Fax: +49-531-391-8206.
E-mail address: k.diethelm@tu-bs.de (K. Diethelm).

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where $(\varepsilon_{k,n})_{n=1}^{\infty}$ *are given for* $k = 0, 1, \ldots, s$. *Then, for every* $f \in C^s[a,b]$ *and every* $\delta > 0$,

$$|H_p[f](t) - H_p[J_n[f]](t)| = \begin{cases} O(n^{p-1-s}) \left(\sum_{k=0}^{p} \varepsilon_{k,n} + \varepsilon_{p-1,n} \ln n \right) & \text{if } p \in \mathbb{N}, \\ O(n^{p-1-s}) \sum_{k=0}^{\lfloor p \rfloor} \varepsilon_{k,n} & \text{if } p \notin \mathbb{N}, \end{cases}$$

uniformly for all $t \in [a+\delta, b-\delta]$. *In the case* $p \in \mathbb{N}$, *the same result holds if we replace* H_p *by* H_p^*.

The proofs can then remain unchanged. As a matter of fact we cannot obtain uniform convergence on the entire interval under the conditions stated in [2] because that would require additional boundary conditions (interpolation conditions with multiple nodes at both end points, the multiplicity depending on the parameter p) in a more or less straightforward generalization of the results of [1, Section 4] for the case $p = 1$.

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References

[1] K. Diethelm, Uniform convergence of optimal-order quadrature rules for Cauchy principal value integrals, J. Comput. Appl. Math. 56 (1994) 321–329.

[2] K. Diethelm, Error bounds for spline-based quadrature methods for strongly singular integrals, J. Comput. Appl. Math. 89 (1998) 257–261.