Research on Calculation Method for Dill Pipe Half-wavelength Based on Energy Method

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Abstract

According to energy conservation, it researched the calculation method for dill pipe half-wavelength from a new angle in this paper, and deduced the calculation equation of half-wavelength, which is important to optimizing drilling parameter and lengthening fatigue life of drill pipe.

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Key words: Half-wavelength; Energy Method; Horizontal Drilling

1. Foreword

During drilling, drill pipe rotate with certain speed, because of the eccentric of drill pipe itself and the bending caused by self-weight unstability, part of drill pipe weight deviate from the rotary center, and the centrifugal force generated by the rotating eccentric-weight will enable the drill pipe more bending. In the function of centrifugal force and gravity, the whole drill pipe wave in the hole, and the peak press to the hole-wall, the frictional resistance could be generated in the course of rotating and drilling.

Half-wavelength of drill pipe is related to bending stress, so it is important to optimizing drilling parameter and lengthening life of drill pipe.

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2. Energy method

Energy conservation is a common law in the nature, the method using energy principle to solve problems is called as energy method.

Before the deformation of drill pipe reach yield value, drill pipe can be regarded as the elastomer, the numerical value of work done by external force is equal to the deformation energy saving in the drill pipe inner.

3. Research on calculation method for deformation energy of drill pipe inner

![Figure 1. Bending of Drill Pipe](image)

### 3.1. Bending Deformation Energy

As is shown in the Figure 1, the half-wavelength of drill pipe is “L”, deflection caused by external bending moment “M” is “f”, axial force is “F”, gravitational load is “q cos α”, infinitesimal in the half-wavelength is “dx”, then the bending deformation energy “u_B” is:

\[
\frac{M^2}{2EI} \, dx
\]

I--Inertia Moment

The bending deformation energy of whole half-wavelength is:

\[
\int_0^L \frac{M^2}{2EI} \, dx = \frac{Ef^2 \pi^4}{4L}
\]

### 3.2. Bending Deformation Energy

As is shown in the Figure 1, the half-wavelength of drill pipe is “L”, deflection caused by external bending moment “M” is “f”, axial force is “F”, gravitational load is “q cos α”, infinitesimal in the half-wavelength is “dx”, then the bending deformation energy “u_B” is:

\[
\frac{M^2}{2EI} \, dx
\]

I--Inertia Moment

The bending deformation energy of whole half-wavelength is:

\[
\int_0^L \frac{M^2}{2EI} \, dx = \frac{Ef^2 \pi^4}{4L}
\]

### 3.3. Bending Deformation Energy

As is shown in the Figure 1, the half-wavelength of drill pipe is “L”, deflection caused by external bending moment “M” is “f”, axial force is “F”, gravitational load is “q cos α”, infinitesimal in the half-wavelength is “dx”, then the bending deformation energy “u_B” is:
\[ du_p = \frac{M^2}{2EI} dx \]  

I--Inertia Moment

The bending deformation energy of whole half-wavelength is:

\[ u_b = \int_0^L \frac{M^2}{2EI} dx = \frac{EI\pi^4}{4L} \]  

(5)  

(6)

3.4. Shearing Deformation Energy

It only consider the effect of the bending moment in Formula 1, there is shearing deformation energy:

\[ u_s = \int_0^L f_s \left( \frac{dV}{dx} \right)^2 dx = \frac{f_s g^2 \cos^2 \alpha L}{2GA} \]  

V- Shearing force
f_s- Shearing coefficient
G- Shear modulus
A- Cross sectional area of drill pipe

(7)

3.5. Twisting formation energy

Drill pipe bears driving torque from power head of drilling rig and resistance moment of bit while rotating, twisting formation energy is:

\[ u_T = \frac{M_n^2 L}{2GJ_p} \]  

\[ M_n- Torque while rotating \]
\[ J_p- Polar moment of inertia \]  

(8)

4. Research on calculation method for the work done by the force of drill pipe

4.1. Work done by axial force

While the drill pipe is bending, the work done by axial force is:

\[ W_a = F \cdot \lambda = \frac{Ff^2 \pi^2}{4L} \]  

\[ \lambda - Difference from deflection curve and chord length \]  

(9)

4.2. Work done by centrifugal force

As is shown in Fig1, infinitesimal in the half-wavelength is “dx”, the angular velocity of rotating is “\( \omega \)”, distance from centerline of drill hole to drill pipe is “\( y \)”, thus, work done by centrifugal force in “dx” is:
\[ dT = \frac{q \cos \alpha}{g} \rho y \]  

\( g \) - Acceleration of gravity

Whole work done by centrifugal force is:

\[
W_c = \int_a^b \frac{1}{2} y dT = \frac{q \alpha^2 f^2 L}{4g} \tag{10}
\]

4.3. Work done by gravity

Work done by gravity is:

\[
W_q = \int_a^b \frac{1}{2} y \cos \alpha dx = \frac{q \cos \alpha f L}{2\pi} \tag{11}
\]

5. Calculation equation of half-wavelength of drill pipe

According to energy conservation, the numerical value of work done by external force is equal to the deformation energy of drill pipe:

\[ u_b + u_x + u_T = W_a + W_c + W_q \tag{12} \]

Thus:

\[
\frac{ELf^2 \pi^4}{2I} + \frac{f_s \alpha^2 \cos^2 \alpha L}{2GA} + \frac{M^2 \pi^4 L}{2GJ_p} = \frac{q \alpha^2 f^2 L}{4L} + \frac{q \alpha^2 f L}{2\pi} \tag{13}
\]

\[
J_p = 2I
\]

\[
f = \frac{4q \cos \alpha L^4}{El \pi^4 \left(1 - \frac{FL}{El \pi^2}\right)}
\]

After sampling:

\[
16(q \cos \alpha)^3 GAgq^2 IL^4 + 8(q \cos \alpha)^2 GAgF \pi^2 IL^6 - g \pi^2 \left[1(q \cos \alpha)^2 \left[8GAEI \pi^2 + 2f, \pi^4 F_3^3\right] + AM^2 \pi^4 F_3^3\right] L^4
\]

\[
+ 2EI \pi^4 \left[2(q \cos \alpha)^2 f, l + AM^2\right] L^2 - g \pi^6 \left(EI \pi^2\right)^2 \left[2(q \cos \alpha)^2 f, l + AM^2\right] = 0
\]

Shearing deformation energy is far less than bending deformation energy while drilling, so it can be ignored. In the proceeding of horizontal drilling, drilling dip angle could be zero approximately. So the equation of half-wavelength while horizontal drilling is:

\[
16Gq^3 \pi^2 IL^4 + 8Ggq^2 F \pi^2 IL^6 - g \pi^2 \left[8Iq^2 GEI \pi^2 + M^2 \pi^4 F_3^3\right] L^4
\]

\[
+ 2EI \pi^3 \pi^4 F_3 M^2 L^2 - g \pi^6 \left(EI \pi^2\right)^2 M^2 = 0
\]

6. Conclusion

It deduced the calculation equation of half-wavelength using energy method, and obtained the calculation equation of half-wavelength while horizontal drilling, which provide theoretical basis for optimizing drilling parameter and lengthening fatigue life of drill pipe.
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