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Study on the Wald-W Method of Uncertain Decision-making

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Abstract

Uncertain decision-making is one of important research areas in the decision-making theory. For a long time five decision standards such as optimism decision standard, pessimism decision standard, compromised decision standard, equality decision standard and regret decision standard have been regarded as a model in all the available literatures. This article put forwards a new type of uncertainty decision-making method, and makes a more systematic study of Wald-W method through the way of solving matrix game.

Keywords: Uncertain decision-making; Decision standards; Matrix game

1. Introduction

Decision-making work can be classified according to the amount of known information. Usually, in the current literatures decision-making is devided into three types including certain decision-making, risk decision-making and uncertain decision-making. In terms of uncertain decision-making, decision-makers have no sufficient information, lack profound understanding of various unborn incidents' nature states and can not predict the probability of nature state. On account of no optimal or satisfactory solution available based on the existing theoretical research findings, this kind of decision mostly depends on decision-makers' subjective judgment Therefore, uncertain decision-making is the most common but also the most difficult problem among all decision-making types.

Shackles of research means and research direction are partly the reason for the serious lag of theoretical research. In fact, as early as in 1950s, A .Wald made a research on relationship between decision theory and game theory and put forward max mini standards (also known as pessimism decision

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standard), one of five standards in uncertain decision-making. However, the further research on this project has been long stopped at the achievements acquired at the time.

This literature discovers that uncertain decision-making problem can be solved by improved method based on the matrix game, namely Wald-W method. Meanwhile, due to popularization of computer techniques, linear programming solution process has become a piece of cake. Taking linear programming as basic method of_solving matrix game makes it easier to obtain mathematical solutions of Wald-W method. Wald-W is a more widely applied method than the current five decision standards.

2. Review of research on uncertain decision-making

For a long time optimism decision standard, compromised decision standard, equality decision standard, regret decision standard and pessimism decision standard have been taken as the pattern in terms of uncertain decision-making research. These five standards are considered as the best solutions in various related literatures. And the subsequent studies are mainly conducted based on these five standards.

2.1. Five classical standards

a. Optimism decision standard

It also goes by the name of Maxi max standard. Optimism decision standard is always aim at optimism for future development, taking the proceeds of the best programs into account. It has a lot of confidence to achieve the most ideal result in each decision program. This reflects optimism and adventure spirit of decision-makers.

Use S_i for strategy set， S_i^* for decision strategy， a_{ij} for return matrix elements(the same below)， its decision strategy can be expressed as $S_i^* \to \max\max(a_{ij})$. Obviously this decision is too risky and requires bearing the risk of the corresponding loss as the price.

b. Compromised decision standard

It is also called optimism coefficient method or Hurwicz method. It is not as risky as optimists and not as conservative as pessimists, but first to identify an optimism coefficient α （$0 \le \alpha \le 1$）according to experience, then to seek a compromised benefit value in each program, last to compare compromised value H_i in all programs to choose the program with the biggest value as the best program. Its decision strategy can be expressed as $S_i^* \to \max\{H_i\} \to \max\{\alpha * a_{i_{\max}} + (1-\alpha) * a_{i_{\min}}\}$. Optimism coefficient mentioned in this method is with great subjectivity due to being determined by persons.

c. Equality decision standard or Laplace method

It sees probability of nature state as equal. If there are n nature states, the probability of each nature state will be $\frac{1}{n}$. Then calculate average value of return E（S_i）in various nature states in each program, identify the biggest value. The program with the biggest average value is the best program. Its decision strategy can be expressed as $S_i^* \to \max\{E(S_i)\} \to \max\left\{\frac{1}{n}(a_{i1} + a_{i2} + \cdots + a_{in})\right\}$. Apparently, it is biased to consider the probability of occurrence of uncertain various nature states as the same average value.

d. Regret decision standard or Savage method

It selects the best value in each state as the ideal goal and defines the difference N_{ij} between the best benefit value and the other value as the regret value when failing to reach the ideal value. Then find out the biggest regret value from each program, identify the program with the smallest regret value as the best program. Its decision strategy can be expressed as follows:

$$S_i^* \to \min \max \{N_{ij}\} \to \min \max \{\max_{1 \le k \le m} a_{kj} - a_{ij}\}$$

It is easy to validate that the best strategy of this standard lacks "independence" while strategy set change. So it is impossible to determine the best strategy.

e. Max min decision standard, also pessimism decision standard or conservative standard

It identifies the program with the biggest benefit value selected among the smallest values generated in each nature state, as the best program. Its decision strategy can be expressed as $S_i^* \to \max\min(a_{ij})$. It can be understood that this standard takes the worst condition into account and makes efforts for the best. Many literatures address that this standard is a more reliable method. The research on uncertain decision-making by A .Wald in early years also involves this problem.

An obvious problem can be found from the statements of five standards. For an uncertain decision-making to be solved, the conclusions resulting from calculation based on the above five standards are different, which usually makes people disoriented. In practice, the action strategy is selected on the basis of personal preferences of different decision-makers, which causes confusion of scientific decision. Due to diversified decision methods and otherness of the conclusion, these solutions are useless.

2.2. Subsequent research results

Ye Zefang (2000) put forward that the current various decision-making methods do not suggest comparison of reliability of decision results. For instance, in an uncertain decision-making problem, if m stands for number of decision scheme, number of nature state is n, the reliability of optimism decision standard will be $\frac{1}{mn}$; for pessimism method, reliability scope will be $\left[\frac{m-1}{mn}, \frac{mn-(n-1)}{mn}\right]$; as for equality decision standard, it will be $\frac{(m-1)!}{m!}$, namely $\frac{1}{m}$; the reliability of regret method is as same as that of pessimism method. Apparently, reliability of regret and pessimism method is better than that of equity method which is better than the reliability of optimism method. This research provides an order in terms of reliability of five decision standards.

Jiang Tao (2007) put forward Robust Optimization method which regarding application of uncertain decision-making. According to Jiang, decision-makers need know benefit values in various states by using various strategies and the difference between each benefit value and the optimum value when the future state is not sole and probability of occurrence of various states is unknown. Robust Optimization method provides optimization tools for search of this kind of strategy. It solves the uncertain decision-making problem resulted from insufficient information. But when it involves more states, more strategies and complication increases, it makes the solving process become very difficult.

Jin Juliang (2003) suggests that solving uncertain decision-making problem can be conducted in the following way. First take the projection of high-dimensional data through a certain combination onto low-dimensional subspace. An at the same time to find the projection value which makes projection function reach optimization by using some mathematic means such as projection index function (namely objective function). Then analyze structural characteristics of high-dimensional data based on this projection value. This is projection pursuit method which shows that when income opportunity risk reflected in profit and loss matrix is bigger than loss opportunity risk, projection pursuit will select the scheme with the greatest return opportunity. This decision result is as same as that of optimism decision standard. When loss opportunity risk is greater than return opportunity risk, decision result is as same as that of max min decision standard. When two risks are equal, decision result is the result of quality decision standard. Projection pursuit method has stronger adequacy than those commonly used methods. Projection pursuit actually puts forward a new calculation method. However, its complex is also enhanced with deepening of its excellence degree.

In 1974 Z. W. Kmietowicz and C. M. Cannon put forward a decision-making method, called "Knowledge incomplete decision" which is applied in case of level probability. They deem that if duplicate strategies know level probability, possible expected max min value of each program will be calculated, then take these the biggest or smallest expected values as decision criteria. It will be much better than decision standard applied for uncertain decision-making under the condition of not knowing probability at all.

If the number of nature state in a decision problem is *n*, probability will be in an order as $P_1 \geq P_2 \geq P_3 \geq \cdots \geq P_n$. If x_j stands for a decision result of a certain scheme probability of which is P_j, the expected value for this scheme will be $E(\theta) = \sum_{j=1}^{n} P_j x_j$. Therefore, solving maximum minimum expected value is to solve the following linear programming problem:

$$\max(\min) E(\theta) = \max(\min) \sum_{j=1}^{n} P_j x_j \tag{1}$$

According to the formula *(1)*, it is possible to find the schemes with the biggest or smallest expected values in each program. It is a more compromised method. When the level probability is known, decision result is much better that that of max min standard and optimism decision standard. However, like many other researches, it still does not solve the uncertain decision-making problem of general sense.

3. Wald-W method research

3.1. Existing Theoretical Foundation

Famous statistician A .Wald is the first person to relate decision theory to game theory. He put statistical decision as two-person zero-sum game between statistician and the nature and proves that Max min strategy should be selected in decision. And this is the max min standard mentioned above.

A .Wald maintains that in a decision problem experimenter hopes to make risk（F，&） be tiny, however, risk is a function including two variables F and &, experimenter can only choose decision function & and can not choose its distribution F. Also, distribution is selected by nature, which unknown to the experimenter. This situation is so similar to two person game.

American famous Bayesian statistician J.O.Berger deems that the essence of max min standard is to attempt to have alert and protection in the worst condition. Suitable fro this situation is: when the situation is decided by an intelligent opponent who will maximize your personal loss. So, decision-makers should expect the worst condition for themselves and then find solution to deal with opponents. Decision research on this situation is called game theory. It can be seen that decision and game are two aspects of one problem.

Therefore, matrix game has provided solid mathematical theory basis for solving uncertain decision-making problem. This literature continues A .Wald's ideas and puts forward more systematic Wald-W method which improves A .Wald's max min standard.

3.2. General mathematic model

Suppose there is a decision matrix of uncertain decision-making in Table 2, the number of decision programs available for decision-makers is m， written as $\{S_i | i = 1,2,\cdots,m\}$； the number of nature state is n, written as $\{E_j | j = 1,2,\cdots n\}$, benefit value for decision matrix is a_{ij}, then decision matrix will be $A = \{a_{ij} | i = 1 \sim m, j = 1 \sim n\}$.

In the above uncertain decision matrix, take decision-maker as player 1, take nature state as player 2, then suppose $X^* = (x_1, \cdots, x_m)$ is mix strategy for player 1, $Y^* = (y_1, \cdots, y_n)$ is mix strategy for player 2, then for player 1, there will be:

$$\begin{cases} \sum_i a_{ij} x_i \geq v, \ j = 1, \cdots, n \\ \sum_i x_i = 1 \\ x_i \geq 0, \ i = 1, \cdots, m \end{cases} \tag{2}$$

Translate *(2)* into a linear programming problem:

$$\begin{cases} \min \ z = \sum_i x_i / v \\ \sum_i a_{ij} x_i \geq v, \ j = 1, \cdots, n \\ x_i \geq 0, \ i = 1, \cdots, m \end{cases}$$

Similarly for player 2, there will be:

$$\begin{cases} \sum_j a_{ij} y_j \leq v, \ i = 1, \cdots, m \\ \sum_j y_j = 1 \\ y_j \geq 0, \ j = 1, \cdots, n \end{cases} \tag{3}$$

Translate *(3)* into a linear programming problem:

$$\begin{cases} \max \ w = \sum_j y_j / v \\ \sum_j a_{ij} y_j \leq v, \ i = 1, \cdots, m \\ y_j \geq 0, \ j = 1, \cdots, n \end{cases} \tag{4}$$

Vector group of $X^* = (x_1, \cdots, x_m)$ & $Y^* = (y_1, \cdots, y_n)$ and expected decision value v can be acquired by solving the above two linear programming problems.

The ultimate results from solving uncertain decision-making problem by applying Wald-W method are two vector groups $X^* = (x_1, \cdots, x_m)$, $Y^* = (y_1, \cdots, y_n)$ and expected decision value v.

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