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A note on embedding non-Abelian finite flavor groups in continuous groups

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ABSTRACT

A non-Abelian finite flavor group $G \subset SO(3)$ can have double covering $G' \subset SU(2)$ such that $G \not\subset G'$. This situation is not contradictory, but quite natural, and we give explicit examples such as $G = D_n$, $G' = Q_{2n}$ and $G = T$, $G' = T'$. This observation can be crucial in particle theory model building.

1. Introduction

One promising direction for extending the Standard Model of particle theory lies in using a flavor symmetry G_F which commutes with the gauge group. The flavor symmetry can lead to predictions for the quarks and lepton mixing angles. Typically G_F is a finite non-Abelian group.

The Standard Model gauge group G_{SM} is comprised of the direct product of continuous groups $G_{SM} \equiv SU(3) \times SU(2) \times U(1)$. The flavor group G_F is itself often a subgroup of a Lie group: $G_F \subset G_L$ Here we focus on the cases $G_L = SO(3)$ and $G_L = SU(2)$.

There can be, as we shall discuss, a pair of candidates G_F^i ($i = 1, 2$) which have the following set of seemingly contradictory properties.

- $G_F^1 \subset SO(3)$.
- $G_F^2 \subset SU(2)$.
- G_F^2 is the double covering of G_F^1 just as $SU(2)$ for $SO(3)$.
- $G_F^1 \not\subset G_F^2$.

The apparent contradiction in the last statement is only that, since $SO(3) \not\subset SU(2)$! The situation described is not uncommon. Indeed, we will show that $G_F^1 \subset G_F^2$ only when G_F^1 is Abelian. First we will set the stage by considering a few explicit examples, including an infinite series of $SU(2)$ subgroups, after a brief and self-contained survey of the small non-Abelian groups.

2. Non-Abelian groups with order $g \leqslant 31$

Some of our examples will be taken from non-Abelian groups with order $g \leqslant 31$. There are 45 such groups. Of the non-Abelian finite groups, the best known are perhaps the permutation groups S_N (with $N \geqslant 3$) of order $N!$ The smallest non-Abelian group is S_3 ($\equiv D_3$), the symmetry of an equilateral triangle with respect to all rotations in a three-dimensional sense. This group is a member of two disparate infinite series, S_N and D_N, distinct for $N > 3$. Both have elementary geometrical significance since the symmetric permutation group S_N is the symmetry of the N-plex in N dimensions while the dihedral group D_N is the symmetry including inversion of the planar N-agon in 3 dimensions.

As a family symmetry, the S_N series (and the even-parity series A_N) becomes less interesting as the order and the dimensions of the representations increase, because S_N contains transformations which independently exchange elements of the fundamental N-plet. Only S_3 and S_4 are of any interest as symmetries associated with the particle spectrum [1]. Also, the order (number of elements) of the S_N groups grow factorially with N, and the order of the dihedral groups increase only linearly with N and their irreducible representations are all one- and two-dimensional. This is reminiscent of the representations of the electroweak $SU(2)_L$ used in Nature.

Each D_N is a subgroup of $O(3)$ and has a counterpart double dihedral group Q_{2N}, of order $4N$, which is a subgroup of the double covering $SU(2)$ of $SO(3)$.

With only the use of D_N, Q_{2N}, S_N and the tetrahedral group $T = A_4$ (of order 12, the even permutations subgroup of S_4), alone and in direct products with Abelian groups, we find 32 of the 45 non-Abelian groups up to order 31. (Note that $D_6 \simeq Z_2 \times D_3$, $D_{10} \simeq Z_2 \times D_5$ and $D_{14} \simeq Z_2 \times D_7$.)

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Table 1
Table for Kronecker products of T' irreps.

$\otimes$	1_1	1_2	1_3	2_1	2_2	2_3	3
1_1	1_1	1_2	1_3	2_1	2_2	2_3	3
1_2	1_2	1_3	1_1	2_2	2_3	2_1	3
1_3	1_3	1_1	1_2	2_3	2_1	2_2	3
2_1	2_1	2_2	2_3	$1+3$	$1'+3$	$1''+3$	$2_1+2_2+2_3$
2_2	2_2	2_3	2_1	$1'+3$	$1''+3$	$1+3$	$2_1+2_2+2_3$
2_3	2_3	2_1	2_2	$1''+3$	$1+3$	$1'+3$	$2_1+2_2+2_3$
3	3	3	3	$2_1+2_2+2_3$	$2_1+2_2+2_3$	$2_1+2_2+2_3$	$1_1+1_2+1_3\ +3+3$

g	
6	$D_3 \equiv S_3$
8	D_4, $Q = Q_4$
10	D_5
12	D_6, Q_6, T
14	D_7
16	D_8, Q_8, $Z_2 \times D_4$, $Z_2 \times Q$
18	D_9, $Z_3 \times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	D_{12}, Q_{12}, $Z_2 \times D_6$, $Z_2 \times Q_6$, $Z_2 \times T$, $Z_3 \times D_4$, $D_3 \times Q$, $Z_4 \times D_3$, S_4
26	D_{13}
28	D_{14}, Q_{14}
30	D_{15}, $D_5 \times Z_3$, $D_3 \times Z_5$

There remain thirteen others formed by "twisted products" of Abelian factors. (Twisted products are a way of constructing certain group extensions; a group extension is a way of describing a group in terms of a normal subgroup and its factor group. We will see later that double covers are also a form of group extension.) Only certain such "twistings" are permissible, namely (completing all $g \leqslant 31$).

g	
16	$Z_2 \tilde{\times} Z_8$ (two, excluding D_8), $Z_4 \tilde{\times} Z_4$, $Z_2 \tilde{\times} (Z_2 \times Z_4)$ (two)
18	$Z_2 \tilde{\times} (Z_3 \times Z_3)$
20	$Z_4 \tilde{\times} Z_7$
21	$Z_3 \tilde{\times} Z_7$
24	$Z_3 \tilde{\times} Q$, $Z_3 \tilde{\times} Z_8$, $Z_3 \tilde{\times} D_4$
27	$Z_9 \tilde{\times} Z_3$, $Z_3 \tilde{\times} (Z_3 \times Z_3)$

It can be shown that these thirteen exhaust the classification of *all* inequivalent non-Abelian groups up to order thirty-one [2].

Of these 45 non-Abelian groups, the dihedrals (D_N) and double dihedrals (Q_{2N}), of order $2N$ and $4N$ respectively, form the simplest sequences. In particular, they can be realized as subgroups of $O(3)$ and $SU(2)$ respectively, the two simplest non-Abelian continuous groups. A finite group is completely described by its presentation as a multiplication table, but it is often more convenient to use a more compact description. One equivalent and compact representation is the character table, and for physical applications this is preferred because it also summarizes the vector representations of the group. For the D_N and Q_{2N}, the tables for Kronecker products, as derivable from the character tables, are simple to express in general [3]. These tables, whose role is to describe the decomposition of tensor products of irreducible vector representations (irreps) as a direct sum of irreps, may be degenerate among different finite groups which have distinct character tables. The character table, on the other hand, just like the presentation as a multiplication table contains the complete information about a group.

Table 2
Table for Kronecker products of T irreps.

$\otimes$	1	$1'$	$1''$	3
1	1	$1'$	$1''$	3
$1'$	$1'$	$1''$	1	3
$1''$	$1''$	1	$1'$	3
3	3	3	3	$1+1'+1''+3+3$

3. *SU*(2) as the double cover of *SO*(3)

Continuous groups which have the same structure for small transformations, described by the Lie algebra, can have various global forms distinguished by their discrete subgroups. For compact Lie groups, each simple Lie algebra corresponds to one global form which is connected and simply-connected; other global forms are obtained through taking a quotient by some subgroup of the center of the group, constructing an extension by a discrete symmetry of the group which is not continuously related to the identity, or some combination of such operations.

In particular, since $SU(2)$ is the double cover of $SO(3)$, their Lie algebras are isomorphic. Conversely, $SO(3)$ is obtained from $SU(2)$ if we identify pairs of elements which differ by a 2π rotation. The irreps of $SO(3)$ have dimensions equal to odd integers $1, 3, 5, \ldots$ while the irreps of $SU(2)$ appear with all positive integers $1, 2, 3, \ldots$. In particular $SU(2)$ has defining irrep the **2** which is a spinor and the corresponding rotations change sign after any odd number of 2π rotations becoming the identity only after an integer multiple of 4π rotations. This is what we mean by saying that $SU(2)$ is the double cover of $SO(3)$: the group manifold is topologically S^3/Z_2 while that of $SU(2)$ is S^3.

4. Finite subgroups of *SO*(3) and *SU*(2)

Let us consider an explicit example. The tetrahedral group T is the symmetry of a regular tetrahedron. As such it is a subgroup of the rotations in three space dimensions: $T \subset SO(3)$. From the visualization it is isomorphic to the even permutations of four objects so $T \subset S_4$. The irreps of T are $1_1, 1_2, 1_3, 3$.

Next consider the binary tetrahedral group T'. This group is more challenging to visualize as a symmetry. From standard treatises we learn that T' is the double cover of T. The irreps of T' are $1_1, 1_2, 1_3, 2_1, 2_2, 2_3, 3$ with the table for Kronecker products shown in Table 1.

We note that the group T has irreps $1_1, 1_2, 1_3, 3$ which are a subset of those of T' and that their table for Kronecker products shown in Table 2 is contained within that (Table 1) for T'.

With all this circumstantial evidence the reader might well suspect from a physical perspective that $T \subset T'$. The present Letter will show, however, that this is interestingly incorrect and will discuss in detail why $T \not\subset T'$. This can first be straightforwardly confirmed by using the representation of T in $SO(3)$ to construct the corresponding rotations in $SU(2)$. In $SU(2)$ these elements do not close by themselves, because a 2π rotation is -1. Instead,

Table 3
Table for Kronecker products of Q_8 irreps.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_2	1_2	1_1	1_4	1_3	2_3	2_2	2_1
1_3	1_3	1_4	1_1	1_2	2_1	2_2	2_3
1_4	1_4	1_3	1_2	1_1	2_3	2_2	2_1
2_1	2_1	2_3	2_1	2_3	$1_1+1_3+2_2$	2_1+2_3	$1_2+1_4+2_2$
2_2	2_2	2_2	2_2	2_2	2_1+2_3	$1_1+1_2+1_3+1_4$	2_1+2_3
2_3	2_3	2_1	2_3	2_1	$1_2+1_4+2_2$	2_1+2_3	$1_1+1_3+2_2$

T' has two subsets of elements which correspond to T and $-T$ (pointwise).

From a more mathematical point of view, the binary tetrahedral group T', and the tetrahedral group T live in a short exact sequence

$$1 \to Z_2 \to T' \to T \to 1,$$

which does not split, hence T' is not a semidirect product of $Z_2 \times T$, i.e., T' has no subgroup isomorphic to T.[1]

Upon further scrutiny we realize that this is also a reflection of the following property. Since the group T has no 2-dimensional irrep, its embedding in $SU(2)$ can only be such that the defining **2** of $SU(2)$ contains two one-dimensional representations of T. In that case, only an Abelian subgroup of T is represented (because any sum of one-dimensional representations is Abelian), and all irreps of $SU(2)$ will also contain only one-dimensional irreps of T. There can exist no nontrivial embedding of T in $SU(2)$. Instead T fits perfectly in $SO(3)$ as expected. Similarly T' cannot be embedded in $SO(3)$ but embeds regularly in $SU(2)$.

For the purposes of particle theory model building T' *acts as if* it contains T as a subgroup only because Table 1 contains Table 2. Strictly speaking $T \not\subset T'$ and any group properties which are more subtle and which must rely on characters will possibly reflect this crucial fact. From another perspective T' *acts as if* it contains T as a subgroup only because T' is a central extension of T. The element -1 is added to T in a nontrivial and consistent way, such that it commutes with all of the elements of T, and the order of each element is doubled.

Is this a peculiar property only of $SO(3) \supset T \not\subset T' \subset SU(2)$? One remark may help clarify the possibilities: $SU(2)$ has exactly one element of order 2 (which is the matrix -1). Therefore it cannot contain any subgroup with more than one element of order 2 (which rules out all of the dihedral groups), or any subgroup whose element of order 2 does not commute with other elements.

We may quickly cite an infinite number of further examples. The dihedral groups D_n are subgroups of $SO(3)$ and their double covers are the dicyclic groups Q_{2n} which are subgroups of $SU(2)$. The irreps for *e.g.* Q_8 are $1_1, 1_2, 1_3, 1_4, 2_1, 2_2, 2_3$ with the table for Kronecker products given in Table 3.

For comparison we note that Q_8 is the double cover of D_4 which has irreps $1_1, 1_2, 1_3, 1_4, 2$. Their table for Kronecker products is shown as Table 4.

We see that Table 4 is contained in Table 3 and that, from [2], $D_4 \not\subset Q_8$.

Thus the case $T \not\subset T'$ is not unique yet it is important to recall this relationship of T to T' when building models of particle theory based on the binary tetrahedral group.

[1] We thank John Ratcliffe and Steve Tschantz for a discussion of this point.

Table 4
Table for Kronecker products of D_4 irreps.

$\otimes$	1_1	1_2	1_3	1_4	2
1_1	1_1	1_2	1_3	1_4	2
1_2	1_2	1_1	1_4	1_3	2
1_3	1_3	1_4	1_1	1_2	2
1_4	1_4	1_3	1_2	1_1	2
2	2	2	2	2	$1_1+1_2+1_3+1_4$

5. Direct products, semidirect products and central extensions

A general group extension can be represented by a short exact sequence

$$1 \to H \to G \to K \to 1,$$

which expresses the conditions that H is a normal subgroup of G and $G/H = K$. We can distinguish three classes of extensions from the rest: direct products, semidirect products, and central extensions.

Direct products are extensions of a trivial sort: H and K are both realized as subgroups of G, and the elements of each subgroup commute with those of the other; more concisely, $G = HK$, $[H,K] = 1$.

A semidirect product is one in which the first property holds, but the multiplication in H is twisted by an action of the elements of K:

$$g = h_1k_1h_2k_2 = h_1\left(k_1h_2k_1^{-1}\right)k_1k_2 = h_1h_2^{k_1}k_1k_2,$$

where the notation $h_2^{k_1}$ is defined by the parenthesized expression.

A central extension is characterized by the property that H is a subgroup of the center of G, $Z(G)$ (which consists of elements commuting with all of G). The point is that an extension can be both central and a semidirect product only if it is in fact a direct product: since $H \subset Z(G)$, $K \subset G$ implies $[H,K] = 1$. Thus $SU(2)$ is a central extension of $SO(3)$, and if $K \subset SO(3)$, the corresponding extension $K' \subset SU(2)$ contains K only if K is Abelian.

Concerning our explicit examples, T is not a subgroup of T' because $SO(3)$ is not a subgroup of $SU(2)$; T' is a double cover of T because $SU(2)$ is a double cover of $SO(3)$; T' is a central extension of T because $SU(2)$ is a central extension of $SO(3)$; T is a central quotient of T' because $SO(3)$ is a central quotient of $SU(2)$. The same statements hold *mutatis mutandis* replacing T, T' respectively by D_n, Q_{2n}.

6. Discussion

The present discussion can stand on its own as an original contribution to the mathematics of group theory.

It is also germane to model building for quarks and leptons. As an example, consider the T' model of quark masses and neutrino masses and mixings discussed in [3]. There the leptons are in singlets and triplets of T', while the quarks are in singlets and doublets. If for some reason one wished to break the T' quark mass relations without disrupting the neutrino sector, i.e., without disturbing tribimaximal mixing (TBM), one could hope to give a VEV to a scalar in a nontrivial T' irrep. The only possibility would appear to be a T' doublet VEV. However, VEVs for doublets do not leave an unbroken A_4 symmetry to protect TBM. Furthermore, not all T' irreps can be decomposed into A_4 irreps, as can easily be seen from studying the character tables of the two groups. (As a specific example, the $2'$ is such a T' irrep.) This means that in the T' model there is no consistent way to reduce the quark symmetry, while temporarily preserving TBM. Other models can be analysed in a similar fashion.

As evidenced by [4], the groups D_n, Q_{2n}, T, T', etc., have been used extensively as flavor symmetries in the literature. Many of these papers contain too hasty statements which can in the future be avoided so thereby the present article contributes to embedding non-Abelian finite flavor groups in continuous groups.

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