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Parallel Dual Tabu Search for Capacitor Placement in Smart Grids

Yoshihiro Ogita^a, Hiroyuki Mori^{a,*}

^aMeiji University, Tama-ku, Kawasaki, 214-8571, Japan

Abstract

This paper proposes parallel dual tabu search (PDTS) for capacitor placement in smart grids. The proposed method makes use of the dual code of the initial value to make solution candidates diverse. The capacitor placement is one of important problems to maintain the nodal voltage profile in the distribution network level of smart grids. The problem formulation of capacitor placement may be expressed as a combinatorial optimization problem of capacitor banks. In this paper, a new method is proposed to improve the performance of tabu search (TS) of meta-heuristics. Although meta-heuristics are not affected by the initial value in smaller systems, they are inclined to give different solutions with different initial values in large-scale problems. In practice, it is very important to set up a good initial value for tabu search although a priori knowledge is not available in general. This paper focuses on the dual code of the initial value to create another initial value systematically. Mathematically, the dual code is orthogonal to the original one so that the solution search is carried out to keep the diversity of solution candidates from two most different directions. The effectiveness of the proposed method is demonstrated in a sample system.

Keywords: Optimization Methods; Dual Code; Parallel Algorithm; Meta-heuristics; Smart Grid

1. Introduction

This paper presents parallel dual tabu search for capacitor placement in the distribution network level of smart grids. According to the global environmental issues and energy diversification, renewable energy has been introduced into energy networks in the world. However, the generation output significantly depends on weather conditions during a short term and brings about new issues. The reverse power flow from renewable energy to the network raises the nodal voltage and creates surplus electricity in case where the renewable energy is connected to the network. In addition, it is necessary to prepare the ancillary service to carry out the network frequency control. In recent years, the concept of Smart Grid is widely spread to carry out the integration of energy networks with information and communication technology (ICT) at both generation and demand sides. It plays a key role to make energy networks more efficient in a way that home, building, and factory consumers are connected to distributed power providers with solar, wind, fuel cells, energy storage systems, *etc.* Unlike the conventional scheme of energy networks, Smart Grid has the function of bilateral networks of energy and information to control transmission and

* Corresponding author to provide phone: +81-44-9347353; fax: +81-44-9347909.
E-mail address: hmori@isc.meiji.ac.jp

distribution networks in real time and keep the balance between the supply and the demand of electricity efficiently. To smooth Smart Grid, advanced optimization methods are required to carry out grid operation and planning.

In this paper, the optimal capacitor placement is discussed to maintain the nodal voltage magnitude in distribution networks. With the significant penetration of distributed generation, voltage control becomes much important in distribution networks. This paper proposes a new algorithm for the optimal capacitor placement in distribution networks. It aims at improving the solution quality for the optimal capacitor placement. The capacitor placement means a combinatorial optimization problem that determines the location and the capacity of capacitor banks in discrete number and minimizes the network losses while satisfying the constraints on the nodal voltage magnitude, *etc.* A lot of studies have been done to improve the performance of the methods. The conventional methods on capacitor placement may be classified as dynamic programming (DP) [1], [2], nonlinear programming (NLP) [3], [4], [5], mixed integer programming [6], *etc.* Mathematical programming is not easy to apply to large-scale problems due to the limitation that it is time-consuming or gives approximate solutions. In fact, the combinations of the location and the banks of capacitors drastically increase with the problem size. The problem to be solved is highly related to the nonlinearity that creates a lot of local minima. Therefore, more efficient methods are required to deal with the optimal capacitor placement in Smart Grid. Recently, meta-heuristics are quite popular for solving combinatorial optimization problems efficiently. It means one of efficient optimization methods that iteratively make use of simple rules or heuristics to obtain better solutions that correspond to a globally optimal solution or their highly approximate ones. As typical meta-heuristics, simulated annealing (SA) [7], genetic algorithms (GA) [8] and tabu search (TS) [9], [10] are well-known and applied to the capacitor placement problem [11-13]. SA is analogous to the heat bath in metal annealing in a way that it introduces a parameter called temperature into the algorithm to control the probability of state transition and converge to a solution at low temperature. GA stems from the natural selection in biology so that a set of solution candidates called population are prepared to carry out multi-point search. It makes use of genetic operators such as cross-over, mutation *etc.* to create new better solutions. The genetic operators are controlled by the probability to keep the diversity of solution candidates. TS is based on combining the hill climbing method of local search with the adaptive memory that some attributes of the solution are fixed for a period while others are updated. In other words, it makes use of the tabu list that temporally stores the attributes for a period. That allows the search process to escape from a local minimum and evaluate better solutions efficiently. In the meta-heuristic methods above, TS is more effective than SA and GA in terms of solution accuracy and computational effort. Furthermore, to improve the performance of the meta-heuristic methods, SA and GA have been modified to the parallel versions such as parallel SA (PSA) [14], parallel GA (PGA) [15] and parallel TS (PTS) [16], respectively. In this paper, a new tabu search is proposed to improve the performance of TS and PTS. Although meta-heuristics are not affected by the initial value in small systems, they are inclined to give different solutions with different initial values in large-scale problems. There is still room for improvement in PTS. To improve the performance, this paper focuses on the dual code of an initial value to create the most distant initial value to keep the diversification of solution candidates. Mathematically, the dual code is orthogonal to the original one. Namely, it is created by inverting each bit at the coded variables. A given initial value and its dual code find out a solution from the most different directions. That allows the solution search to keep the diversity to evaluate better solutions efficiently. In this paper, the proposed method is referred to as parallel dual tabu search (PDTS). The effectiveness of the proposed method is demonstrated in the 69-node distribution system. A comparison is made of SA, GA, TS, DTS, PSA, PGA, PTS and the proposed method in terms of solution accuracy and computational effort.

2. Formulation of optimal capacitor placement in smart grids

In this section, the formulation of optimal capacitor placement is described in smart grids. The capacitor placement is one of important problems to maintain the nodal voltage magnitude of distribution network level of smart grids. It may be expressed as a combinatorial optimization problem with some constraints. Specifically, the objective is to determine the location and the size of capacitor banks that give the minimum cost function with respect to the distribution network losses and installation cost. The constraint on the voltage means that nodal voltage magnitudes are maintained within the upper and the lower bounds. The constraint on the capacitor banks indicates the limitation on the capacity at each node. In this paper, the location, the capacity and the type of capacitors are optimized under the constant loads. The cost function consists of the distribution network losses and the capacitor installation cost. The mathematical formulation may be written as:

Cost function:

$$f_0 = k_p p(\mathbf{x}) + \sum_{i=1}^n \sum_{j=1}^c z_{ij} r_{cij} q_{ij} \rightarrow \text{Min} \quad (1)$$

Constraints

$$\mathbf{g}(\mathbf{x}, \mathbf{q}) = 0 \quad (2)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad (3)$$

$$0 \leq q_{ij} \leq q_i^{\max} \quad (4)$$

where

f_0 : cost function

i : node number

j : capacitor bank number

k_p : cost coefficient for active power distribution network loss

$p(\bullet)$: active power distribution network loss

\mathbf{x} : nodal voltage vector

z_{ij} : 0 (or 1) variables denoting switching-off (or switching-on) of shunt capacitor number j at node i

r_{cij} : cost efficient per unit capacity for shunt capacitors

q_{ij} : amount of reactive power to be installed

n : total number of nodes

c : total number of capacitor banks

$\mathbf{g}(\bullet)$: power flow equation

\mathbf{q} : reactive power vector such that $\mathbf{q} = [q_{11}, q_{12}, \dots, q_{nc}]^T$

V_i : nodal voltage magnitude at node i

$V_i^{\min} (V_i^{\max})$: lower (upper) bound of the nodal voltage magnitude at node i

q_i^{\max} : capacitor size max at node i

The first and the second terms in (1) denote the distribution network loss and the capacitor installation cost, respectively. The loss is determined through the state vector after evaluating binary variable z_{ij} of the switching-on/off conditions of shunt capacitors. Equation (2) shows the equality constraint on the power flow equation in distribution networks. In this paper, DistFlow is used as the power flow equation due to the good performance [5], [6]. Equation (3) gives the inequality constraint on the nodal voltage magnitude at each node. It is necessary to maintain the voltage magnitude within the upper and the lower bounds. Equation (4) means the inequality constraint on the capacitor capacity at each node. To solve (1)-(4), the following cost function is used:

$$f = k_p p(\mathbf{x}) + \sum_{i=1}^n \sum_{j=1}^c z_{ij} r_{cij} q_{ij} + \alpha_1 f_V + \alpha_2 f_Q \rightarrow \text{Min} \quad (5)$$

where

f : cost function

f_V : penalty function on voltage magnitude

f_Q : penalty function on reactive power such

α_1 : penalty factor for f_V

α_2 : penalty factor for f_Q

It should be noted that the above equation is the cost function with the penalty functions of constraints (3)-(4). Specifically, this paper decomposes the problem into three parts:

- to generate new state z_{ij} with a method
- to carry out DistFlow to compute the state vector
- to compute the cost function of (5)

3. Parallel Tabu Search (PTS)

In this section, parallel tabu search (PTS) is described [16]. It has a couple of parallel schemes to improve the performance of TS. One is the decomposition of the neighbourhood to reduce the computational effort while the other is the multiplicity of the tabu length to improve solution accuracy. First, the decomposition of the neighbourhood is described. The calculation of TS is very time-consuming in large-scale problems since it computes

the cost function of solution candidates in the neighbourhood around a given solution or an initial solution. In practice, the large-scale problems often give large neighbourhood even though the neighbourhood is defined as a set of solution candidate with the Hamming distance equal to 1. Therefore, PTS introduces the decomposition of the neighbourhood with parallel processors to reduce computational time in calculating the neighbourhood. If m processors are used, PTS decomposes a neighbourhood into m subneighbourhoods and calculates the solution candidates by each processor, where the subneighbourhoods imply subsets of neighbourhoods. The decomposition scheme allows the algorithm of TS to reduce CPU time to $1/m$. After selecting the best solution in each subneighbourhood, the final solution is eventually selected from the best solutions in subneighbourhoods. Fig. 1 shows the concept of the neighbourhood decomposition, where it is assumed that a solution candidate has 10 bit attributes and two processors handle the decomposition of the neighbourhood and the neighbourhood is defined as a set of solution candidates with the Hamming distance equal to 1. In Fig. 1, the original 10 bits are divided into two groups in which Processors 1 and 2 handle Subneighbourhoods 1 and 2, respectively. In Subneighbourhood 1, the neighbourhood is created for from Attributes #1 to #5 while other attributes #6 -#10 are fixed. On the other hand, in Subneighbourhood 2, Attributes #1 - #5 are fixed while the neighbourhood is created for other attributes #6-#10. Therefore, it is expected that the decomposition of the neighbourhood into subneighbourhoods reduces the computational effort. Next, the multiple tabu lengths are described. Since TS has only one parameter called the tabu length, it is better than SA and GA in turning up the parameters. However, the performance of TS deteriorates in large-scale problems due to the huge search space. It is important to find out better solutions from different directions rather than from only one direction for a long period. Namely, it is effective to make the solution search process more diverse. The multiple tabu lengths mean that n processors with different tabu lengths find out the solution in parallel. PTS with the multiple tabu lengths evaluates better solutions than the conventional TS with only one tabu length. Fig. 2 shows the concept of multiple tabu lengths, where Tabu Lengths a , b and c are used for initial conditions x_0 and neighbourhood N_0 . Since the different tabu lengths give different solutions, the multiple tabu lengths make the search process more diverse to find out better solutions effectively. For convenience, PTS(m,n) that means PTS with m neighbourhoods and n tabu lengths is used in this paper.

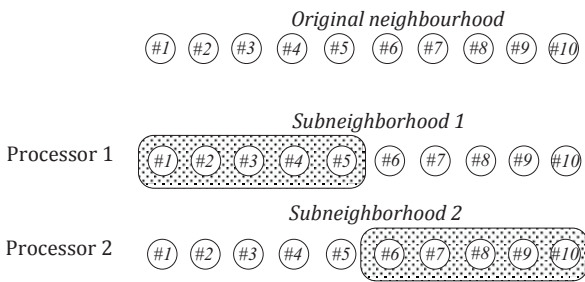


Fig. 1. Concept of neighborhood decomposition

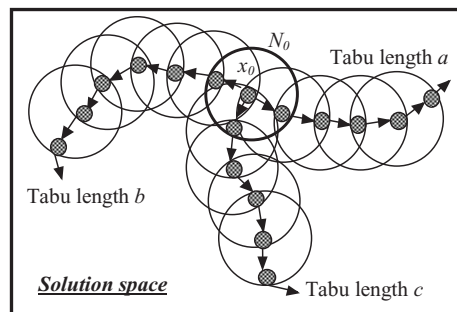


Fig. 2. Search process of PTS

4. Proposed method

This section proposes parallel dual tabu search (PDTs) for capacitor placement in smart grids. The proposed method makes use of the dual code of initial value to evaluate better solutions in PTS. The conventional PTS repeats the deterministic search process as shown in Fig. 2. However, it is very hard to find out better solutions as the problem size becomes larger. Since PTS is inclined to be dependent upon the initial solution in large-scale problems, an appropriate strategy is required to handle the selection of initial value. This paper focuses on the use of dual code that is orthogonal to a given initial vector [17]. Fig. 3 denotes the creation of initial solution, where the alternative initial solution is created by inverting the bits of a given initial solution through the concept of dual code.

Now, suppose that Vector x_1 is given in binary number and define the dual code as x_2 . The relationship between Vectors x_1 and x_2 may be written as

$$\langle x_1, x_2 \rangle = 0 \tag{6}$$

where, $\langle \#, * \rangle$: inner product of # and *

It should be noted that Vector x_2 is the most distant from Vector x_1 . Thus, it is expected that the use of two vectors may improve the solution quality due to the different initial values. Fig. 4 shows the search process of PDTs, where

two different initial values x_1 and x_2 play an import role to make the solution process more diverse to find out better solutions from a standpoint of the most different directions. PDTS is different from PTS as shown in Fig. 2 in a sense that the search process of PDTS becomes more diverse. Therefore, it is expected that PDTS is superior to PTS in terms of solution quality.

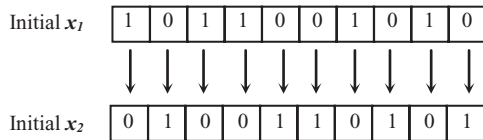


Fig. 3. Creation of initial solution

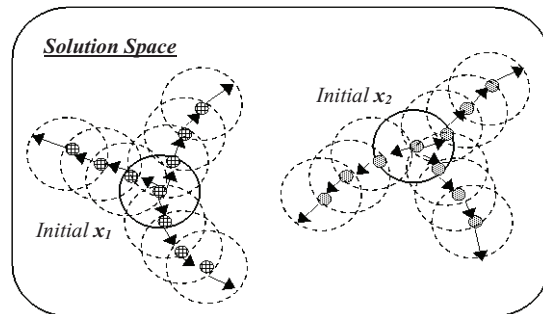


Fig. 4. Search process of PDTS

5. Simulation

5.1 Simulation Conditions

- 1) The proposed method is successfully applied to the 69-node distribution network. Table 1 shows the capacitor type size and the unit cost of available shunt capacitors [18]. It is assumed that at most six capacitor banks may be used at each node. The cost coefficient of the active power distribution network loss is considered to be $k_p=526$. Also, it is assumed that the upper bound of reactive power to be installed at each node is defined as the total amount of reactive power loads. The upper and the lower bounds of the voltage magnitude are set to be 0.90 [p.u.] and 1.10 [p.u.], respectively. According to the preliminary simulation, Penalty Factors α_1 and α_2 are set to be 1000 in (5), respectively.
- 2) Each node has six bits that express the installation conditions of Capacitor Types 1-6. In other words, each bit shows the capacitor type. The combinations of capacitor banks are expressed as 414 bits in the 69-node distribution network, which result in 4.23×10^{124} combinations.
- 3) The proposed method is compared with the conventional SA, PSA, GA, PGA, TS, PTS and DTS. Table 2 shows the parameters of each method, where they are tuned up by the preliminary simulation. TS, PTS, DTS and PDTS make use of the aspiration level as a strategy of TS. The multiplicity of tabu lengths is set to be two and three. The decomposition of the neighborhood needs two and three processors to handle three subneighborhoods.
- 4) The neighborhood of TS, PTS, DTS and PDTS is defined as a set of solution candidates around the solution obtained at the previous iteration. They are created by applying the Hamming distance of unity to it.
- 5) To examine the influence of the initial conditions on the performance of each method, one hundred of initial conditions are prepared for each method. They are created by the uniform random generator.

5.2 Simulation Results

Table 3 shows the results in the 69-node distribution network, where the best, the average, the worst and the standard deviation of the cost functions and the number of feasible solutions are given to understand the performance. Looking at the best cost function, DTS gives better results than TS, GA and SA. Actually, DTS, TS, GA and SA have the cost function of 25.54, 25.57, 34.76 and 84.54, respectively. The trend of the performance is applied to the parallel version of DTS, TS, GA and SA. Parallel methods, PDTS(3,3), PDTS(2,2), PTS(3,3), PTS(2,2), PGA and PSA have the cost function of 23.75, 24.58, 25.57, 25.57, 29.98 and 41.48, respectively, where PDTS(3,3) means PDTS with 3 neighborhoods and 3 tabu lengths and the same as above is applied to the other parallel methods. The trend is applied to the case of the average and the worst cost functions. Regarding the number of the feasible solutions obtained from 100 initial conditions, PDTS(3,3) gives 100 feasible solutions. It is

Table 1. Each capacitor type size and unit cost

Capacitor types	Size[kvar]	Unit cost r_c [\$/kvar]
1	150	0.500
2	300	0.325
3	750	0.276
4	900	0.183
5	1200	0.170
6	1500	0.201

Table 2. Parameters of each method

Method	Parameter	
SA	Cooling schedule	0.999996
	Initial temperature	20000
	Convergence criterion	80000
GA	Population	100
	Crossover rate	0.8
	Mutation rate	0.01
TS/DTS	No. of generations	7000
	Tabu length	41
	Maximum iterations	1500
PSA	Cooling schedule	0.999996
	Initial temperature	20000
	No. of states	30
PGA	Convergence criterion	80000
	Population	300
	No. of Islands	3
PTS/	Crossover rate	0.8
	Mutation rate	0.01
	Migration	100
PDTS	No. of generations	7000
	2 Tabu lengths	41, 43
	3 Tabu lengths	38, 41, 43
	No. of subneighborhoods	2, 3
	Maximum iterations	1500

Table 3. Evaluation of cost functions and CPU time of each method

Method	Cost functions					CPU time	
	Best	Average	Worst	Standard Deviation	No. of feasible solutions	Average (min)	Ratio to the proposed method
SA	84.538	742.112	1528.625	334.510	0	5374	29.69
GA	34.758	52.312	156.450	32.002	10	1433	7.92
TS	25.572	25.615	25.797	0.786	60	517	2.86
PSA	41.484	340.27	614.360	49.825	0	5994	33.12
PGA	29.981	44.269	96.316	25.601	24	1596	8.82
PTS(2,2)	25.568	25.603	25.717	0.693	86	344	1.90
PTS(3,3)	25.567	25.592	25.644	0.653	90	179	0.98
DTS	25.542	25.592	25.950	0.544	86	528	2.92
PDTS(2,2)	24.580	25.120	25.550	0.444	96	338	1.87
PDTS(3,3)	23.750	24.255	24.600	0.389	100	181	1.00

Note) PTS(m, n), and PDTS(m, n) indicate PTS and PDTS with m subneighborhoods and n tabu lengths, respectively.

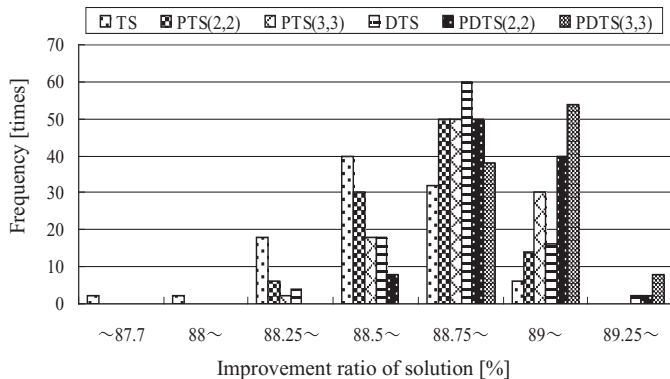


Fig. 5. Frequency distribution of improvement ratio

noteworthy that PDTS succeeded in evaluating the feasible solutions for all the initial conditions. However, PDTS(2,2), DTS, PTS(2,2), PTS(3,3), TS, PSA, SA, PGA and GA provides 96, 86, 86, 90, 60, 0, 0, 24 and 10 feasible solutions, respectively. Looking through the standard deviation of the cost functions, PDTS has much better standard deviation than others. PDTS(3,3) has the standard deviation of 0.389 although TS gives that of

0.786. It can be observed the solution quality is significantly improved from a standpoint of the solution distribution. Fig. 5 shows the improvement ratio of TS, PTS, DTS and PDTS to the original network conditions, where it shows the extent to which they reduce the cost function of (5) with the capacitor banks. It can be seen that PDTS (3,3) improves 89.45 % of the best cost function for initial value. It follows that the duality of initial values makes the search process wider to evaluate better results. In other words, the duality of initial values remarkably improves the solution quality. Therefore, PDTS outperforms others in terms of solution accuracy. Also, Table 3 gives the average CPU time and its ratio to the proposed method. It can be seen that PDTS(3,3) is 1.87-times, 2.92-times, 1.90-times, 0.98-times, 2.86-times, 33.12-times, 29.69-times, 8.82-times and 7.92-times faster than PDTS(2,2), DTS, PTS(2,2), PTS(3,3), TS, PSA, SA, PGA, and GA, respectively. The CPU time of the proposed method is worse than that of PTS (3,3), but it is sufficiently acceptable due to the small difference of 2%. Thus, the proposed is useful for dealing with the optimal capacitor placement in the distribution level of smart grids.

6. Conclusion

This paper has proposed a new tabu search for the optimal capacitor placement in the distribution network level of smart grids. The proposed method made use of the dual code for a given initial value and evaluated better solution with a couple of initial values that were orthogonal to each other. The proposed method has advantage that it diversifies the search space to improve the solution. The proposed method was applied to the sample system. The proposed method was much better than TS and PTS in terms of solution quality. For example, PDTS (3,3) of the proposed method reduced 5.22 % of the average cost function and 40.43 % of the standard deviation of the cost function for the conventional PTS (3,3) in the sample system. The proposed method is more robust in a sense that it is not so influenced by the initial values. Therefore, PDTS is more efficient for evaluating better solutions in the optimal capacitor placement from a standpoint of global optimization.

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