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Existence and global attractivity of positive periodic solutions for the impulsive delay Nicholson's blowflies model

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Abstract

In this paper we shall consider the following nonlinear impulsive delay population model:

$$\begin{cases} x'(t) = -\delta(t)x(t) + p(t)x(t-m\omega)e^{-\alpha(t)x(t-m\omega)} & \text{a.e. } t > 0, \quad t \neq t_k, \\ x(t_k^+) = (1+b_k)x(t_k), \quad k = 1, 2, \ldots, \end{cases} \tag{0.1}$$

where m is a positive integer, $\delta(t)$, $\alpha(t)$ and $p(t)$ are positive periodic continuous functions with period $\omega > 0$. In the nondelay case ($m=0$), we show that (0.1) has a unique positive periodic solution $x^*(t)$ which is globally asymptotically stable. In the delay case, we present sufficient conditions for the global attractivity of $x^*(t)$. Our results imply that under the appropriate linear periodic impulsive perturbations, the impulsive delay equation (0.1) preserves the original periodic property of the nonimpulsive delay equation. In particular, our work extends and improves some known results.

MSC: 34K15; 92D25; 34C25

Keywords: Existence; Global attractivity; Positive periodic solution; Impulsive; Delay differential equation

1. Introduction

The theory of impulsive differential equations has attracted the interest of many researchers in the past twenty years [1,2,11,13,12,14,16,17,26] since they provide a natural description of several real processes subject to certain perturbations whose duration is negligible in comparison with the duration of the process. Such processes are often investigated in various fields of science and technology such as physics, population dynamics, ecology, biological systems, optimal control, etc. For details, see [1,11] and references therein. Recently, the corresponding theory for impulsive functional differential equations has been studied by several authors [2,5,12–14,24].

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Recently, Saker and Agarwal studied the following periodic red blood cells model [23]:

$$N'(t) = -\delta(t)N(t) + p(t)e^{-aN(t-m\omega)},$$

and the delay periodic Hematopoiesis model [18]

$$p'(t) = -\delta(t)p(t) + \frac{\beta(t)}{1+p^n(t-m\omega)}.$$

If we take into account the effect of linear impulsion, then the corresponding impulsive systems of the above two models take the forms

$$\begin{cases} N'(t) = -\delta(t)N(t) + p(t)e^{-aN(t-m\omega)} & \text{a.e. } t > 0, \quad t \neq t_k, \\ N(t_k^+) = (1+b_k)N(t_k), \quad k = 1, 2, \ldots, \end{cases} \tag{1.1}$$

and

$$\begin{cases} p'(t) = -\delta(t)p(t) + \frac{\beta(t)}{1+p^n(t-m\omega)} & \text{a.e. } t > 0, \quad t \neq t_k, \\ p(t_k^+) = (1+b_k)p(t_k), \quad k = 1, 2, \ldots . \end{cases} \tag{1.2}$$

The general form of the former is the impulsive Lasota–Wazewska model

$$\begin{cases} x'(t) = -\alpha(t)x(t) + \sum_{i=1}^{m} \beta_i(t)e^{-\gamma_i(t)x(t-m_i\omega)} & \text{a.e. } t > 0, \quad t \neq t_k, \\ x(t_k^+) = (1+b_k)x(t_k), \quad k = 1, 2, \ldots, \end{cases} \tag{1.3}$$

which was investigated by Yan [26], and he obtained sufficient conditions for the existence and global attractivity of the positive periodic solution.

It is easy to see that the nonlinear terms of (1.1) and (1.2) satisfy

$$\lim_{N\to 0} \frac{e^{-aN}}{N} = \infty \quad \text{and} \quad \lim_{N\to\infty} \frac{e^{-aN}}{N} = 0,$$

and

$$\lim_{p\to 0} \frac{1/(1+p^n)}{p} = \infty \quad \text{and} \quad \lim_{p\to\infty} \frac{1/(1+p^n)}{p} = 0,$$

respectively. In view of this property, Huo et al. [7] considered a more general system

$$\begin{cases} x'(t) + \alpha(t)x(t) = p(t)f(x(t-\sigma(t))) & \text{a.e. } t > 0, \quad t \neq t_k, \\ x(t_k^+) = (1+b_k)x(t_k), \quad k = 1, 2, \ldots, \end{cases} \tag{1.4}$$

under the superlinear condition

$$\lim_{x\to 0} \frac{f(x)}{x} = 0, \quad \lim_{x\to\infty} \frac{f(x)}{x} = \infty, \tag{1.5}$$

or the sublinear condition

$$\lim_{x\to 0} \frac{f(x)}{x} = \infty, \quad \lim_{x\to\infty} \frac{f(x)}{x} = 0, \tag{1.6}$$

and obtained sufficient conditions for the existence and global attractivity of the positive periodic solution of Eq. (1.4).

In 1980, Gurney et al. [6] proposed a mathematical model

$$N'(t) = -\delta N(t) + PN(t-\tau)e^{-aN(t-\tau)}, \tag{1.7}$$

to describe the dynamics of Nicholson's blowflies. Here, $N(t)$ is the size of the population at time t, P is the maximum per capita daily egg production, $1/a$ is the size at which the population reproduces at its maximum rate, δ is the pair capita daily adult death rate, and τ is the generation time. For more details of (1.7) and its discrete analog, see

[8–10,15]. We also note that several other similar interesting models occurring in population dynamics have been studied in [7,19–22,28,29].

Recently, taking into account the effect of a periodically varying environment, Saker and Agarwal [21] considered the following delay periodic Nicholson's Blowflies model:

$$x'(t) = -\delta(t)x(t) + p(t)x(t - m\omega)\mathrm{e}^{-\alpha(t)x(t-m\omega)}, \quad t \geqslant 0, \tag{1.8}$$

and obtained sufficient conditions for existence and global attractivity of positive periodic solution of Eq. (1.8). Obviously,

$$\lim_{x\to 0} \frac{x\mathrm{e}^{-\alpha(t)x}}{x} = 1 \quad \text{and} \quad \lim_{x\to\infty} \frac{x\mathrm{e}^{-\alpha(t)x}}{x} = 0,$$

which does not satisfy the condition (1.5) or (1.6).

Motivated by the above question, in the present paper, we study the existence and global attractivity of positive periodic solutions of the following impulsive delay differential equation

$$\begin{cases} x'(t) = -\delta(t)x(t) + p(t)x(t - m\omega)\mathrm{e}^{-\alpha(t)x(t-m\omega)} & \text{a.e. } t > 0, \quad t \neq t_k, \\ x(t_k^+) = (1 + b_k)x(t_k), \quad k = 1, 2, \ldots, \end{cases} \tag{1.9}$$

where m is a positive integer, $\delta(t)$, $\alpha(t)$ and $p(t)$ are positive periodic continuous functions with period $\omega > 0$. In the nondelay case ($m = 0$), we shall show that (1.9) has a unique positive periodic solution $x^*(t)$, which is global asymptotically stable. In the delay case, we shall present sufficient conditions for persistence of Eq. (1.9), and establish sufficient conditions for the global attractivity of $x^*(t)$. Our results imply that under the appropriate linear periodic impulsive perturbations, the impulsive delay (1.9) preserves the original periodic property of the nonimpulsive delay equation. In particular, our work extends and improves the results of Kulenovic et al. [10] and Saker and Agarwal [22] for the nonimpulsive delay population models.

For system (1.9), we make the following further assumptions:

(A_1) $0 < t_1 < t_2 < \cdots$ are fixed impulsive points with $t_k \to +\infty$ as $k \to \infty$;
(A_2) $\{b_k\}$ is a real sequence and $b_k > -1$, $k = 1, 2, \ldots$;
(A_3) $\prod_{0<t_k<t}(1 + b_k)$ is a periodic function with period $\omega > 0$, $m \geqslant 0$ is an integer.

Throughout this paper we always assume that a product equals unit if the number of factors is zero. And let

$$\overline{f} = \frac{1}{\omega}\int_0^\omega f(s)\,\mathrm{d}s, \quad f_m = \min_{t\in[0,\omega]} f(t) \quad \text{and} \quad f_M = \max_{t\in[0,\omega]} f(t), \tag{1.10}$$

where f is a continuous periodic positive function with period ω. We shall consider Eq. (1.9) with the initial condition

$$x(t) = \varphi(t) \text{ for } t \in [-m\omega, 0]; \quad \varphi(t) \in C([-m\omega, 0], [0, \infty)), \quad \varphi(0) > 0. \tag{1.11}$$

Definition 1.1. A function $x \in ([-m\omega, +\infty), (0, +\infty))$ is said to be a solution of equation(1.9) on $[-m\omega, +\infty)$ if:

(i) $x(t)$ is absolutely continuous on each interval $(0, t_1]$ and $(t_k, t_{k+1}]$, $k = 1, 2, \ldots$;
(ii) for any t_k, $k = 1, 2, \ldots$, $x(t_k^+)$ and $x(t_k^-)$ exist and $x(t_k^-) = x(t_k)$;
(iii) $x(t)$ satisfies the former equation of (1.9) for almost everywhere in $[0, +\infty)\backslash\{t_k\}$ and satisfies the latter equation for every $t = t_k$, $k = 1, 2, \ldots$.

Under the hypotheses (A_1)–(A_3), we consider the nonimpulsive delay differential equation

$$z'(t) = -\delta(t)z(t) + p(t)z(t - m\omega)\mathrm{e}^{-\gamma(t)z(t-m\omega)}, \quad t \geqslant 0, \tag{1.12}$$

with initial condition

$$z(t) = \varphi(t) \text{ for } t \in [-m\omega, 0], \quad \varphi(t) \in C([-m\omega, 0], [0, \infty)), \quad \varphi(0) > 0, \tag{1.13}$$

where

$$\gamma(t) = \prod_{0<t_k<t} (1+b_k)\alpha(t) \quad \text{for } t>0. \tag{1.14}$$

By a solution of (1.12) and (1.13) we mean an absolutely continuous function $z(t)$ defined on $[-m\omega, +\infty)$ satisfying (1.12) a.e. for $t \geqslant 0$ and $z(t) = \varphi(t)$ on $[-m\omega, 0]$.

The following lemma will be useful to prove our results. The proof is similar to that of Theorem 1 in [27]. For the sake of completeness, we list it here.

Lemma 1.2. *Assume that* (A_1)–(A_3) *hold. Then*

(i) *if* $z(t)$ *is a solution of* (1.12) *on* $[-m\omega, +\infty)$, *then* $x(t)=\prod_{0<t_k<t}(1+b_k)z(t)$ *is a solution of* (1.9) *on* $[-m\omega, +\infty)$;
(ii) *if* $x(t)$ *is a solution of* (1.9) *on* $[-m\omega, +\infty)$, *then* $z(t) = \prod_{0<t_k<t}(1+b_k)^{-1}x(t)$ *is a solution of* (1.12) *on* $[-m\omega, +\infty)$.

Proof. We only prove the case (i), the second case can be proved similarly.

Suppose that $z(t)$ is a solution of (1.12) on $[-m\omega, +\infty)$, then we have

$$\begin{aligned} x'(t) &= \prod_{0<t_k<t}(1+b_k)z'(t) \\ &= \prod_{0<t_k<t}(1+b_k)(-\delta(t)z(t) + p(t)z(t-m\omega)e^{-\gamma(t)z(t-m\omega)}) \\ &= -\delta(t)x(t) + \prod_{0<t_k<t}(1+b_k)p(t)z(t-m\omega)e^{-\gamma(t)z(t-m\omega)}. \end{aligned}$$

By (A_3), we have

$$\prod_{0<t_k<t-m\omega}(1+b_k) = \prod_{0<t_k<t}(1+b_k),$$

and hence

$$x(t-m\omega) = \prod_{0<t_k<t-m\omega}(1+b_k)z(t-m\omega) = \prod_{0<t_k<t}(1+b_k)z(t-m\omega).$$

Therefore,

$$x'(t) = -\delta(t)x(t) + p(t)x(t-m\omega)e^{-\alpha(t)x(t-m\omega)},$$

which implies that $x(t)=\prod_{0<t_k<t}(1+b_k)z(t)$ satisfies the first equation of (1.9) for almost everywhere in $[0, +\infty)\backslash\{t_k\}$.

On the other hand, for every $t = t_k$, $k = 1, 2, \ldots,$

$$x(t_k^+) = \lim_{t\to t_k^+} \prod_{0<t_j<t}(1+b_j)z(t) = \prod_{0<t_j\leqslant t_k}(1+b_j)z(t_k),$$

and

$$x(t_k) = \prod_{0<t_j<t_k}(1+b_j)z(t_k).$$

This means that, for every $k = 1, 2, \ldots,$

$$x(t_k^+) = (1+b_k)x(t_k).$$

From the above analysis, we know that the conclusion of Lemma 1.2 is true. The proof is complete. □

Lemma 1.3. *Assume that* (A_1)–(A_3) *hold. Then the solutions of* (1.9) *are defined on* $[-m\omega, +\infty)$ *and are positive on* $[0, \infty)$.

Proof. By Lemma 1.2, we only need to prove that the solutions of (1.12) and (1.13) are defined on $[-m\omega, +\infty)$ and are positive on $[0, +\infty)$. From (1.12) and (1.13) we have that for any $\varphi(t) \in C([-m\omega, 0], [0, \infty))$ and $t > 0$,

$$z(t) = \varphi(0)\mathrm{e}^{-\int_0^t \delta(t)\,\mathrm{d}t} + \int_0^t p(s)z(s - m\omega)\mathrm{e}^{-\int_s^t \delta(r)\,\mathrm{d}r}\mathrm{e}^{-\gamma(s)z(s-m\omega)}\,\mathrm{d}s.$$

Hence, $z(t)$ is defined on $[-m\omega, \infty)$ and is positive on $[0, \infty)$. The proof is complete. □

2. Results in the Nondelay case

In this section, we study the periodic and asymptotic behavior of all positive solutions of (1.12) without delay. We shall prove that there exists a unique positive periodic solution $z^*(t)$ which is globally asymptotically stable.

Now, we consider (1.12) without delay, i.e.,

$$z'(t) = -\delta(t)z(t) + p(t)z(t)\mathrm{e}^{-\gamma(t)z(t)}, \quad t \geqslant 0. \tag{2.1}$$

Lemma 2.1. *Assume that* (A_1)–(A_3) *hold and* $p(t) \leqslant \delta(t)$ *for* $t \in [0, \omega]$. *Then Eq.* (2.1) *has no positive periodic solutions.*

Proof. By Lemma 1.3, we know that any solution of Eq. (2.1) with initial condition (1.13) remains positive, then from (2.1) we have

$$z'(t) = -\delta(t)z(t) + p(t)z(t)\mathrm{e}^{-\gamma(t)z(t)} < -\delta(t)z(t) + p(t)z(t) \leqslant 0, \tag{2.2}$$

which implies that system (2.1) has no positive periodic solutions. The proof is complete. □

Lemma 2.2. *Assume that* (A_1)–(A_3) *hold and* $p(t) \leqslant \delta(t)$ *for* $t \in [0, \omega]$. *Then the zero solution of Eq.* (2.1) *is globally asymptotically stable.*

Proof. Let $z(t)$ be a positive solution of Eq. (2.1). From the proof of Lemma 2.1, it is easy to see that if $p(t) \leqslant \delta(t)$ for $t \in [0, \omega]$, then $z(t)$ is decreasing, therefore, $\lim_{t\to\infty} z(t) = \rho \in [0, +\infty)$. We only need to show that $\rho = 0$. Otherwise, if $\rho > 0$, then there exists a $T > m\omega$ such that $z(t) > \rho/2$ for $t \geqslant T$. Now we consider two cases: $p(t) \equiv \delta(t)$ for $t \in [0, \omega]$ and $p(t) \leqslant (\not\equiv) \delta(t)$ for $t \in [0, \omega]$.

If the former case holds, then from (2.2) we have

$$\begin{aligned} z'(t) &= -\delta(t)z(t) + \delta(t)z(t)\mathrm{e}^{-\gamma(t)z(t)} = -\delta(t)z(t)(1 - \mathrm{e}^{-\gamma(t)z(t)}) \\ &\leqslant -\delta(t)z(t)(1 - \mathrm{e}^{-\gamma(t)\rho/2}) \leqslant -\delta_m z(t)(1 - \mathrm{e}^{-\gamma_m \rho/2}), \end{aligned}$$

and so

$$z(t) \leqslant z(0)\exp\left\{-\int_0^t \delta_m[1 - \mathrm{e}^{-\gamma_m\rho/2}]\,\mathrm{d}s\right\}.$$

Let $t \to \infty$, then $z(t) \to 0$, which contradicts the fact $\rho > 0$.

Either if the latter case holds, then from (2.2) we have

$$z'(t) < -\delta(t)z(t) + p(t)z(t) < (p(t) - \delta(t))\frac{\rho}{2},$$

and so

$$
\begin{aligned}
z(t) &\leqslant z(T) - \frac{\rho}{2}\int_T^t (\delta(s) - p(s))\,\mathrm{d}s \\
&\leqslant z(T) - \frac{\rho}{2}\sum_{k=1}^{[(t-T)/\omega]}\int_{T+(k-1)\omega}^{T+k\omega} (\delta(s) - p(s))\,\mathrm{d}s \\
&= z(T) - \frac{\rho}{2}\left[\frac{t-T}{\omega}\right](\overline{\delta} - \overline{p})\omega,
\end{aligned}
$$

where $[(t-T)/\omega]$ is the greatest integer parts of $(t-T)/\omega$. Let $t \to \infty$, then $z(t) \to -\infty$. This contradiction shows that $\rho = 0$, i.e., $\lim_{t\to\infty} z(t) = 0$. The proof is complete. □

In the sequel, we shall consider the case

$$
p(t) > \delta(t) \quad \text{for } t \in [0, \omega]. \tag{2.3}
$$

In order to establish the uniqueness and global attractivity of the positive periodic solution of (2.1), we need to obtain certain upper and lower bounds.

Lemma 2.3. *Assume that* (A_1)–(A_3) *and* (2.3) *hold. Let* $z(t)$ *be a positive solution of* (2.1). *Then*

$$
\left(\frac{p(t) - \delta(t)}{p(t)\gamma(t)}\right)_m \leqslant z(t) \leqslant \left(\frac{p(t)}{\gamma(t)\delta(t)}\right)_M.
$$

Proof. By the inequality $\mathrm{e}^{-au} \geqslant 1 - au (a > 0, u \geqslant 0)$, and also in view of (2.1), we have

$$
\begin{aligned}
z'(t) &= -\delta(t)z(t) + p(t)z(t)\mathrm{e}^{-\gamma(t)z(t)} \geqslant -\delta(t)z(t) + p(t)z(t)[1 - \gamma(t)z(t)] \\
&= z(t)[(p(t) - \delta(t)) - p(t)\gamma(t)z(t)],
\end{aligned}
$$

then from comparison theorem [4, Lemma 2.2, 25], we have

$$
z(t) \geqslant \left(\frac{p(t) - \delta(t)}{p(t)\gamma(t)}\right)_m.
$$

On the other hand, from (2.1) and the inequality $u\mathrm{e}^{-au} \leqslant 1/ae$ $(a > 0, u \geqslant 0)$, we have

$$
z'(t) = -\delta(t)z(t) + p(t)z(t)\mathrm{e}^{-\gamma(t)z(t)} \leqslant -\delta(t)z(t) + \frac{p(t)}{\gamma(t)e},
$$

and so

$$
z(t) \leqslant z(0)\mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} + \mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s}\int_0^t \frac{p(s)}{\gamma(s)e}\mathrm{e}^{\int_0^s \delta(v)\,\mathrm{d}v}\,\mathrm{d}s.
$$

Thus

$$
\begin{aligned}
z(t) &\leqslant z(0)\mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} + \mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} \int_0^t \frac{p(s)}{\gamma(s)e} \mathrm{e}^{\int_0^s \delta(v)\,\mathrm{d}v}\,\mathrm{d}s \\
&= z(0)\mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} + \mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} \int_0^t \frac{p(s)}{\gamma(s)\delta(s)e} \delta(s)\mathrm{e}^{\int_0^s \delta(v)\,\mathrm{d}v}\,\mathrm{d}s \\
&\leqslant z(0)\mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} + \mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} \left(\frac{p(s)}{\gamma(s)\delta(s)e}\right)_M \int_0^t \delta(s)\mathrm{e}^{\int_0^s \delta(v)\,\mathrm{d}v}\,\mathrm{d}s \\
&= z(0)\mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} + \mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} \left(\frac{p(s)}{\gamma(s)\delta(s)e}\right)_M \int_0^t (\mathrm{e}^{\int_0^s \delta(v)\,\mathrm{d}v})'\,\mathrm{d}s \\
&= z(0)\mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} + \mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} \left(\frac{p(s)}{\gamma(s)\delta(s)e}\right)_M (\mathrm{e}^{\int_0^t \delta(s)\,\mathrm{d}s} - 1) \\
&= z(0)\mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s} + \left(\frac{p(s)}{\gamma(s)\delta(s)e}\right)_M (1 - \mathrm{e}^{-\int_0^t \delta(s)\,\mathrm{d}s}) \\
&\overset{t\to\infty}{\rightarrow} \left(\frac{p(s)}{\gamma(s)\delta(s)e}\right)_M < \left(\frac{p(t)}{\gamma(t)\delta(t)}\right)_M.
\end{aligned}
$$

Hence for t sufficiently large,

$$
z(t) \leqslant \left(\frac{p(t)}{\gamma(t)\delta(t)}\right)_M.
$$

The proof is complete. □

Theorem 2.4. *Assume that* (A_1)–(A_3) *and* (2.3) *hold. Then*

(a) *there exists a unique* ω*-periodic positive solution* $z^*(t)$ *of* (2.1), *and*
(b) *for every other positive solution* $z(t)$ *of* (2.1),

$$
\lim_{t\to\infty} (z(t) - z^*(t)) = 0. \tag{2.4}
$$

Proof. First, we prove (a), set

$$
z_1 = \left(\frac{p(t) - \delta(t)}{p(t)\gamma(t)}\right)_m \quad \text{and} \quad z_2 = \left(\frac{p(t)}{\gamma(t)\delta(t)}\right)_M.
$$

Suppose that $z(t) = z(t, 0, z_0)$ where $z_0 > 0$ is the unique solution of (2.1) which passes through $(0, z_0)$. We claim that if $z_0 \in [z_1, z_2]$, then $z(t) \in [z_1, z_2]$ for all $t \geqslant 0$. Otherwise, let $t_* = \inf\{t > 0 : z(t) > z_2\}$, then there exists a $t^* \geqslant t_*$ such that $z(t^*) > z_2$ and $z'(t^*) \geqslant 0$. However,

$$
\begin{aligned}
z'(t^*) &= -\delta(t^*)z(t^*) + p(t)z(t^*)\mathrm{e}^{-\gamma(t^*)z(t^*)} \\
&\leqslant -\delta(t^*)z(t^*) + \frac{p(t^*)}{\gamma(t^*)e} < -\delta(t^*)z_2 + \frac{p(t^*)}{\gamma(t^*)} < 0,
\end{aligned}
$$

which is a contradiction. By a similar argument, we can show that $z(t) \geqslant z_1$ for all $t \geqslant 0$. Hence, we have in particular that $z_\omega = z(\omega, 0, z_0) \in [z_1, z_2]$ for $z_0 \in [z_1, z_2]$. Define a mapping $F : [z_1, z_2] \to [z_1, z_2]$ as follows: for each $z_0 \in [z_1, z_2]$, $F(z_0) = z_\omega$. Since the solution $z(t, 0, z_0)$ depends continuously on the initial value z_0, it follows that F is continuous and maps the interval $[z_1, z_2]$ into itself. Therefore, F has a fixed point z_0^*. Then $z(t, 0, z_0^*) := z^*(t)$ is an ω-periodic positive solution of (2.1). The proof of (a) is complete. □

Now we prove (b). Assume that $z(t) \geqslant z^*(t)$ for t sufficiently large (the proof when $z(t) < z^*(t)$ is similar and will be omitted). Set

$$z(t) = z^*(t)\exp\{x(t)\}. \tag{2.5}$$

Then, $x(t) \geqslant 0$ for t sufficiently large,

$$z'(t) = -\delta(t)z^*(t)\exp\{x(t)\} + p(t)z^*(t)\exp\{x(t)\}\mathrm{e}^{-\gamma(t)z^*(t)\exp\{x(t)\}},$$

and

$$\begin{aligned} z'(t) &= z^{*\prime}(t)\exp\{x(t)\} + z^*(t)\exp\{x(t)\}x'(t) \\ &= -\delta(t)z^*(t)\exp\{x(t)\} + p(t)z^*(t)\exp\{x(t)\}\mathrm{e}^{-\gamma(t)z^*(t)} + z^*(t)\exp\{x(t)\}x'(t). \end{aligned}$$

Combination of the above two equalities leads to

$$\begin{aligned} x'(t) &= p(t)[\mathrm{e}^{-\gamma(t)z^*(t)\exp\{x(t)\}} - \mathrm{e}^{-\gamma(t)z^*(t)}] \\ &= p(t)\mathrm{e}^{-\gamma(t)z^*(t)}[\mathrm{e}^{-\gamma(t)z^*(t)[\exp\{x(t)\}-1]} - 1]. \end{aligned}$$

Thus, to prove (2.4), it is sufficient to show that

$$\lim_{t\to+\infty} x(t) = 0. \tag{2.6}$$

Now we define a Lyapunov function V for Eq. (2.1) in the form

$$V(t) = V(x(t)) = (\mathrm{e}^{x(t)} - 1)^2, \quad t \geqslant 0.$$

The derivative of $V(t)$ along a solution of Eq. (2.1) can be written as

$$V'(t) = 2(\mathrm{e}^{x(t)} - 1)\mathrm{e}^{x(t)}p(t)\mathrm{e}^{-\gamma(t)z^*(t)}[\mathrm{e}^{-\gamma(t)z^*(t)[\exp\{x(t)\}-1]} - 1] \triangleq -f(t). \tag{2.7}$$

Obviously, $f(t) \geqslant 0$. Integrating (2.7) from 0 to t yields

$$V(t) + \int_0^t f(s)\,\mathrm{d}s \leqslant V(0) < \infty,$$

which implies that $f \in L_1[0, \infty)$. Since $z(t)$ and $z^*(t)$ are both absolutely continuous functions, $x(t)$ is also absolutely continuous on $[0, \infty)$. By Barbalat's lemma [3], we can obtain (2.6), which follows that $\lim_{t\to\infty}(z(t) - z^*(t)) = 0$. This complete the proof of (b). □

Remark 2.5. If $\gamma(t) = a$ ($a > 0$ is a constant), then Eq. (2.1) reduces to the equation

$$z'(t) = -\delta(t)z(t) + p(t)z(t)\mathrm{e}^{-az(t)}, \quad t \geqslant 0, \tag{2.8}$$

which has been studied by Saker and Agarwal [22]. Clearly, our Theorem 2.4 improves Theorem 2.1 of Saker and Agarwal [22] by the weaker condition $p(t) > \delta(t)$ for $t \in [0, \omega]$ than $p_m > \delta_M$ in [22].

Remark 2.6. For Eq. (2.8), Saker and Agarwal [22] proved that (2.4) holds under the further condition $a(z^*(t))_m > 1$. However, our Theorem 2.4 means that this condition is not necessary.

Remark 2.7. From the proof of Theorem 2.4, it follows that the unique ω-periodic solution $z^*(t)$ of (2.1) satisfies $z_1 \leqslant z^*(t) \leqslant z_2$. Thus, an interval for the location of $z^*(t)$ is readily available.

By Lemmas 1.2, 2.1, 2.2 and Theorem 2.4, we can easily obtain the following result.

Theorem 2.8. *Assume that* $m = 0$ *and* (A_1)–(A_3) *hold.*

(i) *If* $p(t) \leqslant \delta(t)$ *for* $t \in [0, \omega]$, *then Eq.* (1.9) *has no positive periodic solutions, and for every other positive solution* $x(t)$ *of* (1.9), $\lim_{t\to\infty} x(t) = 0$.

(ii) *If* $p(t) > \delta(t)$ *for* $t \in [0, \omega]$, *then Eq.* (1.9) *has a unique positive* ω*-periodic solution* $x^*(t)$, *and for every other positive solution* $x(t)$ *of* (1.9), $\lim_{t\to\infty}(x(t) - x^*(t)) = 0$.

3. Results in the delay case

In this section, we shall derive sufficient conditions for $x^*(t)$ to be a global attractor of all the other positive solutions of Eq. (1.9).

It is clear that if $z^*(t)$ is an ω-periodic positive solution of (2.1), then it is also an ω-periodic positive solution of (1.12). Conversely, if (1.12) and (1.13) has an ω-periodic positive solution $z^*(t)$, then $z^*(t)$ is an ω-periodic positive solution of (2.1). Hence, Eq. (1.12) has a unique ω-periodic positive solution $z^*(t)$ under the assumptions of Theorem 2.4. By Lemma 1.2, Eq. (1.9) has a unique ω-periodic positive solution $x^*(t) := \prod_{0<t_k<t}(1 + b_k)z^*(t)$ under the assumptions of Theorem 2.4. In the nondelay case, we have seen in Theorem 2.4 (b) that every positive solution of Eq. (1.12) converges to the unique ω-periodic positive solution $z^*(t)$. In the delay case, by a similar process as that in the proof of Theorem 2.4 (b), it is easy to see that the following result holds.

Theorem 3.1. *Assume that* (A_1)–(A_3) *and* (2.3) *hold. Then any solution* $z(t)$ *of* (1.12) *which does not oscillate about* $z^*(t)$ *satisfies*

$$\lim_{t\to\infty}(z(t) - z^*(t)) = 0. \tag{3.1}$$

To show that $x^*(t)$ is a global attractor of (1.9), by Lemma 1.2, we only need to prove that $z^*(t)$ is a global attractor of (1.12). In order to obtain the global attractivity of $z^*(t)$ for Eq. (1.12), we need the following lemmas.

Lemma 3.2. *Assume that c and d are positive constants. The following statements hold.*

(i) *If* $cd < 1$, *then the algebraic equation* $g(u) = (u + c)e^{-du} - c$ *has two real roots* $u_1 = 0$, $u_2 > 0$ *and* $g(u) > 0$ *for* $u \in (0, u_2)$, $g(u) < 0$ *for* $u \in (-\infty, 0) \cup (u_2, +\infty)$.
(ii) *If* $cd = 1$, *then the algebraic equation* $g(u) = (u + c)e^{-du} - c$ *has only one real root* $u_1 = 0$ *and* $g(u) \leqslant 0$ *for* $u \in (-\infty, +\infty)$.
(iii) *If* $cd > 1$, *then the algebraic equation* $g(u) = (u + c)e^{-du} - c$ *has two real roots* $u_1 = 0$, $u_2 < 0$ *and* $g(u) > 0$ *for* $u \in (u_2, 0)$, $g(u) < 0$ *for* $u \in (-\infty, u_2) \cup (0, +\infty)$.
(iv) *If* $cd < 1$, *then* $g(u) \leqslant (1 - cd)u$ *for* $u \geqslant 0$.

The proof of Lemma 3.2 is trivial, we omit it here.

Lemma 3.3. *Assume that* (A_1)–(A_3) *and* (2.3) *hold. If*

$$p(t)e^{-\gamma(t)z^*(t)}(1 - \gamma(t)z^*(t)) < \delta(t) \quad \text{for } t \in [0, \omega]. \tag{3.2}$$

Then

$$e^{\int_t^{t-\sigma}\delta(s)\,ds} + \int_{t-\sigma}^{t} p(s)e^{-\gamma(s)z^*(s)}(1 - \gamma(s)z^*(s))e^{\int_t^s \delta(\zeta)d\zeta}\,ds < 1$$

for any $t \in (0, +\infty)$, $\sigma > 0$ *and* $\gamma(s)z^*(s) < 1$ *for* $s \in [t - \sigma, t]$.

Proof. By condition (3.2), it is easy to see that

$$\begin{aligned}
&e^{\int_t^{t-\sigma}\delta(s)\,ds} + \int_{t-\sigma}^{t} p(s)e^{-\gamma(s)z^*(s)}(1 - \gamma(s)z^*(s))e^{\int_t^s \delta(v)\,dv}\,ds \\
&\quad < e^{\int_t^{t-\sigma}\delta(s)\,ds} + \int_{t-\sigma}^{t} \delta(s)e^{\int_t^s \delta(v)\,dv}\,ds = 1.
\end{aligned}$$

This completes the proof. □

Theorem 3.4. *Assume that* (A_1)–(A_3), (2.3) *and* (3.2) *hold. Suppose further that*

$$\limsup_{t\to\infty} \int_{t-m\omega}^{t} p(s)\exp\left(\int_t^s \delta(v)\,\mathrm{d}v\right)\mathrm{d}s = \theta < 1. \tag{3.3}$$

Then any positive solution $z(t)$ *of* (1.12) *satisfies* (3.1).

Proof. We have already established the asymptotic stability for the solution $z(t)$ of (1.12) which does not oscillate about $z^*(t)$ in Theorem 3.1, so we only need to prove the solution $z(t)$ of (1.12) strictly oscillating about $z^*(t)$ also satisfies (3.1).

Set

$$w(t) = z(t) - z^*(t), \tag{3.4}$$

then we can obtain

$$w'(t) = -\delta(t)w(t) + p(t)\mathrm{e}^{-\gamma(t)z^*(t)}[(w(t-m\omega) + z^*(t))\mathrm{e}^{-\gamma(t)w(t-m\omega)} - z^*(t)]. \tag{3.5}$$

From (3.4), we know that the solution $z(t)$ of (1.12) oscillates about $z^*(t)$ if and only if the solution $w(t)$ of (3.5) oscillates about zero. Let $w(t)$ be an arbitrary solution of (3.5) and $m\omega \leqslant \xi_1 < \xi_2 < \cdots < \xi_n < \cdots$ be a sequence of zero points of $w(t)$ with $\lim_{n\to\infty}\xi_n = \infty$ such that $w(t)$ has both positive and negative values in each interval (ξ_n, ξ_{n+1}). Let t_n and s_n be the points in (ξ_n, ξ_{n+1}) such that $w(t_n) = \max_{\xi_n \leqslant t \leqslant \xi_{n+1}} w(t)$ and $w(s_n) = \min_{\xi_n \leqslant t \leqslant \xi_{n+1}} w(t)$. Then

$$w(t_n) > 0, \quad w'(t_n) = 0; \qquad w(s_n) < 0, \quad w'(s_n) = 0, \quad \text{for all } n \geqslant 1. \tag{3.6}$$

If we denote

$$\mu = \limsup_{t\to\infty} w(t) \quad \text{and} \quad \lambda = \liminf_{t\to\infty} w(t),$$

then

$$\mu = \limsup_{n\to\infty} w(t_n) \quad \text{and} \quad \lambda = \liminf_{n\to\infty} w(s_n).$$

We claim that for each $n \geqslant 1$,

$$w(t) \text{ has a zero point } T_n \in [t_n - m\omega, t_n), \tag{3.7}$$

and

$$w(t) \text{ has a zero point } S_n \in [s_n - m\omega, s_n). \tag{3.8}$$

First, we show that (3.7) is true. We divide the proof into two cases.

Case 1: If $t_n - m\omega \leqslant \xi_n$, then the conclusion is obvious. In fact, we can choose $T_n = \xi_n$.

Case 2: If $t_n - m\omega > \xi_n$, then according to the definition of t_n, we have

$$w(t_n) \geqslant w(t_n - m\omega).$$

By (3.5) and (3.6), we have

$$0 = w'(t_n) = -\delta(t_n)w(t_n) + p(t_n)\mathrm{e}^{-\gamma(t_n)z^*(t_n)}\{[w(t_n - m\omega) + z^*(t_n)]\mathrm{e}^{-\gamma(t_n)w(t_n-m\omega)} - z^*(t_n)\},$$

which implies that

$$[w(t_n - m\omega) + z^*(t_n)]\mathrm{e}^{-\gamma(t_n)w(t_n-m\omega)} - z^*(t_n) = \delta(t_n)w(t_n) > 0.$$

Consider the function

$$g(u) = (u + c)\mathrm{e}^{-\mathrm{d}u} - c,$$

where

$$c = z^*(t_n) > 0 \quad \text{and} \quad d = \gamma(t_n) > 0.$$

In terms of Lemma 3.2, we divide the proof of Case 2 into three subcases.

Subcase 1: If $cd > 1$, then $u_2 < w(t_n - m\omega) < 0$. Notice that $w(t_n) > 0$, thus (3.7) holds.

Subcase 2: If $cd = 1$, then this case is impossible.

Subcase 3: If $cd < 1$, then $0 < w(t_n - m\omega) < u_2$. Assume that (3.7) does not hold true, then we can obtain $w(t) > 0$ for $t \in [t_n - m\omega, t_n]$. Notice that $\gamma(t_n)z^*(t_n) < 1$, then there exists a sufficiently small positive real number $\tau > 0$ such that

$$w(t) \geqslant w(t - m\omega), \quad w(t) > 0, \quad w(t - m\omega) > 0, \quad \gamma(t)z^*(t) < 1 \ \text{ for } t \in [t_n - \tau, t_n].$$

Multiplying Eq. (3.5) by $\exp(\int_0^t \delta(s)\,\mathrm{d}s)$ and then integrating it over $[t_n - \tau, t_n]$, we get

$$w(t_n) = w(t_n - \tau)\exp\left(\int_{t_n}^{t_n - \tau} \delta(s)\,\mathrm{d}s\right) + \int_{t_n-\tau}^{t_n} p(s)\mathrm{e}^{-\gamma(s)z^*(s)}\exp\left(\int_{t_n}^{s} \delta(s)\,\mathrm{d}s\right)[(w(s - m\omega) + z^*(s))\mathrm{e}^{-\gamma(s)w(s-m\omega)} - z^*(s)]. \tag{3.9}$$

By Lemma 3.2 (iv), we have

$$(w(s - m\omega) + z^*(s))\mathrm{e}^{-\gamma(s)w(s-m\omega)} - z^*(s) \leqslant (1 - \gamma(s)z^*(s))w(s - m\omega) \quad \text{for } s \in [t_n - \tau, t_n].$$

Then (3.9) implies

$$\begin{aligned} w(t_n) &\leqslant w(t_n)\exp\left(\int_{t_n}^{t_n-\tau} \delta(s)\,\mathrm{d}s\right) + \int_{t_n-\tau}^{t_n} p(s)\mathrm{e}^{-\gamma(s)z^*(s)}\exp\left(\int_{t_n}^{s} \delta(s)\,\mathrm{d}s\right)(1 - \gamma(s)z^*(s))w(s - m\omega)\,\mathrm{d}s \\ &\leqslant w(t_n)\exp\left(\int_{t_n}^{t_n-\tau} \delta(s)\,\mathrm{d}s\right) + w(t_n)\int_{t_n-\tau}^{t_n} p(s)\mathrm{e}^{-\gamma(s)z^*(s)}\exp\left(\int_{t_n}^{s} \delta(s)\,\mathrm{d}s\right)(1 - \gamma(s)z^*(s))\,\mathrm{d}s. \end{aligned}$$

Thus

$$\exp\left(\int_{t_n}^{t_n-\tau} \delta(s)\,\mathrm{d}s\right) + \int_{t_n-\tau}^{t_n} p(s)\mathrm{e}^{-\gamma(s)z^*(s)}\exp\left(\int_{t_n}^{s} \delta(s)\,\mathrm{d}s\right)(1 - \gamma(s)z^*(s))\,\mathrm{d}s \geqslant 1$$

for $\gamma(s)z^*(s) < 1$, $s \in [t_n - \tau, t_n]$. By Lemma 3.3, this is a contradiction. Hence we reach the conclusion (3.7).

Similarly, we can prove (3.8).

Now, let

$$G(t, u) = \mathrm{e}^{-\gamma(t)z^*(t)}\{[u + z^*(t)]\mathrm{e}^{-\gamma(t)\ u} - z^*(t)\}.$$

Then

$$\frac{\partial G(t,u)}{\partial u} = \exp(-\gamma(t)z^*(t))(1 - \gamma(t)z^*(t) - \gamma(t)u)\mathrm{e}^{-\gamma(t)u}.$$

Clearly, Eq. (3.5) can be written as

$$w'(t) = -\delta(t)w(t) + p(t)F(t)w(t - m\omega), \tag{3.10}$$

where

$$F(t) = \left.\frac{\partial G(t,u)}{\partial u}\right|_{u=\xi(t)} = \exp(-\gamma(t)z^*(t))(1 - \gamma(t)z^*(t) - \gamma(t)\xi(t))\mathrm{e}^{-\gamma(t)\xi(t)},$$

and $\xi(t)$ lies between 0 and $w(t-m\omega)$. Set

$$h(t,v)=\mathrm{e}^{-\gamma(t)v}(1-\gamma(t)v),$$

where

$$v=z^*(t)+\xi(t).$$

Since $\xi(t)$ lies between 0 and $w(t-m\omega)$, v lies between $z^*(t-m\omega)$ and $z(t-m\omega)$, then

$$|h(t,v)|=\mathrm{e}^{-\gamma(t)v}|1-\gamma(t)v|\leqslant 1,$$

which implies

$$|F(t)|\leqslant 1.$$

Multiplying Eq. (3.10) by $\exp(\int_0^t \delta(s)\,\mathrm{d}s)$ and then integrating it over $[T_n, t_n]$ and $[S_n, s_n]$, respectively, we have

$$w(t_n)=\int_{T_n}^{t_n} p(s)F(s)\exp\left(\int_{t_n}^{s}\delta(\xi)\,\mathrm{d}\xi\right)w(s-m\omega)\,\mathrm{d}s, \tag{3.11}$$

and

$$-w(s_n)=\int_{S_n}^{s_n} -p(s)F(s)\exp\left(\int_{t_n}^{s}\delta(\xi)\,\mathrm{d}\xi\right)w(s-m\omega)\,\mathrm{d}s. \tag{3.12}$$

Now we can conclude that for any given $\varepsilon>0$, there exists a sufficiently large n_0 such that for all $t\geqslant t_{n_0}$, $w(t-m\omega)<\mu+\varepsilon$ and $-w(t-m\omega)<-\lambda+\varepsilon$. Hence, $|w(t-m\omega)|\leqslant H+\varepsilon$ for $t\geqslant t_{n_0}$, where $H=\max\{\mu,-\lambda\}$. Then from (3.11) and (3.12), we have

$$0\leqslant w(t_n)\leqslant\left[\int_{t_n-m\omega}^{t_n} p(s)\exp\left(\int_{t_n}^{s}\delta(s)\,\mathrm{d}s\right)\mathrm{d}s\right](H+\varepsilon), \tag{3.13}$$

and

$$0\leqslant -w(s_n)\leqslant\left[\int_{s_n-m\omega}^{s_n} p(s)\exp\left(\int_{t_n}^{s}\delta(s)\,\mathrm{d}s\right)\mathrm{d}s\right](H+\varepsilon). \tag{3.14}$$

In view of (3.3), for sufficiently large n, inequalities (3.13) and (3.14) imply

$$0\leqslant w(t_n)\leqslant\theta(H+\varepsilon),$$

and

$$0\leqslant -w(s_n)\leqslant\theta(H+\varepsilon).$$

Notice that $\varepsilon>0$ is arbitrary, we have

$$0\leqslant\mu\leqslant\theta H, \tag{3.15}$$

and

$$0\leqslant -\lambda\leqslant\theta H. \tag{3.16}$$

Since $\theta<1$ and $H=\max\{\mu,-\lambda\}$, (3.15) and (3.16) hold if and only if $\mu=\lambda=0$. Therefore, $\lim_{t\to\infty}w(t)=0$. By the medium of the transformation (3.4), we immediately reach the conclusion (3.1). The proof is complete. □

By the medium of Lemma 1.2, we state the stability result for system (1.9).

Theorem 3.5. *Assume that* (A_1)–(A_3), (2.3) *and*

$$p(t)e^{-\alpha(t)x^*(t)}(1-\alpha(t)x^*(t)) < \delta(t) \quad \text{for } t \in [0, \omega]$$

hold. Suppose further that

$$\limsup_{t\to\infty} \int_{t-m\omega}^{t} p(s) \exp\left(\int_t^s \delta(\xi)\,d\xi\right) ds = \theta < 1.$$

Then any positive solution $x(t)$ *of* (1.9) *satisfies*

$$\lim_{t\to\infty}(x(t)-x^*(t)) = 0.$$

Remark 3.6. If $\gamma(t)=a$, then Eq. (1.12) reduces to the equation

$$z'(t) = -\delta(t)z(t) + p(t)z(t-m\omega)e^{-az(t-m\omega)}, \quad t \geqslant 0, \tag{3.17}$$

which has been studied by Saker and Agarwal [22]. Clearly, our Theorem 3.4 improves Theorem 3.6 of Saker and Agarwal [22] under the weaker condition $p(t) > \delta(t)$ for $t \in [0, \omega]$. In particular, our result drops the condition

$$a(z^*(t))_m > 1$$

and replace the Saker and Agarwal's condition $p(t)e^{-az^*(t)} \leqslant \delta(t)$, $t \in [0,\omega]$ by the weaker condition $p(t)e^{-az^*(t)}(1-az^*(t)) < \delta(t)$ for $t \in [0, \omega]$.

Remark 3.7. By Lemma 2.3, the unique ω-periodic positive solution $z^*(t)$ of Eq. (1.12) satisfies

$$\left(\frac{p(t)-\delta(t)}{p(t)\gamma(t)}\right)_m \leqslant z^*(t) \leqslant \left(\frac{p(t)}{\gamma(t)\delta(t)}\right)_M,$$

and so, we have

$$p(t)\exp\left\{-\gamma(t)\left(\frac{p(t)-\delta(t)}{p(t)\gamma(t)}\right)_m\right\}\left(1-\gamma(t)\left(\frac{p(t)-\delta(t)}{p(t)\gamma(t)}\right)_m\right)$$
$$\geqslant p(t)\exp\{-\gamma(t)z^*(t)\}(1-\gamma(t)z^*(t)) \quad \text{for } t \in [0,\omega].$$

Hence, the condition (3.2) can be replaced by the following condition:

$$p(t)\exp\left\{-\gamma(t)\left(\frac{p(t)-\delta(t)}{p(t)\gamma(t)}\right)_m\right\}\left(1-\gamma(t)\left(\frac{p(t)-\delta(t)}{p(t)\gamma(t)}\right)_m\right) < \delta(t) \quad \text{for } t \in [0,\omega].$$

4. Discussion

In [10], Kulenovic et al. studied the equation

$$N'(t) = -\delta N(t) + pN(t-\tau)e^{-aN(t-\tau)}, \tag{4.1}$$

and showed that when $p > \delta$, every positive solution of (4.1) tends to $N^* = (1/a)\ln(p/\delta)$ as $t \to \infty$ provided that

$$(e^{\delta\tau}-1)\left(\frac{p}{\delta}-1\right) < 1.$$

Recently, Saker and Agarwal [22, Theorem 3.6 or Remark 3.7] extends the above result of Kulenovic et al. [10] for Eq. (4.1) under the condition $a(z^*(t))_m > 1$. That is to say, their Theorem 3.6 extends the result in [10] only under the condition $p > e\delta$. From Theorem 3.4, it is easy to see that our result improves the known result of Kulenovic et al. [10] for Eq. (4.1) under the weaker condition $p > \delta$.

On the other hand, we should point out the conditions for the global attractivity of Saker and Agarwal [22, Theorem 3.6] is so strict that under these conditions the periodic solution of (3.17) may only be a constant solution of (3.17). We demonstrate it in the sequel.

The condition

$$p(t)e^{-az^*(t)} \leqslant \delta(t) \quad \text{for } t \in [0, \omega]$$

implies that

$$z^*(t) \geqslant \frac{1}{a} \ln\left(\frac{p(t)}{\delta(t)}\right) \quad \text{for } t \in [0, \omega].$$

Then from Eq. (3.17), we have

$$z^{*\prime}(t) = -\delta(t)z^*(t) + p(t)z^*(t)e^{-az^*(t)} \leqslant z^*(t)(-\delta(t) + p(t)e^{-a\cdot(1/a)\ln(p(t)/\delta(t))}) = 0,$$

which implies that if $z^*(t)$ is a positive periodic solution of (3.17), then it is a decreasing function and so it must be a constant. Therefore, our result improves Theorem 3.6 in [22].

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