Application of Mean Field Annealing Algorithms to GPS-Based Attitude Determination

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Abstract: This paper researches the attitude determination of a body based on GPS carrier phase measurement, and the algorithms used is the mean field annealing neural network (MFANN), which has the advantages of high accuracy and rapidity. The MFANN approach is a combination of the competitive Hopfield neural network and the stochastic simulated annealing technique, which is efficacious in resolving the optimal attitude determination. Firstly, the fundamental principle of GPS attitude determination is described, then a test platform of attitude determination is set up, and the MFANN algorithm is verified to resolve integer ambiguity and azimuth angles. Lastly, the experimental example is presented by means of MFANN, and it shows that this method is effective.

Key words: GPS; neural network; MFANN; integer ambiguity; attitude determination

Used as a new generation satellite navigation and positioning system, GPS not only has the ability of global, all-day and continuous three-dimensional navigation and positioning, but also has the capabilities of anti-jamming and secrecy. Compared with inertial navigation system, GPS receiver is very cheap and small, especially it has no error accumulation[1]. It has been some time for the researchers to study the attitude determination on based GPS carrier phase. Several papers discussed attitude determination by using single, double and even triple difference of GPS signal [2-5]. On this paper, a new algorithm of mean field annealing neural network [6,7] is used to determine the attitude of body, which is combined with both the competitive Hopfield neural network [8] and the stochastic simulated annealing technique. The MFANN has not only the advantage of global optimization but also the competence of single epoch of observation as well. Compared with the previous method, it has the advantage of higher accuracy and rapidity.

1 The Principle of Attitude Determination

The carrier phase measurement equation of the $i$th satellite with respect to the $m$th receiver antenna can be written as

$$\Phi_{i, m}(t) = \frac{L}{c} \Phi_{i, m} + N_{im} + \left[ f \left( \delta_{i, m}(t) - \delta_{m}(t) \right) - \frac{L}{c} k_i \right]$$

where $\Phi_{i, m}$ is the carrier phase measurement; $f$ is the L1 carrier frequency which is 1575.42MHz; $c$ is the speed of light; $\Phi_{m} (t)$ is the distance be-
between the $m$th antenna and the $i$th satellite; $\delta_m(t)$ and $\delta_i(t)$ are the receiver clock error and satellite clock error respectively; $N_{im}$ is the associated integer ambiguity; and $k_i$ is the ionospheric and tropospheric errors along satellite signal transmission.

The pseudorange between the $m$th antenna and the $i$th satellite is

$$\rho_{m, i}(t) = \mathbf{C}_{bi} \mathbf{a}_{m, b} - (s_{i, e} - r_{i, e})$$  \hspace{1cm} (2)

where $\mathbf{C}_{bi}$ is the transfer matrix from the body-fixed to earth-fixed coordinate system; $\mathbf{a}_{m, b}$ is the position vector of the $m$th antenna in body-fixed frame; and $s_{i, e}$ is the position vector offset by $r_i$ of the $i$th satellite in the earth-fixed coordinate system.

Substituting Eq. (2) into Eq. (1), the measurement equation can be gained

$$\Psi_{i, m}(t) = \frac{1}{c} \mathbf{C}_{bi} \mathbf{a}_{m, b} - s_{i, e} = \frac{\rho_{i, m}(t)}{c} + N_{i, m} + \epsilon_{m}(t) + g_i(t)$$ \hspace{1cm} (3)

where $\epsilon_{m}(t)$ and $g_i(t)$ represent the errors associated with the $m$th antenna and the $i$th satellite respectively.

The measurement equation of $\Psi_{i, m}(t)$ from the $n$th antenna to the $i$th satellite can also be got accordingly. Taking a single difference between the two antennas $m$ and $n$, the following can be obtained

$$\Psi_{i, n}(t) - \Psi_{i, m}(t) = \frac{\rho_{i, n}(t) - \rho_{i, m}(t)}{c} + N_{i, n} - N_{i, m} + \epsilon_{n}(t) - \epsilon_{m}(t)$$ \hspace{1cm} (4)

Hence, some common errors associated with satellite can be removed by taking the single difference. At the same time, the difference between $\rho_{i, m}(t)$ and $\rho_{i, n}(t)$ can be got, that is

$$\rho_{i, m}(t) = (\mathbf{a}_{m, b} - \mathbf{a}_{n, b})^T \mathbf{C}_{bi} \mathbf{a}_{m, b} + \frac{\mathbf{a}_{n, b}}{2} - s_{i, e} = \frac{\mathbf{C}_{bi}^T (\mathbf{a}_{m, b} + \mathbf{a}_{n, b})}{2} - s_{i, e}$$ \hspace{1cm} (5)

Substituting Eq. (5) into Eq. (4) yields

$$\Psi_{i, n}(t) - \Psi_{i, m}(t) = \frac{\rho_{i, n}(t) - \rho_{i, m}(t)}{c} = \frac{\mathbf{C}_{bi}^T (\mathbf{a}_{m, b} + \mathbf{a}_{n, b})}{2} - s_{i, e}$$ \hspace{1cm} (6)

Taking the double difference between the $i$th satellite and the $j$th satellite, then substituting into Eq. (6), also since $(\mathbf{a}_{m, b} + \mathbf{a}_{n, b}) / 2$ is much smaller than $s_{j, e}$, it leads to

$$\varphi_{jmn} = \frac{\rho_{j, m}(t) - \rho_{j, n}(t)}{c} \mathbf{C}_{be} \mathbf{a}_{j, b} = \left( \frac{s_{j, e}}{s_{i, e}} - \frac{s_{j, e}}{s_{i, e}} \right) + N_{ijmn}$$ \hspace{1cm} (7)

The double difference Eq. (7) can be recasted as a matrix equation

$$\Phi = \mathbf{A} \mathbf{C}_{be} \mathbf{S} + N$$ \hspace{1cm} (8)

2 Construct System Model

Assume the vectors of two antennas be located at body-fixed reference frame as

$$\mathbf{a}_{m, b} = \begin{bmatrix} - \frac{d}{2} \\ 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{a}_{n, b} = \begin{bmatrix} - \frac{d}{2} \\ 0 \\ 0 \end{bmatrix}$$

where $d$ is the baseline length between the two antennas. Substituting the coordinate transfer matrix $\mathbf{C}_{be}$ and the two antenna vectors into Eq. (7) yields

$$\varphi_{jmn} = \frac{\rho_{j, m}(t) - \rho_{j, n}(t)}{c} \mathbf{C}_{be} \mathbf{a}_{j, b} = \left( \frac{s_{j, e}}{s_{i, e}} - \frac{s_{j, e}}{s_{i, e}} \right) + N_{ijmn}$$ \hspace{1cm} (9)

where $\lambda$ and $\Phi$ are, respectively, the latitude and longitude at the test site and that can be set before hand; $s_{j, e}$ and $s_{i, e}$ are obtained from the ephemeris data; and $\varphi_{jmn}$ is the double difference of the GPS carrier phase measurements. Hence, the unknowns are the azimuth angle $\Phi$ and the integer ambiguity $N_{ijmn}$.

Suppose that

$$P'(t_k) = \frac{\rho_{j, m}(t_k) - \rho_{j, n}(t_k)}{c} \mathbf{C}_{be} \mathbf{a}_{j, b} = \left( \frac{s_{j, e}}{s_{i, e}} - \frac{s_{j, e}}{s_{i, e}} \right)$$

and

$$Q'(t_k) = \frac{\rho_{j, m}(t_k) - \rho_{j, n}(t_k)}{c} \mathbf{C}_{be} \mathbf{a}_{j, b} = \left( \frac{s_{j, e}}{s_{i, e}} - \frac{s_{j, e}}{s_{i, e}} \right)$$

then, Eq. (9) can be rewritten as

$$\varphi_{jmn}(t_k) = P'(t_k) \mathbf{C} + Q'(t_k)$$ \hspace{1cm} (10)

Thus, the attitude determination model in-
cluding unknown parameter as azimuth angle \( \phi \) and the integer ambiguity \( N_{ijmn} \) has been set up.

3 Mean Field Annealing Algorithms

From the double difference Eq. (10), the following can be got,
\[
\sqrt{(P(t_k))^2 + (Q(t_k))^2} (\sin \theta \cos \phi + \cos \theta \sin \phi) = \Phi_{ijmn} - N_{ijmn}
\]
where the angle \( \theta \) can be resolved from following equation
\[
\theta = \arctan \frac{P(t_k)}{Q(t_k)}
\]
(12)
Thus, for any fixed integer \( N_{ijmn} \) there are two azimuth angles \( \phi \) which satisfy the double difference Eq. (12). Assume that
\[
\xi = \arcsin \frac{\Phi_{ijmn} - N_{ijmn}}{\sqrt{(P(t_k))^2 + (Q(t_k))^2}}
\]
then the solutions of azimuth angle \( \phi \) are
\[
\phi = \xi - \theta \mod (2\pi)
\]
and
\[
\phi = \pi - \xi - \theta \mod (2\pi)
\]
(14)

In order to ensure that \( \xi \) is exists, the following equation must be held,
\[
- \sqrt{(P(t_k))^2 + (Q(t_k))^2} \leq \Phi_{ijmn} - N_{ijmn} \leq \sqrt{(P(t_k))^2 + (Q(t_k))^2}
\]
(15)
Eq. (15) then governs the range of the integer ambiguity \( N_{ijmn} \) can lie. Also because
\[
(P(t_k))^2 + (Q(t_k))^2 \leq \left(\frac{df}{c}\right)^2 \cdot \left(\frac{s_i, e}{\|s_{i,e}\|} - \frac{s_j, e}{\|s_{j,e}\|}\right)^2 \leq 4 \left(\frac{df}{c}\right)^2
\]
(16)
substituting Eq. (16) into Eq. (15), the bounds of the double difference ambiguity \( N_{ijmn} \) can be obtained, that is
\[
\Phi_{ijmn}(t_k) - 2 \frac{df}{c} \leq N_{ijmn} \leq \Phi_{ijmn}(t_k) + 2 \frac{df}{c}
\]
(17)
For each baseline combination \( m,n \), suppose that there are \( k \) possible integer values \( N_{ijmn} \). Now construct a matrix \( H \), in which the \((k, l)\)th entry represents: the \( k \)th value of integer ambiguity \( N_{ijmn} \) and the \( l \)th double difference measurement of \( \Phi_{ijmn} \). Calculating the azimuth angle \( \phi_{ka} \) and \( \phi_{kb} \) through Eq. (14) and Eq. (15), the following matrix can be gained,
\[
H = [h_{kl}] = \\
\begin{bmatrix}
\phi_{t1a} & \phi_{t1b} & \phi_{t2a} & \phi_{t2b} \\
\phi_{t3a} & \phi_{t3b} & \phi_{t4a} & \phi_{t4b} \\
\phi_{t5a} & \phi_{t5b} & \phi_{t6a} & \phi_{t6b} \\
\end{bmatrix}
\]
(18)
Up to now the problem is reduced to finding the right azimuth angle \( \phi \) from matrix \( H \). The right attitude angle exists in each column of \( H \) and only one. Then, the solution of the azimuth is the double difference integer ambiguity. Although each entry in the matrix \( H \) contains two angles, the problem can be dealt with easily by augmenting the matrix into a larger dimension \( 2k \times (n_o - 1) \). Then the attitude determination problem is changed to a minimax type optimization problem
\[
\min \left( \max \left( \min | h_{kl} - \phi | \right) \right)
\]
(19)
In the method of MFANN, instead of searching for the angle \( \phi \) directly, try to construct a permutation matrix \( V = \{ v_{kl} \} \) of the same size as \( H \), and then, substitute the membership of each entry in \( H \). At first, the initial value of each state variable \( v_{kl} \) in the permutation matrix \( V \) is chosen in the range of 0 to 1, that is, \( 0 \leq v_{kl} \leq 1 \). Each state variable in \( V \) is close to 0 or 1 by dynamic iterative process. When \( v_{kl} = 1 \), the \( l \)th double difference measurement corresponding to the \( k \)th candidate is the right integer ambiguity that is searched for.

Here the neural network selected includes \( 2k \times (n_o - 1) \) nerve cells connected to each other. Let \( r_{is} \) denote the output of the \((r, s)\)th nerve cell, and \( w_{ijkl} \) represent the juncture between the \((r, s)\)th and the \((k, l)\)th nerve cells. In the network, the \((k, l)\)th nerve cell accepts the input of the \((k, l)\)th nerve cell added by \( w_{ijkl} \). Considering the offset \( i_{rs} \), the total input of the \((r, s)\)th nerve cell can be written as
\[
net_{rs} = \sum_k \sum_j w_{ijkl} r_{is} + i_{rs}
\]
(20)
where \( i_{rs} \) is the offset of output. Let \( i_{rs} = 0 \), then, the energy function can be written as
\[ E = \frac{1}{2} \sum_{i} \sum_{j} \sum_{k} \sum_{l} \nabla v_{ijkl} w_{rs} v_{ijkl} \]  

(21)

where \( \nabla v_{ijkl} = | h_{rs} - 1 \). At the same time, with the theory of mean field as supplement, the energy function will be expressed as

\[ \Delta E_{rs} = E(v, t) - E(v, 0) = \frac{1}{2} \sum_{i} \sum_{j} \sum_{k} | h_{rs} - 1 | v_{ijkl} \]  

(22)

In the algorithms of MFANN, the starting value of temperature \( T_0 \) is several times of the maximal energy in all the possible state combination. In the iterative process, \( T \) reduces ceaselessly, and the reduced rate has the following form

\[ T(t) = T_{rate} \cdot T(t_{0}) \]  

(23)

where \( T_{rate} \) is the rate of temperature reducing, and \( T_{rate} < 1 \). In order to get a better effect of annealing, this paper introduces the following method

\[ T_{rate}(t_k) = \frac{C}{\log((t_k + 1)^3)} \]  

(24)

The constant \( C \) should be selected to keep the iterative times \( t_k \) in a proper range. Here \( C \) is selected as \( T_{rate} \in [0.05, 0.96] \).

Given the definite temperature and assuming the state of nerve cell distribution as Boltzmann, then, \( v_{rs} \) will represent the probability of right azimuth angle which is ensured by the \( r \)th double difference and the \( s \)th integer ambiguity, that is

\[ v_{rs} = \frac{\exp(-\Delta E_{rs}/T)}{\sum_{s} \exp(-\Delta E_{rs}/T)} \]  

(25)

After the initial state and temperature are set up, let the neural network begin to run until the energy function reaches the minimum or near the minimum. Then only one state becomes 1 in each entry of matrix \( V \), and the other states will become 0. From the item in \( H \) corresponding to the item which is 1 in \( V \), the azimuth angle and integer ambiguity can be obtained, thus the right optimization is obtained.

4 Experimental Results

In order to assess the algorithm of MFANN, the actual GPS satellite data are collected, which is carried out in some place on the top of 15# building in NUAA and the orientation is known beforehand. The receiver used are two GG-24, and seven GPS satellites (prn6, 10, 13, 17, 24, 26, 27) can be observed during the collection. The start epoch is 124000 and the collecting frequency is 1Hz. The double differences of carrier phase with visible satellites are shown in Fig.1.

Considering the frequency of carrier wave L1 and the base length 1m, it is obtained that

\[ n(P(t_k)) = 10 \cdot 256 \]  

So the bounds of integer ambiguity should be in \([-10, 11]\). Then the azimuth angle matrix \( H \) and the output matrix \( V \) are constructed. In \( V \) the initial value is random in the range of \((0, 1)\). After the network runs for a certain, the result is got as Eq. (26). Hence, the solutions of the integer ambiguity as well as the azimuth angle can be obtained.

The integer ambiguities are shown in Table 1 and the azimuth angle is found to be \(-97.6386^\circ\). In order to validate the legitimacy of the solution, combinations of data in some other epochs are selected for calculation, and the same results are obtained.

\[
V = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  

(26)

This paper introduces a new attitude determination method based on GPS carrier phase signal named as MFANN, which has the advantages of high accuracy and rapidity in solving the integer ambiguity and azimuth angle.

For the MFANN, it can give the body’s attitude by using the data of only one epoch. Although it needs $2k \times (n_s - 1)^2$ circulations to construct the azimuth matrix $H$ and initialize the output matrix $V$, and usually needs to lower the temperature for 9 to 10 times to find azimuth angle, also it consumes $2k \times (n_s - 1)^2$ circulations to calculate the mean energy of each nerve cell at every temperature, the real-time is still effective because the MFANN is parallel and global in scope. In the aspect of convergence, the annealing temperature $T$ and the rate of annealing $T_{rate}$ are very important, so they should be selected reasonably. In the aspect of optimization, the MFANN can ensure the solution to be optimized or to approach the global optimization because of the principle of itself.

5 Conclusions

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References


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