Ain Shams Engineering Journal (2014) 5, 193–203



Ain Shams University

www.elsevier.com/locate/asej

Ain Shams Engineering Journal



MECHANICAL ENGINEERING

Development of reliability index for combined cycle power plant using graph theoretic approach

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Received 22 October 2012; revised 5 August 2013; accepted 15 September 2013 Available online 20 October 2013

KEYWORDS

CCPP; Systems; Graph theory; Digraph; Permanent function; Matrix **Abstract** A systematic approach based on graph theory and matrix method is developed ingeniously for the evaluation of reliability index for a Combined Cycle Power Plant (CCPP). In present work CCPP system is divided into six subsystems. Consideration of all these subsystems and their interrelations are rudiment in evaluating the index. Reliability of CCPP is modeled in terms of a Reliability Attributes Digraph. Nodes in digraph represent system reliability and reliability of interrelations is represented by edges. The digraph is converted into one-to-one matrix called as Variable System Reliability Permanent Matrix (VPM-r). A procedure is defined to develop variable permanent function for reliability (VPF-r) from VPM-r. Reliability index of CCPP system is obtained from the permanent of the matrix by substituting numerical values of the attributes and their interrelations. A higher value of index implies better reliability of the system. The proposed methodology is illustrated step-by-step with the help of two examples.

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1. Introduction

Reliability analysis is an innate aspect of combined cycle power plant design and plays considerable role throughout the plant operation in terms of expenses (operating and maintenance) and optimal maintenance scheduling of its

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Peer review under responsibility of Ain Shams University.

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equipments. Reliability may be defined as the ability of an equipment, component, product, system, etc., to function under designated operating state of affairs for a specified period of time or number of cycles [1]. For a large and complex electricity generating system such as CCPP, reliability is the probability of generating electricity under operational conditions for a definite period of time. Reliability of a CCPP is function of maintenance (scheduled or forced) cost, which in turns depends upon the Mean Time Between Failures (MTBF) and Mean Time To Repair (MTTR) of equipments or systems, and which are further dependent on complexity in design, state, age of the equipment or system and to some extent on the availability of spare parts.

Recurring failures that lead to complete power plant outage need repair and proactive maintenance to invigorate power

2090-4479 © 2013 Production and hosting by Elsevier B.V. on behalf of Ain Shams University. http://dx.doi.org/10.1016/j.asej.2013.09.010 plant performance and reduction monetary losses. Downtime losses and maintenance cost of a CCPP can be reduced by adopting a proper mix of maintenance and repair strategies. In the worst situation, unavailability of an equipment or system affects whole plant and plant trips in this case. But in general, the failure of an equipment or system may not affect the complete plant and therefore its criticality is at some intermediate value. In that case reliability of system comes down and its effect on reliability of other systems is also observed. The criticality level decides the importance of the equipment or system and choice of appropriate maintenance and repair strategy so that reliability may be maintained up to a mark.

In the literature both qualitative and quantitative methods for assessing the reliability of complex systems are available. The most commonly used qualitative methods are Fault Tree Analysis (FTA), Failure Modes, Effects and Criticality Analysis (FMECA), Failure Modes and Effects Analysis (FMEA), Root Cause Analysis (RCA), Root Cause Failure Analysis (RCFA), Fish Bone Analysis (FBA), Event Tree Analysis (ETA), and Predictive Failure Analysis (PFA). Block diagram analysis, Markov chain, and Monte Carlo simulation are some of the quantitative methods of reliability analysis available in the literature.

Various attempts have been made by researchers in developing procedures for the evaluation of the reliability of various systems [2–10]. The two-state Markov model is the mainly used outage model in power system reliability analysis [11].

Eti et al. [12] integrated reliability and risk analysis for maintenance policies of a thermal power plant. Need to integrate RAMS (reliability, availability, maintainability and supportability) centered maintenance along with risk analysis was stressed, although results expected or obtained with the application of those concepts were not explained.

A staircase function was introduced by Ji et al. [11] to approximate the aging failure rate in power systems and a component renewal process outage model based on a timevarying failure rate was proposed. The model reflected the effects of component aging and repair activities on the aging failure rate.

Markov method was used by Haghifam and Manbachi [13] to model reliability, availability and mean-time-to-failure indices of combined heat and power (CHP) systems based on interactions between electricity generation, fuel-distribution and heat-generation subsystems. The proposed model can be useful in feasibility studies of CHP systems and in determining their optimal design, placement and operational parameters.

Carpaneto et al. [14] carried out Monte Carlo simulation for identifying long, medium and short-term time frames by incorporating uncertainty at large-scale and small-scale for cogeneration system. Availability coefficient assumed to be independent of year, scenario and control strategy was defined for unavailability of the CHP units, due to scheduled maintenance and reliability aspects, taking into account. Large-scale uncertainty referred to the evolution of energy prices and loads and relevant to the long-term time frame was addressed within multi-year scenario analysis. Small-scale uncertainty relevant to both short-term and medium-term time frames was addressed through probabilistic models and Monte Carlo simulations [15].

Mohan et al. [2] calculated RTRI (real-time reliability index) for a SPP (steam power plant) using graph theory. Integration of systems and subsystems and interaction among them were considered for the reliability analysis and the proposed methodology can be applied for obtaining availability and maintainability; including optimum selection, bench marking, and sensitivity analysis of SPP. Tang [16] proposed a new method based on the combination of graph theory and Boolean function for assessing reliability of mechanical systems. Graph theory was used for modeling system level reliability and Boolean analysis for interactions. The combination of graph theory and Boolean function bring into being an effective way to evaluate the reliability of a large, complex mechanical system. Garg et al. [5] developed a graph theoretical model to compare various technical and economical features of wind, hydro and thermal power plants.

Performance analysis of coal based steam power plant boiler was carried out by Mohan et al. [3] using graph theory and step-by-step methodology for the evaluation was also proposed. Further graph theory was applied to calculate real-time efficiency index (RTEI) defined as the ratio of the values of variable permanent system structure function (VPF) in realtime (RT) situation to its achievable design value [4] and in this connection graph theory was used to recommend the an appropriate maintenance strategy for power plants [6].

The reliability and availability of a CCPP depend on the perfect operation of all its systems (e.g., gas turbine, heat recovery steam generator, steam turbine and cooling system) [17]. So far researchers evaluated combined cycle power plant system reliability only at system level without making an allowance for the interactions of systems, and subsystems. Therefore, there is a need for extending the compass of reliability analysis for combined cycle power plants by taking care of interaction among different systems and subsystems.

A number of approaches and methodologies developed by researchers are available in the literature to model the various systems and their elements. Graph theory is one of such methodologies. It synthesizes the inter-relationship among different parameters and systems to evaluate score for the entire system. Because of its inherent simplicity, graph theory and matrix method have wide range of applications in engineering, science and in numerous other areas [22]. Several examples of its use have appeared in the literature [2–6,21–24] to model the various systems.

This paper presents a mathematical model using graph theoretic systems approach that enables the prediction of CCPP reliability in terms of an index by taking into account various systems and interactions between them.

2. System structure graph of a combined cycle power plant

System structure development is imperative for understanding and analysis of its performance [25] and a combined cycle power plant is no exception. System structure is of two types: abstract and physical. Abstract structure involves performance contributing events and their interrelations or interdependencies. The physical structure of a system implies subsystems, assemblies, components and their interconnections. A CCPP is a combination of a Compressed Natural Gas (CNG) fired gas turbine with Heat Recovery Steam Generator (HRSG) and a steam powered turbine. These plants are very large, typically rated in the hundreds of mega-watts. Combined cycle power plant considered for the present analysis is shown in Fig. 1.



Figure 1 Schematic flow diagram of combined cycle power plant.

Ambient air at Normal Temperature and Pressure (NTP) is compressed by the air compressor to decrease its volume. Air at elevated temperature and pressure is directed to the combustion chamber. The compressed air is mixed with CNG from the fuel supply system to produce hot combustion gas in combustion chamber at constant pressure. Hot combustion gas enters the gas turbine where power is generated. HRSG is the link between the gas turbine and the steam turbine process, whose function is to transfer heat energy from exhaust gases to high pressure water and produces high pressure steam. The steam is separated in the boiler drum and supplied to the super heater section and boiler condenser section. The super heated steam produced in the super heater then enters into steam turbine through the turbine stop valve. After expansion in the turbine the exhaust steam is condensed in the condenser. In the cooling water system, heat recovered from the steam turbine exhaust is carried by the circulating water to the cooling tower, which rejects the heat to the atmosphere.

For the Graph Theoretic Analysis (GTA) large and complex system such as combined cycle power plant must be divided into small subsystems for the convenience of analysis. GTA takes care of inheritance and interdependencies of the subsystems. Further it gives a quantitative measure of system reliability which is helpful in comparing the present reliability with the design value. Six subsystems identified for a CCPP are as follows:

- 1. Air compressor system (S_1) .
- 2. Combustion chamber system (S_2) .
- 3. Gas turbine system (S_3) .
- 4. Heat recovery steam generator system (S_4) .
- 5. Steam turbine system (S_5) .
- 6. Water system (S_6) .

Division of combined cycle power plant in these subsystems is based on the working of different components and subsystems can be divided further into sub-subsystems. Combined cycle power plant is a combination of gas turbine cycle and steam turbine cycle. Gas turbine cycle comprises of air compressor system, combustion chamber system and gas turbine system. Output of gas turbine cycle that is flue gases at high temperature is the driving energy for steam turbine cycle.

Therefore, steam turbine cycle is dependent upon the gas turbine cycle and unavailability of gas turbine cycle trip the whole plant. In case steam turbine cycle is unavailable then flue gases may be sent directly to atmosphere through bypass stack. In this way gas turbine cycle is not dependent on the steam turbine cycle. Failure of a particular component does not mean that whole plant is not working but its reliability is decreased with respect to design value. In case of failure of air compressor or combustion chamber or gas turbine, whole plant is tripped. If any of the HRSG, steam turbine and water system fail then steam cycle will not be working. Therefore, taking in view of this combined cycle power plant is divided into the six subsystems as explained above. Components or systems affecting their reliability are considered their part. Reliability of these subsystems and their interaction will decide the reliability of CCPP as they are connected with each other physically or indirectly. As these subsystems are also very big, therefore, hereafter they are also referred as systems. Let each of the six systems of plant be represented by vertices S_i 's (i = 1, 2, ...3, 4, 5, 6) and interconnection between two systems (S_i, S_i) is represented by edges c_{ii} 's $(i, j = 1, 2, 3, 4, 5, 6 \text{ and } i \neq j)$ connecting the two vertices S_i and S_j . All six systems are connected by flow of air, flue gases, water, steam, heat and work. This flow is shown in Fig. 2 with the help of vertex and edges. This representation is called as System Structure Graph (SSG). This is based upon the functioning of combined cycle power plant as per the following:

- 1. Air compressor and gas turbine are attached to each other with rigid shaft for continuous power supply to compressor for pressurizing the ambient air. It is represented by the edge c_{31} .
- 2. Compressed air surging from compressor to the combustion chamber is represented by edge c_{12} . Fuel is injected in the combustion chamber and chemical reaction of fuel with air is at constant pressure. Fuel injection system is considered to be a part of combustion chamber system.
- 3. Gas turbine blades are cooled by being made hollow so that coolant air, obtained directly from the compressor, can circulate through it. Edge c_{13} represents the air bypassing the combustion chamber.



Figure 2 System structural graph of combined cycle power plant.

- 4. Flue gases surging from combustion chamber to gas turbine is shown by edge c_{23} .
- 5. HRSG is used for (i) partial heating of compressed air leaving the compressor (c_{42}) , (ii) feed water heating (c_{46}) , (iii) steam generation at dual or multi-pressure.
- 6. Flue gases coming out of gas turbine and entering to HRSG system is shown by the edge c_{34} .
- 7. Superheated steam generated in HRSG and flowing to steam turbine is shown by edge c_{45} .
- 8. Edge c_{56} represents the flow of steam from steam turbine to the condenser.
- 9. De-mineralized (DM) water injection to superheated and reheated steam (as an attemperation spray) is represented by the edge c_{65} .
- 10. DM water supplied to HRSG (as feed water) is represented by edge c_{64} .

The system structure graph shown in Fig. 2 represents the internal structure of the CCPP at system level.

3. GTA for reliability analysis of combined cycle power plant

Reliability has two connotations; probabilistic and deterministic. Probabilistic approach is based upon statistical failure modeling, without research and itemizing causes of failure. Deterministic approach concentrates on understanding how and why a component or system fails, and how it can be designed, repaired and tested to prevent such failure from occurrence or recurrence. In the present analysis, probabilistic approach in conjunction with GTA is applied for combined cycle power plant reliability analysis. GTA consists of following three steps [21]:

- Diagraph representation.
- Matrix representation.
- Development of permanent function.

The digraph characterizes the visual representation of the systems and their interdependence. The matrix converts the

digraph into mathematical form and the permanent function is a mathematical model that helps determine the reliability index. It may be noted here that development of permanent function is not merely the determinant of the matrix. It is developed in such a manner that no information regarding the system reliability is lost. For this purpose, a step-by-step methodology is proposed hereafter with the help of two examples.

3.1. Digraph representation of CCPP reliability system

In Fig. 2 it is explained how the air, flue gases, water, steam, heat and work flows from one system to another system. For the GTA it is not necessary to represent these physical interactions by different types of lines. It is required to know whether a system is connected to other systems. If yes then interaction is represented by a line and arrow haggard at the end shows the direction of flow of physical property. A pictogram of systems and their interdependencies in terms of nodes and edges is called digraph. Let nodes (S_i 's) represent systems and edges (c_{ij} 's) symbolize their interactions. S_i indicates the inheritance of systems and c_{ij} indicates degree of dependence of *j*th system on the *i*th system. The digraph signifies the proposed CCPP systems and interrelations and represents the system in a simplified manner. Diagraph for the SSG (Fig. 2) can be represented as shown in Fig. 3.

Graph theoretic models have adaptability to model any of the RAM (Reliability, Availability and Maintenance) characteristics by associating suitable attributes and interdependencies to the nodes and edges of the SSG [9]. For example, if the node R_i represents the reliability of *i*th system and r_{ij} represents the reliability of the interconnection between *i*th and *j*th systems (nodes) of CCPP; then, systems reliability graph or digraph (SRD) can be obtained from the SSG of a CCPP (Fig. 2 or Fig. 3).

The digraph model (SRD) provides the system structure reliability unequivocally. Reliability of the connection between two systems is considered if the systems are connected either by rigid or imaginary links such as connection between turbine



Figure 3 System structure digraph for combined cycle power plant.

and generator rotors through a mechanical shaft or between combustion chamber system and water system of boiler, through flue gases.

3.2. Matrix representation of CCPP system

Although a digraph is very convenient for a visual study, other representations are better for computer processing. A matrix is a convenient and useful way of representing a digraph to a computer. Matrices lend themselves easily to mechanical manipulations. Many results of matrix algebra can be readily applied to study the structural properties of graphs from an algebraic point of view. The starting point in matrix representation is the adjacency matrix.

3.2.1. System reliability adjacency matrix

Consider a case of CCPP system having N systems leading to symmetric adjacency matrix $\{0, 1\}$ of order $N \times N$. Let r_{ij} represents the reliability of interconnection between system *i* and *j* such that $r_{ij} = 1$, if the reliability of *i*th system depends on the *j*th system, (in the digraph, this is represented by an edge (r_{ij}) between node *i* and *j*) and is equal to zero, otherwise.

The adjacency matrix A_c for the corresponding digraph (Fig. 4) is as follows:

	1	2	3	4	5	6	Systems		
	0	1	1	0	0	0	1		
	0	0	1	0	0	0	2		
$A_c =$	1	0	0	1	0	0	3	(1)
	0	1	0	0	1	1	4		
	0	0	0	0	0	1	5		
	0	0	0	1	1	0	6		

The off-diagonal elements of this matrix (r_{ij}) represent reliability of interconnection between system *i* and *j*. Moreover, this matrix considers only the reliability of connections be-



Figure 4 System reliability digraph for combined cycle power plant.

tween the systems without taking the effect of systems reliability. To consider this effect, another matrix known as "Characteristic System Reliability Matrix" is defined.

3.2.2. Characteristic system reliability matrix

The presence of different systems of the CCPP is realized by defining a characteristic system reliability matrix $B_c = \{RI - A_c\}$. This matrix for system reliability digraph of CCPP (Fig. 4) is expressed as follows:

$$B_{c} = \{RI - A_{c}\} = \begin{bmatrix} R & -1 & -1 & 0 & 0 & 0 \\ 0 & R & -1 & 0 & 0 & 0 \\ -1 & 0 & R & -1 & 0 & 0 \\ 0 & -1 & 0 & R & -1 & -1 \\ 0 & 0 & 0 & 0 & R & -1 \\ 0 & 0 & 0 & -1 & -1 & R \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \\ 3 \\ 5 \\ 6 \end{bmatrix}$$
(2)

where I is the identity matrix and R represents its reliability of systems. Characteristic System Reliability Matrix is analogous to characteristic matrix in the graph theory [2]. Characteristic System Reliability Matrix does not include information about reliability of interdependencies among different systems. The determinant of characteristic system reliability matrix called as characteristic system reliability polynomial, is written as follows:

$$\det\{B_c\} = R^6 + 4R^4 - 3R^3 + 3R^2 - 4R + 1 \tag{3}$$

The characteristic system reliability polynomial which is derived above is invariant of the system and it does not change by altering the labeling of systems. It is a reliability of the system.

In the above matrix, value of R is taken to be same for all the diagonal elements representing that all systems are considered to be identical. Due to this reason reliability polynomial is nonunique. It has been reported in the literature [3] that many graphs may have same characteristic polynomial that is cospectral graph. In practice, in a CCPP all the six systems do not possess the same reliability. To incorporate distinct systems reliability and reliability of interconnections between them, a matrix called variable characteristic system reliability matrix is proposed.

3.2.3. Variable Characteristic System Reliability Matrix (VCSRM) of CCPP

A variable characteristic system reliability matrix T_a for a combined cycle power plant is defined taking into account reliability of systems and interconnection defined by the system reliability digraph (Fig. 4). This matrix is the combination of two matrices D_a and F_a . Let the off-diagonal elements of a matrix, F_a , representing the reliability of connection between systems is denoted by r_{ij} instead of 1, whenever system *i* is connected to system *j* with *i*, *j* = 1, 2, 3, 4, 5, 6 and 0 otherwise. Diagonal elements of the matrix F_a are 0. Another matrix D_a , is a diagonal matrix with its variable diagonal elements R_i (*i* = 1, 2, 3, 4, 5, 6) representing the reliability of six systems and all the non-diagonal elements are 0. For system reliability digraph of CCPP (Fig. 4) the VCSRM $T_a = [D_a - F_a]$ abbreviated as VCM-r is written as

$$T_{a} = [D_{a} - F_{a}] = \begin{bmatrix} R_{1} & -r_{12} & -r_{13} & 0 & 0 & 0\\ 0 & R_{2} & -r_{23} & 0 & 0 & 0\\ -r_{31} & 0 & R_{3} & -r_{34} & 0 & 0\\ 0 & -r_{42} & 0 & R_{4} & -r_{45} & -r_{46}\\ 0 & 0 & 0 & 0 & R_{5} & -r_{56}\\ 0 & 0 & 0 & -r_{64} & -r_{65} & R_{6} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 6 \end{bmatrix}$$
(4)

The determinant of VCSRM is called variable characteristic system reliability multinomial, denoted as VCF-r for matrix (4), and is written as follows:

$$Per[T_{a}] = [R_{1}R_{2}R_{3}R_{4}R_{5}R_{6} + (r_{13}r_{31})(R_{2}R_{4}R_{5}R_{6}) \\ + (r_{46}r_{64})(R_{1}R_{2}R_{3}R_{5}) + (r_{56}r_{65})(R_{1}R_{2}R_{3}R_{4}) \\ - R_{1}R_{2}R_{3}(r_{45}r_{56}r_{64}) - R_{1}R_{5}R_{6}(r_{23}r_{34}r_{42}) \\ - R_{4}R_{5}R_{6}(r_{12}r_{23}r_{31}) + R_{2}R_{4}(r_{13}r_{31})(r_{56}r_{65}) \\ + R_{2}R_{5}(r_{13}r_{31})(r_{46}r_{64}) - R_{1}(r_{56}r_{65})(r_{23}r_{34}r_{42}) \\ - R_{2}(r_{13}r_{31})(r_{45}r_{56}r_{64}) - R_{4}(r_{56}r_{65})(r_{12}r_{23}r_{31}) \\ - R_{5}(r_{46}r_{64})(r_{12}r_{23}r_{31}) + (r_{12}r_{23}r_{31})(r_{45}r_{56}r_{64})]$$
(5)

Every term in the $Per[T_a]$ is representing the part of the systems and interrelations. For example the $R_1R_2R_3R_4R_5R_6$ shows that all six systems are linked to each other. Failure of any of the system will trip the plant. But the multinomial (5) is also unsuitable for reliability analysis and Variable permanent system reliability matrix is defined for better interpretation of the results.

3.3. Variable permanent system reliability matrix (VPSRM) for CCPP

The negative sign in Eq. (5) indicates subtraction of reliability information about loops of systems and does not give a true picture of the CCPP reliability. Taking into consideration this fact, a variable permanent system reliability matrix (VPSRM) T_c abbreviated as VPM-r for the combined cycle power plant is written as follows:

$$T_{c} = [D_{c} + F_{c}] = \begin{bmatrix} R_{1} & r_{12} & r_{13} & 0 & 0 & 0\\ 0 & R_{2} & r_{23} & 0 & 0 & 0\\ r_{31} & 0 & R_{3} & r_{34} & 0 & 0\\ 0 & r_{42} & 0 & R_{4} & r_{45} & r_{46}\\ 0 & 0 & 0 & 0 & R_{5} & r_{56}\\ 0 & 0 & 0 & r_{64} & r_{65} & R_{6} \end{bmatrix} \begin{bmatrix} 1\\ 2\\ 3\\ 4\\ 6\\ 5\\ 6\end{bmatrix}$$
(6)

Where the $R_{i\delta}$, $r_{ij\delta}$, D_c , and F_c are with same meaning as in the matrix of expression (4).

The permanent of VPSRM is called the variable permanent system reliability function and is abbreviated as VPF-r. The only difference between matrices (4) and (6) is in the signs of the off-diagonal elements. In the VCSRM, expression (4), the off-diagonal elements r_{ij} have negative signs, while these are positive in the VPSRM of expression (6). VPF-r for matrix (6) is written as:

$$Per[T_c] = [R_1 R_2 R_3 R_4 R_5 R_6 + (r_{13} r_{31}) (R_2 R_4 R_5 R_6) + (r_{46} r_{64}) (R_1 R_2 R_3 R_5) + (r_{56} r_{65}) (R_1 R_2 R_3 R_4) + R_1 R_2 R_3 (r_{45} r_{56} r_{64}) + R_1 R_5 R_6 (r_{23} r_{34} r_{42}) + R_4 R_5 R_6 (r_{12} r_{23} r_{31}) + R_2 R_4 (r_{13} r_{31}) (r_{56} r_{65}) + R_2 R_5 (r_{13} r_{31}) (r_{46} r_{64}) + R_1 (r_{56} r_{65}) (r_{23} r_{34} r_{42}) + R_2 (r_{13} r_{31}) (r_{45} r_{56} r_{64}) + R_4 (r_{56} r_{65}) (r_{12} r_{23} r_{31}) + R_5 (r_{46} r_{64}) (r_{12} r_{23} r_{31}) + (r_{12} r_{23} r_{31}) (r_{45} r_{56} r_{64})]$$
(7)

Comparing expressions (5) and (7), that is, the VCF-r and VPF-r, respectively, for the CCPP systems of Fig. 2, it may be noted that all the terms are exactly the same in both expressions. However, they differ in their signs. In VPF-r expression (Eq. (7)), all the terms carry positive signs; while in the VCM-r of expression (5) both positive and negative signs appear in the multinomial. This multinomial (Eq. (7)) uniquely represents the reliability of CCPP system of Fig. 2 and includes all the information regarding various constituents as systems and interactions among them.

A physical meaning is associated with each term of permanent function [3]. Permanent function for Fig. 4 is written as Eq. (7) and graphical representation of different terms is shown in Fig. 5.

A computer program is developed using C⁺⁺ language for calculating the values of permanent function for square matrix of $N \times N$ matrix.

3.4. Combined cycle power plant real-time reliability index (*RTRI_{CCPP}*)

Concept of real-time reliability index (RTRI) was proposed first time for a steam power plant (SPP) by Mohan et al. [2]. It was defined as the ratio of its reliability under real-time conditions to the reliability under its designed conditions. The reliability of a CCPP decreases regularly with time due to various reasons such as non-availability of some of the systems/subsystems or due to aging effect, etc. Performance of a combined cycle power plant in any case cannot be higher than its designed value. Therefore, for all practical purposes, real-time performance of a CCPP is judged with respect to its designed performance. In view of this, the RTRI for combined cycle power plant is defined as the ratio of real-time reliability, i.e. (Reliability)_{*RT*} to the designed reliability that is (Reliability)_{*D*}. Mathematical expression is as following:

$$RTRI_{CCPP} = \frac{(Reliability)_{RT}}{(Reliability)_D} = \frac{(VPF-r)_{RT}}{(VPF-r)_D}$$
(8)

To calculate this index, the values of R_i and r_{ij} are required to be replaced in Eq. (7). Faisal et al. [18] explained that if data regarding the variables from some previous research or field study are available then it can be used to determine the index. But in case no quantitative values are available and in order to avoid complexity at system or subsystem level, then values for inheritance and interrelation may be taken from Table 1 and 2 respectively. From the literature it has been found that Wani and Gandhi [19] have used data from previous research for selecting the values of the variables while Kulkarni [20] had used a questionnaire to measure each attribute in terms of weightage to arrive at the values of the variables.



Figure 5 Graphical representation of permanent function (Eq. (7)) of CCPP corresponds to digraph (Fig. 4).

S. no.	Qualitative measure of parameters affecting combined cycle reliability	Assigned value of parameter (R_i)		
1	One failure in 8 h	1		
2	One failure in 24 h	2		
3	One failure in 80 h	3		
4	One failure in 350 h	4		
5	One failure in 1000 h	5		
6	One failure in 2500 h	6		
7	One failure in 5000 h	7		
8	One failure in 10,000 h	8		
9	One failure in 25,000 h	9		

Table	1	Quantification	of	factors	affecting	combined	cycle
power	pla	nt reliability.					

It is worth to notice that one can choose any scale for R_i or r_{ij} [18,19]. The user may opt for an appropriate scale, for example, 0–5, 0–10, 0–50 or 0–100 for R_i 's and r_{ij} 's, but the final

Table 2	Quantification	of	interdependencies/off-diagonal
elements.			

•••••••••			
S. no.	Qualitative measure of interdependencies	Assigned value of <i>r_{ij}</i>	
1	Very strong	5	
2	Strong	4	
3	Medium	3	
4	Weak	2	
5	Very weak	1	

ranking is not affected as these are relative values. However, lower scale value is desirable to obtain a manageable value of $RTRI_{CCPP}$ and also to reduce partisanship. Index value may differ from plant to plant because every system and interdependency has different values. In this way, different power

plants may be arranged in ascending or descending order, according to their reliability index value.

4. Step-by-step procedure for determining RTRI_{CCPP}

A methodology is the key for the evaluation of RTRI_{CCPP} for different combined cycle power plants. In the present work, methodology based on graph theory and matrix method is developed for evaluating current value of combined cycle power plant reliability and it may be compared either with an ideal case or with any other real life operating plant. The main steps of the proposed methodology are as follows:

- Step 1. Consider a combined cycle power plant. If it seems to be very large system then divide it into smaller subsystems (e.g., air compressor system, combustion chamber system, gas turbine system, HRSG system, steam turbine system, and water system). Identify the various system categories affecting the CCPP reliability.
- *Step 2*. Develop system structure graph for the reliability of CCPP system based upon the interaction among different subsystems.
- *Step 3.* Convert the system structure graph of CCPP into corresponding system reliability digraph with systems reliability as nodes and edges for the reliability of interconnections.
- Step 4. Develop the CCPP system reliability matrix corresponding to the CCPP system reliability digraph. This will be $N \times N$ matrix with diagonal elements of R_i and off-diagonal elements of r_{ij} . The value of inheritance R_i (diagonal elements) for each subsystem is decided by experts or data available in literature. The values of reliability of interactions r_{ij} (off-diagonal elements) are to be determined by the experts or data available in literature.
- Step 5. Calculate the permanent function of CCPP system reliability matrix for values of real-time reliability (Reliability)_{*RT*} and designed reliability (Reliability)_{*D*}.
- *Step 6*. Calculate the ratio of real-time reliability and designed reliability as in Eq. (8). This is the value of RTRI_{CCPP} which mathematically characterizes the reliability of any combined cycle power plant based on the different systems and their interdependencies.
- *Step 7*. Record the results of this study and document them for future analysis.

5. Illustrative examples

Step-by-step methodology explained in the last section is helpful in estimating the reliability at system level which may be extended to the subsystems level. For the demonstration of proposed methodology two examples are taken into consideration.

5.1. Example 1

Failure of bearing lubricating oil cooler in a combined cycle power plant is taken as first example. For smooth revolution of turbo generator (TG), the bearings are lubricated through the lube oil system. The hot oil from the bearing is cooled through water cooler before feeding back into the lube oil tank. Suppose four coolers are used in series for this purpose. In the present analysis TG is considered a part of the steam turbine system. Now if one of the cooler is not available then the value of $RTRI_{CCPP}$ for the present case will be computed as follows:

- *Step 1*. Consider the combined cycle power plant shown in Fig. 1.
- *Step 2*. Block diagram for the reliability of CCPP system is shown in Fig. 2.
- *Step 3*. System reliability digraph (SRD) corresponding to the block diagram of CCPP (Fig. 2) is shown in Fig. 4.
- Step 4. Under design conditions, it is presumed that all the six systems and components are available at their designed reliabilities during plant operation. Let reliability of these six systems be $(R_i)_d$ (i = 1, 2, ..., 6). Let reliability of interconnections under designed conditions is denoted by $(r_{ij})_d$ (i,j = 1, 2, ..., 6 and $i \neq j$). It is also assumed that all these interconnections are also available during operation at designed reliability. Then the variable permanent system designed reliability matrix for combined cycle power plant under consideration, i.e. $(T_c)_d$ will be corresponding to matrix T_c (Eq. (6)).

$$(T_c)_d = \begin{bmatrix} R_{1d} & r_{12d} & r_{13d} & 0 & 0 & 0\\ 0 & R_{2d} & r_{23d} & 0 & 0 & 0\\ r_{31d} & 0 & R_{3d} & r_{34d} & 0 & 0\\ 0 & r_{42d} & 0 & R_{4d} & r_{45d} & r_{46d} \\ 0 & 0 & 0 & 0 & R_{5d} & r_{56d} \\ 0 & 0 & 0 & r_{64d} & r_{65d} & R_{6d} \end{bmatrix}$$
(9)

The matrices T_c and $(T_c)_d$ are similar and number of nodes and interconnections among the nodes are same. If $(R_i)_d = R_i$ and $(r_{ij})_d = r_{ij}$, then the values of $Per(T_c)$ and $Per(T_c)_d$ will be equal. Therefore in this case $RTRI_{CCPP} = 1$. The $Per(T_c)_d$ value gives the measure of designed reliability of combined cycle power plant, i.e. under the conditions when all its systems and subsystems, and the interconnection between them are available at their designed reliability. This condition exists only during the performance guarantee tests, which are conducted at the time of handing over a newly commissioned power plant [2]. Thereafter, the reliability of various systems and subsystems during operation starts falling below their designed values, and is required to be restored back by adopting proper maintenance strategies [6]. On the other hand, the $Per(T_c)_{RT}$ represents reliability function of combined cycle power plant under normal operating conditions or real-time conditions when all the systems and subsystems are available but may not be operating at their designed reliabilities.

In real-time situation, if one of the four coolers are out, real-time reliability gets reduced to three-fourth of its design value and correspondingly the value of R_{5d} will also get reduced to its three-fourth. The real-time value of $(T_c)_{RT}$ in this case will be obtained by replacing in matrix (Eq. (9)), the values of R_{5d} and r_{56d} , r_{64d} and r_{65d} by their three-fourth values: three-fourth R_{5d} , three-fourth r_{56d} , three-fourth r_{64d} , and three-fourth r_{65d} , respectively. Since the reliability, R_{5d} , of steam turbine system is getting reduced; it will correspondingly limit the reliability of the interconnections connected with this

system (Fig. 4), e.g., r_{56d} , r_{64d} and r_{65d} . Then from the matrix, Eq. (9), the value of $(T_c)_{RT}$ is

	1	2	3	4	5	6	Systems	
	R_{1d}	r_{12d}	r_{13d}	0	0	0	1	
	0	R_{2d}	r_{23d}	0	0	0	2	
$(T_c)_{RT} =$	r_{31d}	0	R_{3d}	r_{34d}	0	0	3	(10)
	0	r_{42d}	0	R_{4d}	r_{45d}	r_{46d}	4	
	0	0	0	0	$0.75R_{5d}$	$0.75r_{56d}$	5	
	0	0	0	$0.75r_{64d}$	$0.75r_{65d}$	R_{6d}	6	

- Step 5. Assuming that the reliability of R_{ids} and r_{ijd} is equal to unity then value for permanent function of matrix in Eq. (9), will be equal to 12 and matrix in Eq. (10) will be equal to 9.94.
- Step 6. RTRI is the ratio of real-time reliability, i.e. (Reliability)_{RT} to the designed reliability that is (Reliability)_D and is expressed as

$$RTRI_{CCPP} = \frac{(Reliability)_{RT}}{(Reliability)_D} = \frac{(VPF-r)_{RT}}{(VPF-r)_D}$$

Therefore the RTRI for this case is = 9.94/12 = 0.828 of its designed value.

Based on the value of RTRI, the operating staff can adjust the process performance, e.g., reduce the electricity generation so as to match with the real-time reliability value. In case plant is allowed to run above the reliability index, it will lead to inefficient and unsafe operations which may lead to safety hazards or complete shutdown at a later stage.

• *Step 7*. Record the results of this study and document them for future analysis.

5.2. Example 2

Fouling, erosion and rubbing wear are responsible for physical changes in the compressor blade geometry which causes the performance deterioration. Fouling is the accumulation of deposits on the blade surfaces causing an increase in surface roughness. The accumulation of deposits increases rapidly during the accumulation of operating time/cycles and then levels off to a fairly constant value where the aerodynamic forces prevent any further accumulation. Increased pressure losses due to fouling can be reduced by washing the engine periodically. But frequent engine washing increases engine erosion. Erosion is the removal of material from the blades surface by solid particles colliding with the blades. This material removal causes increased tip clearances and reduced chord lengths. Erosion has been observed to be more severe in the tip region at the rear of the compressor due to centrifugal forces causing the migration of solid particles to the outer diameter [26]. Rubbing wear is the removal of material from the rotor blade tips and knife edge seals due to contact between static and rotating parts. Typically rubbing wear occurs when compressor blade tips rub with the compressor casing. This is usually the result of the engine rotor flexing during heavy operating loads, or in a mismatch of thermal growth between the rotors and casing [27]. The increased loss due to this effect is more a function of engine cycles than total engine operating hours. The rate of increase in clearances is dependent upon the operating loads imposed on the engine early in its operation. Mass flow and efficiency penalties for fouling, erosion, and wear are calculated with the help of sensors. In general practice the mass flow penalty for erosion and wear is then distributed for each stage across the entire compressor. The efficiency penalty is applied to the compressor section as a whole. A new pressure ratio is adjusted to maintain constant output. Fouling, erosion and rubbing wear affects the reliable availability of compressor and it comes down from the design reliability as mentioned by manufacturer. For a site condition, suppose no data is available regarding the effect of these three factors on compressor performance then an operator with his experience may estimate the intensity of these factors qualitatively based on surrounding and operating conditions. For example, if air is dustier then fouling and erosion are comparatively higher. Further these factors are independent from each other. For the reliability estimation of air compressor at offdesign condition, GTA can be applied at subsystem level with the help of the methodology discussed in Section 4.

Under design conditions, it is presumed that all these factors (fouling, erosion, and rubbing wear) are absent. Let these factors are represented by F_i (i = 1, 2, 3) and their interdependency is denoted by f_{ij} (i,j = 1, 2, 3 and $i \neq j$). Here, the value of F_i is considered to be varying in-between 1 and 0. When any factor F_i is absent, which is the design condition, and it is not affecting the compressor performance then its value is 1. If the presence of factor F_i is so high that compressor stops working then its criticality is highest then it is assigned a value of zero. In other conditions a value in-between 1 and 0 may be assigned based on the observations of site conditions and operating experience. Then cause effect design matrix for these factors is represented as:

$$E_D = \begin{bmatrix} F_{1D} & f_{12D} & f_{13D} \\ f_{21D} & F_{2D} & f_{23D} \\ f_{31D} & f_{32D} & F_{3D} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(11)

As all the factors are independent, therefore, all non-diagonal elements are zero. In real life operating system, at some time operator observes the presence of fouling, erosion and rubbing wear then in the absence of relevant practical data some qualitative value on the scale of 1–0 may be assigned as per the discussion in Section 3.4. Then suppose based on the observation the real-time cause and effect matrix becomes

$$E_{RT} = \begin{bmatrix} F_1 & f_{12} & f_{13} \\ f_{21} & F_2 & f_{23} \\ f_{31} & f_{32} & F_3 \end{bmatrix} = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 0.92 \end{bmatrix}$$
(12)

Corresponding to expression (8) RTRI for compressor comes out to be as:

$$\mathbf{RTRI}_{\text{Compressor}} = \frac{\mathbf{Per}(E_{RT})}{\mathbf{Per}(E_D)} = \frac{0.7866}{1} = 0.7866$$
(13)

Therefore, RTRI for the compressor comes out to be 78.66% of its designed value and it will provide guidance for the quantification of inheritance of compressor in expression (9).

With the help of sensors and control systems, performance of the CCPP may achieve to its design value even if some of its components are deteriorated. But scheduled or unscheduled maintenance has to be done to maintain the reliability up to mark to circumvent complete shutdown. Further reliable working of the sensors and control systems are also dependent on some other factors such as temperature. For measuring their reliability in real life operating conditions, the procedure adopted in example 2 may be extended. A digraph representing factors and interdependencies can be developed. After converting digraph into matrix form quantification may be done. If practical data is not available then some qualitative values based on experience may be assigned to inheritance and interdependencies.

Two examples discussed in this section, reveals that the proposed methodology can be applied for the development of reliability index in a real life operating power plant.

6. Conclusion

In this paper, graph theoretic approach has been applied to obtain real-time reliability index for a combined cycle power plant. For this purpose, CCPP has been divided into six systems as the CCPP is a very large system. Systems/subsystems affecting the reliable availability of power plant are identified. For successful implementation of graph theoretical methodology, it is required to develop digraph, matrix and permanent function based on reliability of systems and reliability of interconnections. The approach helps to express CCPP reliability in quantitative terms. Using this procedure, an appropriate maintenance strategy for any combined cycle power plant can also be recommended.

The proposed structural approach for the evaluation of real-time reliability index for a CCPP has the following features:

- Reliability assessment of power plant is more accurate with graph theory as quantitative measure of interrelations among different systems is taken care of.
- Graph theoretic model is flexible enough to add on different systems, subsystems of and interaction among them in reliability analysis of a CCPP.
- The methodology is proficient in quantifying the influence of various system, subsystems and parameters on reliability of power plant.
- The value of real-time reliability index is useful for designers in selecting an optimum design in terms of reliability from available alternatives.
- The real-time reliability index enables the plant manager to know the reliable availability of power plant on real-time basis which will help them to take commercial decision on real-time basis.
- Sensitivity analysis may be carried out to identify the critical component or system affecting the power plant reliability.

Practical implementation of the proposed methodology in a systematic manner will help power generation industry to identify, categorize, analyze and evaluate parameters responsible for CCPP reliability. Thus, CCPP reliability index will help an organization to carry out SWOT (strength-weaknessopportunities-threats) of their system and take strategic decision to achieve profitability through productivity.

Similarly, methodology can be developed for obtaining other RAM indices: availability and maintainability; including optimum selection, benchmarking, and sensitivity analysis for combined cycle power plant.

References

- De Souza GFM. Thermal power plant performance analysis. London: Elsevier Butterworth-Heinemann; 2012.
- [2] Mohan M, Gandhi OP, Agrawal VP. Real-time reliability index of a steam power plant: a systems approach. Proc Inst Mech Eng Part A: J Power Energy 2008;222:355–69.
- [3] Mohan M, Gandhi OP, Agrawal VP. Systems modeling of a coal based steam power plant. Proc Inst Mech Eng Part A: J Power Energy 2003;217:259–77.
- [4] Mohan M, Gandhi OP, Agrawal VP. Real-time efficiency index of a steam power plant: a systems approach. Proc Inst Mech Eng Part A: J Power Energy 2006;220:103–31.
- [5] Garg RK, Agrawal VP, Gupta VK. Selection of power plants by evaluation and comparison using graph theoretical methodology. Electr Power Energy Syst 2006;28:429–35.
- [6] Mohan M, Gandhi OP, Agrawal VP. Maintenance criticality index of a steam power plant: a graph theoretic approach. Proc Inst Mech Eng Part A: J Power Energy 2004;218:619–36.
- [7] Goode KB, Moore J, Roylance BJ. Plant machinery working life prediction method utilising reliability and condition monitoring data. Proc Inst Mech Eng Part E: J Process Mech Eng 2000;214:109–22.
- [8] Bradt D. Use of reliability, availability and maintainability techniques to optimise system operation. Hydrocarbon Process 1997;76:63–5.
- [9] Gandhi OP, Agrawal VP, Shishodia KS. Reliability analysis and evaluation of systems. Reliab Eng Syst Safety 1991;32:283–305.
- [10] Carlier S, Coindoz M, Deneuville L, Garbellini L, Altavilla A. Evaluation of reliability, availability, maintainability and safety requirements for manned space vehicles with extended on-orbit stay time. Acta Astronautica 1996;38(2):115–23.
- [11] Ji G, Wu W, Zhang B, Sun H. A renewal-process-based component outage model considering the effects of aging and maintenance. Electr Power Energy Syst 2013;44:52–9.
- [12] Eti M, Ogaji S, Probert S. Integrating reliability, availability, maintainability and supportability with risk analysis for improved operation of the AFAM thermal power-station. Appl Energy 2007;84:202–21.
- [13] Haghifam MR, Manbachi M. Reliability and availability modelling of combined heat and power (CHP) systems. Electr Power Energy Syst 2011;33:385–93.
- [14] Carpaneto E, Chicco G, Mancarella P, Russo A. Cogeneration planning under uncertainty. Part I: Multiple time frame approach. Appl Energy 2011;88:1059–67.
- [15] Carpaneto E, Chicco G, Mancarella P, Russo A. Cogeneration planning under uncertainty. Part II: Decision theory-based assessment of planning alternatives. Appl Energy 2011;88:1075–83.
- [16] Tang J. Mechanical system reliability analysis using a combination of graph theory and Boolean function. Reliab Eng Syst Safety 2001;72:21–30.
- [17] Kehlhofer RH, Warner J, Nielsen H, Bachmann R. Combined cycle gas and steam turbine power plants. Tulsa: PennWell; 1999.
- [18] Faisal MN, Banwet DK, Shankar R. Quantification of risk mitigation environment of supply chains using graph theory and matrix methods. Euro J Ind Eng 2007;1(1):22–39.
- [19] Wani MF, Gandhi OP. Development of maintainability index for mechanical systems. Reliab Eng Syst Safety 1999;65:259–70.
- [20] Kulkarni S. Graph theory and matrix approach for performance evaluation of TQM in Indian industries. TQM Mag 2005;17(6):509–26.
- [21] Dev N, Samsher, Kachhwaha SS, Attri R. GTA-based framework for evaluating the role of design parameters in cogeneration cycle power plant efficiency. Ain Shams Eng J 2013;4(2):273–84.
- [22] Deo N. Graph theory with applications to engineering and computer science. New Delhi: Prentice Hall; 2007.

- [23] Raj T, Attri R. Quantifying barriers to implementing Total Quality Management (TQM). Euro J Ind Eng 2010;4(3):308–35.
- [24] Dev N, Samsher, Kachhwaha SS. System modeling and analysis of a combined cycle power plant. Int J Syst Assur Eng Manage 2012, doi: http://dx.doi.org/10.1007/s13198-012-0112-y.
- [25] Koenigsberger F, Tlusty J. Machine tool structures. London: Pergamon; 1970.
- [26] Tabakoff W. Compressor erosion and performance deterioration. A1AA/ASME 4th joint fluid mechanics, plasma dynamics, and lasers conference, vol. 37. Atlanta (GA): ASME Publication FED; 1986.
- [27] Zaita AV, Buley G, Karlsons G. Performance deterioration modeling in aircraft gas turbine engines. J Eng Gas Turb Power 1998;120:344–9.



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