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Influence of data transformation on multicriteria evaluation result

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Abstract

The main idea of quantitative multiple criteria decision-making methods (MCDM) is comprising values of a chosen set of criteria into a single cumulative criterion of evaluation. Units of measurement can be different: per cent, ranks, grades, money units, physical units, etc. Consequently, their incorporation into a single evaluation criterion is possible if values of criteria are independent of units of measurement. Such dimensionless values are obtained by normalizing the values. Criteria can be both minimizing and maximizing. Some MCDA methods imply transformation of minimizing criteria into maximizing ones. Moreover, values of criteria can me negative (profit, growth rate, etc.), but some MCDA methods can use only positive criteria. Therefore, majority of MCDA methods use both normalization and transformation of criteria with negative values. There are different formulae available. Even in the same method different transformation and normalization formulae can be used. Nevertheless, using different transformation and normalization formulae can lead to differences in results of evaluation. In this paper it is shown that different types of transformation and normalization of data applied to popular MCDA methods, such as SAW or TOPSIS may produce considerable differences in evaluation. Consequently, attention has to be paid to making a choice of the type of normalization, which reveals preferences of decision-maker. Dependence of evaluation results on the chosen type of transformation or normalization is demonstrated. A case-study is provided.

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Keywords: Normalization, transformation, MCDA, multiple criteria, SAW, TOPSIS

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1. Introduction

In decision-aid systems various quantitative multiple criteria decision-aid (MCDA) methods are widely used. The purpose of the methods is to help a decision-maker to evaluate several alternatives or processes in respect to an objective of evaluation by indicating the best alternative/process or by providing a ranking of the alternatives by the level of their attractiveness. The whole plethora of the MCDA methods exist with varying degree of popularity. It is logical to conclude that neither single universal method, which most accurately would reflect characters of decision-makers and process given data, is not yet identified.

A core feature of any decision-aid method is to reveal and embed preferences of decision-makers with an acceptable level of fidelity into a decision-aid methodology for the purpose of reflecting opinions of participating experts. Majority of the methods use only weights of criteria for this purpose. Criteria describing characteristics of evaluated objects in respect of the goal of evaluation must be chosen in the initial step of the analysis. Values of criteria are used in mentioned methodologies together with weights, which measure preferences of expert decision-makers in terms of importance of chosen criteria for evaluation. Values of criteria describing characteristics of the alternatives can be derived from existing statists or be estimated by experts. An MCDA method comprises both components, namely values of criteria, which characterize the evaluated objects or processes, and weights elicited from experts. Finally, both weights and values of criteria are comprised into a single criterion of the MCDA method. Such an aggregation is possible only if criteria are measured in the same dimension. This is achieved by making values of criteria dimensionless. This is achieved by normalization of values of each criteria, which is a mapping of values of criteria into the set of dimensionless real numbers.

Usually, several types of normalization are simultaneously available for certain methods, choices of ways of normalization can be made. Contrary, a few methods, such as TOPSIS or COPRAS, use only one proprietary normalization [1]. Different types of normalization naturally may influence the result of evaluation as they map values of criteria into the set of real numbers different way.

Several types of criteria exist. Maximizing and minimizing criteria are widely used in vast majority of cases. Maximizing criteria are such that the higher their value is associated with reflection of an underlying better appreciation of the alternative by a decision-maker. Contrary, smaller values of minimizing criteria mean that the corresponding alternative should be reflected as more attractive. Some methods use exclusively maximizing criteria. This implies that an additional transformation of minimizing values to maximizing values is required in order to use such a method.

Virtually, a normalization type should attempt to adequately account opinions of participating experts on each criterion. Therefore, using different types of normalization for different criteria is a natural solution in such cases, when perception of values of different criteria is different by participating experts. There could be cases, when neither above-mentioned type of normalization fits. In such cases, when an expert decision-maker would be inclined to opt for a particular value of a criterion and has certain preferences, say, for a certain size of a house, age of an employee, etc., using neither maximizing nor minimizing normalization would be plausible. Using a normalization suitable for a maximizing criterion in such a case would produce a distortion. For example, in a case of choosing a house of a particular size, say, of 100 square meters, which will be discussed below in more detail, such a normalization designed for a maximizing criterion would produce a higher normalized value for 200 or 250 square meter house, even if a buyer would not prefer a bigger house because of higher heating bills to pay, more cleaning effort required, less efficient accessibility of all locations of the house, more floors, etc.

In the case, when an option for a good available in the market is considered by a buyer, it is plausible to consider the normalization, which maps the worst value to zero, and the best value to 1. This normalization often can represent perception of the situation in the market by a buyer decision-maker. Even in case if a value of certain characteristic of the worst commodity in the market is rather good, this characteristic would often be assessed by the lowest grade (in our case it is 0), while the characteristic of the best commodity by the best grade (in our case 1).

Unfortunately, availability of different methods of normalization yet do help to achieve a better matching of decision-maker's preferences. Usually, MCDA methods use the same type of normalization for the entire set of criteria. And criteria can be perceived in a different way. Also, research on proper matching of type of normalization in accordance with preferences of the decision-maker is scarce.

In this paper it is shown that there could be cases, when improper choice of type of normalization is considerably altering the result of evaluation. A simple case of choosing a house by a buyer is created and studied. A case, where different methods of normalization simultaneously are used, is proposed. The paper suggests an approach of putting more emphasis on making a choice of a method of normalization and on creation of more appropriate types of normalization to cover the whole variety of the ways of how decision-makers may perceive values of a criterion in accordance to his/her utility. Such an approach may improve mapping of criteria to the set of normalized criteria making it more adequate.

2. Some popular MCDA methods, which use normalization

2.1. The SAW (Simple Additive Weighing) method

The method is one of the most popular [1, 2]. It exposes core ideas of the MCDA methods to comprise normalized values of criteria and their weights of importance to a single criterion of the method by the following formula (1) [4, 5]:

$$S_j = \sum_{i=1}^{m} \omega_i \tilde{r}_{ij} \tag{1}$$

where S_j is the cumulative criterion; ω_i are weights of criteria; m is the number of chosen criteria; $\tilde{r}_{ij}$ are normalized values of criteria; i – index for criteria; j – index for alternatives. Normalization of data is a mapping of function between the set of values of criteria, which have a certain dimension or measurement, to the set of real numbers. The initial idea of such a mapping is to make values of criteria dimensionless, nevertheless a mapping for applied to a certain criterion does not necessarily reflect opinion of participating experts

The criterion S_j is calculated for each alternative and shows the level of their attractiveness in the quantitative way. The larger is the cumulative criterion S_j, the more attractive is the alternative.

The SAW method uses only maximizing criteria and only positive values. Therefore, in the case if minimizing criteria are present, they have to be transformed into the maximizing ones. There are several ways to do this.

The normalization mapping values of criteria into the interval [0,1] could be used, which allows to transform minimizing criteria into maximizing in one step. Normalization of maximizing criteria is carried out by the formula (2):

$$\tilde{r}_{ij} = \frac{r_{ij} - \min_j r_{ij}}{\max_j r_{ij} - \min_j r_{ij}} \tag{2}$$

While normalization of minimizing criteria is carried out by the formula (3):

$$\tilde{r}_{ij} = \frac{\max_j r_{ij} - r_{ij}}{\max_j r_{ij} - \min_j r_{ij}}. \tag{3}$$

Other popular type of normalization as shown by the formula (4):

$$\bar{r}_{ij} = \frac{r_{ij}}{\max_j r_{ij}} \tag{4}$$

does not require additional two transformation of minimizing criteria into maximizing ones as it is carried out by using the inverse formula (5) for minimizing criteria instead of the formula (4):

$$\bar{r}_{ij} = \frac{\min_j r_{ij}}{r_{ij}} \qquad (5)$$

The normalization, which maps values of criteria to such normalized values, which make up one in sum as is shown by the formula (6) requires additional transformation of minimizing values into maximizing ones. Such a normalization can be carried out by the following formula (5), nevertheless it introduces distortions [6].

$$\tilde{r}_{ij} = \frac{r_{ij}}{\sum_{j=1}^{m} r_{ij}} \qquad (6)$$

In case there are negative values present, both above-mentioned types of normalization require an a-priori transformation of negative values to positive ones. It could be carried out by the formula (7) [7]:

$$\hat{r}_{ij} = 1 + r_{ij} + \min_j r_{ij} \qquad (7)$$

We note that such a transformation introduces distortions as the result depends on the magnitude of the shift of the set of values of criteria.

2.2. The TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method

The TOPSIS method uses a proprietary vector normalization. It normalizes values of criteria by formula (8) in a way that the resulting normalized vector is of the unitary length [8, 9]. The normalized vector is constructed for each criterion. Its co-ordinates consist of values of the criteria for each alternative, divided by the normalizing constant:

$$\tilde{r}_{ij} = \frac{r_{ij}}{\sqrt{\sum_{j=1}^{n} r_{ij}^2}} \qquad (8)$$

In the original method the normalization is uniform for all the criteria. The original method does not allow to alter any normalization and to use an alternative normalization in the case if preferences of an expert decision-maker differ.

3. Case study: a problem of choosing the best house

Suppose a decision-maker encounters a problem of choosing the best house. Let him choose a set of criteria, which describe a house as follows:

1) The size of the house, sq. m.;
2) Distance to public transport, km;
3) Adjacent land area, ares;
4) Distance to city center
5) Air pollution, grade;
6) Distance to a grocery store, km.

Suppose, the decision-maker has a ‘proprietary’ preference for the size of the house 100 square meters because of the following reasons already mentioned above: heating bills to pay, cleaning effort required, efficient accessibility of all locations of the house, prevailing single floor for this size.

Neither of described above methods of normalization would correctly map values of corresponding criterion representing the size of the house in case the decision-maker is considering the following alternatives of houses A, B, C, and D with the following values of criteria. For the purpose of the case-study we outline the following weights to the criteria, as is shown in Table 1.

Table 1. Values of criteria, which represent alternative houses considered.

		Alternatives:	A	B	C	D	Weights
No. of criterion	Criteria	type					
1.	Size, sq. m.	max or proprietary	110	60	200	250	0.32
2.	Distance to public transport, km	min	5	2	1.5	7	0.27
3.	Adjacent land area, ares	max	5.5	5	6.5	6	0.13
4.	Distance to city center, km	min	12	7	20	18	0.11
5.	Air pollution, grade	min	3	7	2	1	0.12
6.	Distance to a grocery store, km	min	2	0.5	3	10	0.05

Transformed values for the SAW method by formulae (5)–(6) are presented in Table 2.

Table 2. Values of normalized criteria for the SAW method.

	Alternatives:	A	B	C	D
No. of criterion	Criteria				
1.	Size, sq. m.	0.177	0.097	0.323	0.403
2.	Distance to public transport, km	0.132	0.331	0.442	0.095
3.	Adjacent land area, ares	0.239	0.217	0.283	0.261
4.	Distance to city center, km	0.251	0.431	0.151	0.167
5.	Air pollution, grade	0.169	0.072	0.253	0.506
6.	Distance to a grocery store, km	0.170	0.682	0.114	0.034

Transformed values for the TOPSIS method by formula (8) are presented in Table 3.

Table 3. Values of normalized criteria for the TOPSIS method.

	Alternatives:	A	B	C	D
No. of criterion	Criteria				
1.	Size, sq. m.	0.320	0.175	0.582	0.727
2.	Distance to public transport, km	0.558	0.223	0.167	0.781
3.	Adjacent land area, ares	0.476	0.433	0.563	0.519
4.	Distance to city center, km	0.396	0.231	0.660	0.594
5.	Air pollution, grade	0.378	0.882	0.252	0.126
6.	Distance to a grocery store, km	0.188	0.047	0.282	0.940

As the criterion 1 should reflect preference of the decision-maker for the house of 100 square meters, ‘proprietary’ transformation (9) should be used, values of which is presented in Table 4.

$$\tilde{r}_{ij} = \exp^{-z^2/2}, \quad \text{where} \quad z = \frac{(r_{ij} - r_i^0)}{\sigma_i}, \tag{9}$$

The mean r_i^0 is the most desirable value of the criterion, and σ_i is perceived standard deviation of the mapping of values of criteria into the set of real numbers.

We used 100 square meters as the mean, and rather large number 50 for standard deviation to replicate a possibly quite high level of indifference of the buyer to the size of the house.

Table 4. Values of normalized criterion 1 by formula (9).

	Alternatives:	A	B	C	D
No. of criterion	Criteria				
1.	Size, sq. m.	0.980	0.726	0.135	0.011

This case of transformation will provide alternative values of the criterion 2, which will be used to calculate values of SAW and TOPSIS alternative cumulative criteria to compare them with the cumulative criteria, obtained from normalized values from Table 2 and 3. Results are shown in Table 5 and 6.

Table 5. Values of cumulative criterion of the SAW method.

Alternatives:	A	B	C	D
Cumulative criterion				
obtained from classic normalization	0.180	0.239	0.312	0.269
obtained from 'proprietary' normalization	0.293	0.333	0.232	0.142

Table 6. Values of cumulative criterion of the TOPSIS method.

Alternatives:	A	B	C	D
Cumulative criterion				
obtained from classic normalization	0.387	0.451	0.767	0.530
obtained from 'proprietary' normalization	0.707	0.684	0.455	0.229

Comparison of results obtained using both SAW and TOPSIS methods shows huge discrepancies between obtained rankings of the alternatives considered (Table 7). This clearly suggests necessity to pay much higher attention to the choice of types of normalization.

Table 7. Comparison of rankings of alternatives obtained using SAW and TOPSIS methods.

Alternatives:	A	B	C	D
Cumulative criterion				
obtained using classic normalization, SAW	4	3	1	2
obtained using classic normalization, TOPSIS	4	3	1	2
obtained using 'proprietary' normalization, SAW	2	1	3	4
obtained using 'proprietary' normalization, TOPSIS	1	2	3	4

The normalization (9), which sustains the preference of the decision-maker for a smaller house, considerably adjusts results in favor of the alternatives 1 and 2. Adjustment is visible because of the rather high weight assigned to the criterion 2. Values of the cumulative criterion of the TOPSIS method for the two alternatives differ by only 3%, which make the two alternatives of similar attractiveness.

4. Conclusions

A core feature of any decision-aid method is to reveal and embed preferences of decision-makers with an acceptable level of fidelity into a decision-aid methodology for the purpose of reflecting opinions of participating experts. Majority of the methods use only weights of criteria for this purpose. Different types of transformation and normalization of data are available for popular MCDA methods, such as SAW or TOPSIS. If different methods are applied, it may produce considerable differences in evaluation. Consequently, much more attention has to be paid to making a choice of the type of normalization, which should reveal preferences of decision-maker in the best possible way.

Different types of normalization naturally may influence the result of evaluation as they map values of criteria into the set of real numbers different way. Using different types of normalization for different criteria is a natural solution in such cases, when perception of values of different criteria is different by participating experts. There could be cases, when neither maximizing, nor minimizing type of normalization fits. In such cases, when an expert decision-maker would be inclined to opt for a particular value of a criterion and has certain preferences, say, for a certain size of a house, age of an employee, etc., a special 'proprietary' normalization should be used.

In this paper it is shown that there could be cases, when improper choice of type of normalization is considerably altering the result of evaluation. A case study of choosing a house by a buyer is created and studied. A case, where different methods of normalization simultaneously are used, is proposed. The paper suggests an approach of putting more emphasis on making a choice of a method of normalization and on creation of more appropriate types of normalization to cover the whole variety of the ways of how decision-makers may perceive values of a criterion in accordance to his/her utility. Such an approach may improve mapping of criteria to the set of normalized criteria making it more adequate and could provide an additional reporting tool for MCDA methods [10].

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