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The derivative of the topological susceptibility at zero momentum and an estimate of η' mass in the chiral limit

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Abstract

The anomaly-anomaly correlator is studied using QCD sum rules. Using the matrix elements of anomaly between vacuum and pseudoscalars π, η and η', the derivative of correlator $\chi'(0)$ is evaluated and found to be $\approx 1.82 \times 10^{-3}$ GeV^2. Assuming that $\chi'(0)$ has no significant dependence on quark masses, the mass of η' in the chiral limit is found to be ≈ 723 MeV. The same calculation also yields for the singlet pseudoscalar decay constant in the chiral limit a value of ≈ 178 MeV.

In QCD, it is known that the vacuum gauge field configurations are significant, in particular the instanton solutions corresponding to self-dual fields ($G^a_{\mu\nu} = \pm(1/2)\tilde{G}^a_{\mu\nu}$) play a role in solving the so-called $U(1)$ problem, i.e., the ninth pseudoscalar η' remains massive even in the chiral limit when all quark masses are zero. The axial vector current in QCD has an anomaly

$$\partial^\mu \bar{q}\gamma_\mu\gamma_5 q = 2im_q\bar{q}\gamma_5 q - \frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}, \quad \text{where } \tilde{G}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}. \tag{1}$$

The topological susceptibility $\chi(q^2)$ defined by

$$\chi(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|T\{Q(x), Q(0)\}|0\rangle, \quad \text{with } Q(x) = \frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}, \tag{2}$$

is of considerable theoretical interest and has been studied using a variety of theoretical tools like lattice gauge theory, QCD sum rules, chiral perturbation theory, etc. In particular, the derivative of the susceptibility at $q^2 = 0$

$$\chi'(0) = \left.\frac{d\chi(q^2)}{dq^2}\right|_{q^2=0} \tag{3}$$

enters in the discussion of the proton-spin problem [1–5]. In the QCD sum rule approach, one can determine $\chi'(0)$ as follows. Using dispersion relation one can write

$$\frac{\chi'(q^2)}{q^2} - \frac{\chi'(0)}{q^2} = \frac{1}{\pi}\int ds\, \Im(\chi(s))\left[\frac{1}{s(s-q^2)^2} + \frac{1}{s^2(s-q^2)}\right] + \text{subtractions}. \tag{4}$$

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Defining the Borel transform of a function $f(q^2)$ by

$$\hat{B}f\big(q^2\big) = -q^2 \lim_{n\to\infty}\left[\frac{(-q^2)^{n+1}}{n!}\left(\frac{d}{dq^2}\right)^n f\big(q^2\big)\right]_{-q^2/n=M^2}, \tag{5}$$

one gets from Eq. (4)

$$\chi'(0) = \frac{1}{\pi}\int ds\, \frac{\Im(\chi(s))}{s^2}\left(1+\frac{s}{M^2}\right)e^{-s/M^2} - \hat{B}\left[\frac{\chi'(q^2)}{q^2}\right]. \tag{6}$$

According to Eq. (2), $\Im(\chi(s))$ receives contribution from all states $|n\rangle$ such that $\langle 0|Q|n\rangle \neq 0$. In particular, we have [6]

$$\langle 0|Q|\pi^0\rangle = f_\pi m_\pi^2 \frac{m_d - m_u}{m_d + m_u}\frac{1}{2\sqrt{2}}. \tag{7}$$

The matrix elements, when $|n\rangle$ is $|\eta\rangle$ or $|\eta'\rangle$, can be determined as follows. It is known from both theoretical considerations based on chiral perturbation theory as well as phenomenological analysis that one needs two mixing angles θ_8 and θ_0 to describe the coupling of the octet and singlet axial vector currents to η and η' [7–9]. Introduce the definition

$$\langle 0|J^a_{\mu 5}|P(p)\rangle = i f^a_P p_\mu, \quad a = 0, 8, \quad P = \eta, \eta', \tag{8}$$

where $J^{8,0}_{\mu 5}$ are the octet and singlet axial currents:

$$J^8_{\mu 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s), \tag{9}$$

$$J^0_{\mu 5} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s). \tag{10}$$

The $|P(p)\rangle$ represents either η or η' with momentum p_μ. The couplings f^a_P can be equivalently represented by two couplings f_8, f_0 and two mixing angles θ_8 and θ_0 by the matrix identity

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f_8\cos\theta_8 & -f_0\sin\theta_0 \\ f_8\sin\theta_8 & f_0\cos\theta_0 \end{pmatrix}. \tag{11}$$

Phenomenological analysis of the various decays of η and η' to determine f^a_P has been carried out by a number of authors [7–9]. In a recent analysis [9] Escribano and Frere find, with

$$f_8 = 1.28 f_\pi \quad (f_\pi = 130.7\ \text{MeV}), \tag{12}$$

the other three parameters to be

$$\theta_8 = (-22.2 \pm 1.8)^\circ, \qquad \theta_0 = (-8.7 \pm 2.1)^\circ, \qquad f_0 = (1.18 \pm 0.04) f_\pi. \tag{13}$$

The divergence of the axial currents are given by

$$\partial^\mu J^8_{\mu 5} = \frac{i2}{\sqrt{6}}(m_u\bar{u}\gamma_5 u + m_d\bar{d}\gamma_5 d - 2m_s\bar{s}\gamma_5 s), \tag{14}$$

$$\partial^\mu J^0_{\mu 5} = \frac{i2}{\sqrt{3}}(m_u\bar{u}\gamma_5 u + m_d\bar{d}\gamma_5 d + m_s\bar{s}\gamma_5 s) - \frac{1}{\sqrt{3}}\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}. \tag{15}$$

Since $m_u, m_d \ll m_s$, one can neglect them [10] to obtain

$$\langle 0|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}|\eta\rangle = \sqrt{\frac{3}{2}}m_\eta^2\big(f_8\cos\theta_8 - \sqrt{2}f_0\sin\theta_0\big), \tag{16}$$

$$\langle 0|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}|\eta'\rangle = \sqrt{\frac{3}{2}}m_{\eta'}^2\big(f_8\sin\theta_8 + \sqrt{2}f_0\cos\theta_0\big). \tag{17}$$

Using Eqs. (7), (16) and (17) we get the representation of $\chi(q^2)$ in terms of physical states as

$$\chi\big(q^2\big) = -\frac{m_\pi^4}{8(q^2 - m_\pi^2)}f_\pi^2\left(\frac{m_d - m_u}{m_d + m_u}\right)^2 - \frac{m_\eta^4}{24(q^2 - m_\eta^2)}\big(f_8\cos\theta_8 - \sqrt{2}f_0\sin\theta_0\big)^2$$
$$- \frac{m_{\eta'}^4}{24(q^2 - m_{\eta'}^2)}\big(f_8\sin\theta_8 + \sqrt{2}f_0\cos\theta_0\big)^2 + \text{higher mass states}. \tag{18}$$

On the other hand, $\chi(q^2)$ has an operator product expansion [1,5,11,12]

$$\chi(q^2)_{\mathrm{OPE}} = -\left(\frac{\alpha_s}{8\pi}\right)^2 \frac{2}{\pi^2} q^4 \ln\left(\frac{-q^2}{\mu^2}\right)\left[1+\frac{\alpha_s}{\pi}\left(\frac{83}{4}-\frac{9}{4}\ln\left(\frac{-q^2}{\mu^2}\right)\right)\right] - \frac{1}{16}\frac{\alpha_s}{\pi}\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle\left(1-\frac{9}{4}\frac{\alpha_s}{\pi}\ln\left(\frac{-q^2}{\mu^2}\right)\right)$$
$$+\frac{1}{8q^2}\frac{\alpha_s}{\pi}\langle 0|\frac{\alpha_s}{\pi}g_s G^3|0\rangle - \frac{15}{128}\frac{\pi\alpha_s}{q^4}\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle^2 + 16\left(\frac{\alpha_s}{4\pi}\right)^3 \sum_{i=u,d,s} m_i\langle\bar{q}_i q_i\rangle\left[\ln\left(\frac{-q^2}{\mu^2}\right)+\frac{1}{2}\right]$$
$$-\left[\frac{q^4}{2}\int d\rho\, n(\rho)\rho^4 K_2^2(Q\rho) + \text{screening correction to the direct instantons}\right]. \tag{19}$$

In Eq. (19), the first term arises from the perturbative gluon loop with radiative corrections [12], the second, third and fourth terms are from the vacuum expectation values of G^2, G^3 and G^4. The $\langle 0|G^4|0\rangle$ term has been expressed as $\langle 0|G^2|0\rangle^2$ using factorization [11]. The fifth term proportional to the quark mass has been computed by us and is indeed quite small compared to other terms numerically. Finally, the last two terms represent the contribution to $\chi(q^2)$ from the direct instantons [11]. $n(\rho)$ is the density of instanton of size ρ, K_2 is the McDonald function and $Q^2 = -q^2$. In a recent work [13], Forkel has emphasized the importance of screening correction which almost cancels the direct instanton contribution (cf., especially, Fig. 8 and Sections V and VI of Ref. [13]). For this reason we shall disregard the direct instanton term and screening correction for the present and return to it later.

From Eq. (6), we now obtain

$$\chi'(0) = \frac{f_\pi^2}{8}\left(\frac{m_d-m_u}{m_d+m_u}\right)^2\left(1+\frac{m_\pi^2}{M^2}\right)e^{-m_\pi^2/M^2} + \frac{1}{24}\left(f_8\cos\theta_8 - \sqrt{2}f_0\sin\theta_0\right)^2\left(1+\frac{m_\eta^2}{M^2}\right)e^{-m_\eta^2/M^2}$$
$$+\frac{1}{24}\left(f_8\sin\theta_8+\sqrt{2}f_0\cos\theta_0\right)^2\left(1+\frac{m_{\eta'}^2}{M^2}\right)e^{-m_{\eta'}^2/M^2}$$
$$-\left(\frac{\alpha_s}{4\pi}\right)^2\frac{1}{\pi^2}M^2E_0(W^2/M^2)\left[1+\frac{\alpha_s}{\pi}\frac{74}{4}+\frac{\alpha_s}{\pi}\frac{9}{2}\left(\gamma-\ln\frac{M^2}{\mu^2}\right)\right]$$
$$-16\left(\frac{\alpha_s}{4\pi}\right)^3\frac{1}{M^2}\sum_{i=u,d,s}m_i\langle\bar{q}_i q_i\rangle - \frac{9}{64}\frac{1}{M^2}\left(\frac{\alpha_s}{\pi}\right)^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{1}{16}\frac{1}{M^4}\frac{\alpha_s}{\pi}\left\langle g_s\frac{\alpha_s}{\pi}G^3\right\rangle - \frac{5}{128}\frac{\pi^2}{M^6}\frac{\alpha_s}{\pi}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle^2. \tag{20}$$

Here $E_0(x) = 1 - e^{-x}$ and takes into account the contribution of higher mass states, which has been summed using duality to the perturbative term in χ_{OPE}, and W is the effective continuum threshold. We take $W^2 = 2.3\ \mathrm{GeV}^2$, and in Fig. 1 plot the r.h.s. of Eq. (20) as a function of M^2. We take $\alpha_s = 0.5$ for $\mu = 1$ GeV and

$$\langle 0|g_s^2G^2|0\rangle = 0.5\ \mathrm{GeV}^2, \qquad \langle 0|\bar{s}s|0\rangle = 0.8\langle 0|\bar{u}u|0\rangle \quad \text{with } \langle 0|\bar{u}u|0\rangle = -(240\ \mathrm{MeV})^3,$$
$$m_s = 150\ \mathrm{MeV}, \quad m_u/m_d \approx 0.5. \tag{21}$$

Writing

$$\langle 0|g_s^3G^3|0\rangle = \frac{\epsilon}{2}\langle 0|g_s^2G^2|0\rangle, \tag{22}$$

as in Ref. [5], we take $\epsilon = 1\ \mathrm{GeV}^2$. We also have the PCAC relation,

$$-2(m_u+m_d)\langle 0|\bar{u}u|0\rangle = f_\pi^2 m_\pi^2. \tag{23}$$

For f_0, f_8, θ_8 and θ_0 we use the central values given in Eqs. (12) and (13).

Let us now examine how the various terms in the r.h.s. of Eq. (20) add up to remain a constant. The pion term is small and has little variation because of its low mass, η and η' are significantly larger and η is even larger that η'. In Fig. 1 the upper line gives the combined contribution of π, η and η' which we denote as χ'_{poles} and it is seen that it has a gentle increase with M^2. The OPE term given by the last two lines in Eq. (20), which we denote by χ'_{OPE}, so that

$$\chi'(0) = \chi'_{\mathrm{poles}} - \chi'_{\mathrm{OPE}}$$

is also plotted in Fig. 1. It is seen that χ'_{OPE} is roughly about 25% of χ'_{poles} also increases with M^2, with the result that $\chi'(0)$ is nearly a constant w.r.t. M^2.

We expect this trend of compensating variation in χ'_{poles} and χ'_{OPE} to be maintained when variation in χ'_{poles} due to uncertainties in θ_8, θ_0, f_8, f_0 (see Eqs. (12) and (13)) and the variations in χ'_{OPE} due to uncertainties in the estimates of the vacuum condensates are taken into account. We can then obtain, from Fig. 1, the value

$$\chi'(0) \approx 1.82\times 10^{-3}\ \mathrm{GeV}^2. \tag{24}$$

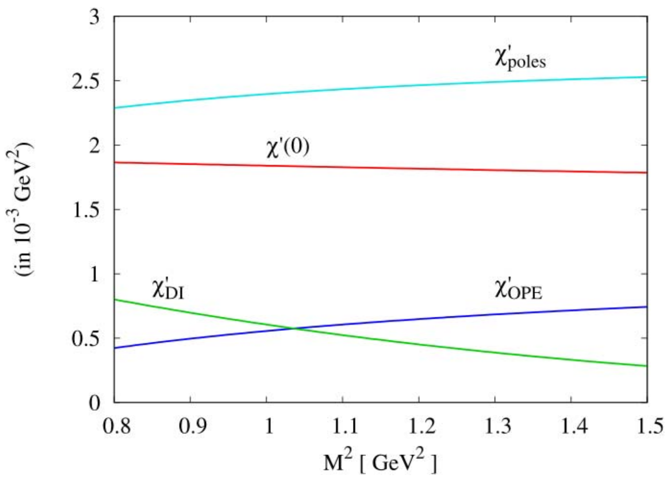

Fig. 1. Various terms contributing to $\chi'(0)$, Eq. (20). The value of $\chi'(0)$ is the one obtained without the direct instantons. The latter, see Eq. (29), is given by $\chi'_{\rm DI}$, which is larger than $\chi'_{\rm OPE}$ and also has the wrong M^2 behaviour suggesting that screening corrections are important.

We note that the above determination, Eq. (24), is in agreement with an entirely different calculation by two of us [14] from the study of the correlator of isoscalar axial vector currents

$$\begin{aligned}
\Pi^{I=0}_{\mu\nu} &= \frac{i}{2}\int d^4x\, e^{iq\cdot x}\langle 0|\{\bar u\gamma_\mu\gamma_5 u(x)+\bar d\gamma_\mu\gamma_5 d(x), \bar u\gamma_\mu\gamma_5 u(0)+\bar d\gamma_\mu\gamma_5 d(0)\}|0\rangle,\\
\Pi^{I=0}_{\mu\nu} &= -\Pi^{I=0}_1(q^2)g_{\mu\nu}+\Pi^{I=0}_2(q^2)q_\mu q_\nu.
\end{aligned} \tag{25}$$

$\Pi^{I=0}_1(q^2=0)$ can be computed from the spectrum of axial vector mesons. In Ref. [14] a value

$$\Pi^{I=0}_1(q^2=0) = -0.0152\ \text{GeV}^2 \tag{26}$$

was obtained. It is not difficult to see that when $m_u = m_d = 0$

$$\chi'(0) = -\frac{1}{8}\Pi^{I=0}_1(q^2=0). \tag{27}$$

From Eqs. (26) and (27) we get $\chi'(0)\approx 1.9\times 10^{-3}\ \text{GeV}^2$, which is consistent with Eq. (24). Let us now return to Eq. (19) and consider the effect of incorporating the direct instanton term in Eq. (20) in the spike approximation [5]:

$$n(\rho) = n_0\delta(\rho-\rho_c), \tag{28}$$

with $n_0 = 0.75\times 10^{-3}\ \text{GeV}^4$ and $\rho_c = 1.5\ \text{GeV}^{-1}$. The contribution of the direct instanton to $\hat B[\chi'(q^2)/q^2]$ can be found using the asymptotic expansion for $K_2(z)$ and $K'_2(z)$ and we find it to be

$$\chi'_{\rm DI} = \frac{n_0}{4}\sqrt{\pi}\rho_c^4 M^2\left[M\rho_c+\frac{9}{4}\frac{1}{M\rho_c}+\frac{45}{32}\frac{1}{M^3\rho_c^3}\right]e^{-M^2\rho_c^2}. \tag{29}$$

We have plotted this term separately in Fig. 1. We note that unlike $\chi'_{\rm poles}$ and $\chi'_{\rm OPE}$, which increase with M^2 and therefore compensate each other, the contribution of $\chi'_{\rm DI}$, Eq. (29), decreases rapidly with M^2. It is not difficult to see that $\chi'(0)$ will no longer remain constant. This strongly suggests that screening corrections to $\chi'_{\rm DI}$ are important just as they are for $[\chi(q^2)/q^2]$ as found by Forkel [13].

We now turn to an estimate of η' mass in the chiral limit: $m_u = m_d = m_s = 0$. In this limit $SU(3)$ flavor symmetry is exact and, we have $m_\pi = m_\eta = 0$ while η' is a singlet. Let us denote by $\eta_\chi = \eta'(m_s=0)$ and $m_\chi = m_{\eta'}(m_s=0)$, the singlet particle and its mass in the chiral limit. Returning to Eq. (19), we first note that the explicitly quark mass dependent term in $\chi_{\rm OPE}$

$$-16\left(\frac{\alpha_s}{4\pi}\right)^3\sum_{i=u,d,s} m_i\langle\bar q_i q_i\rangle \approx 1.9\times 10^{-6}\ \text{GeV}^4$$

is numerically much smaller than, for example,

$$\frac{9}{64}\left(\frac{\alpha_s}{\pi}\right)^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \approx 4.5\times 10^{-5}\ \text{GeV}^4,$$

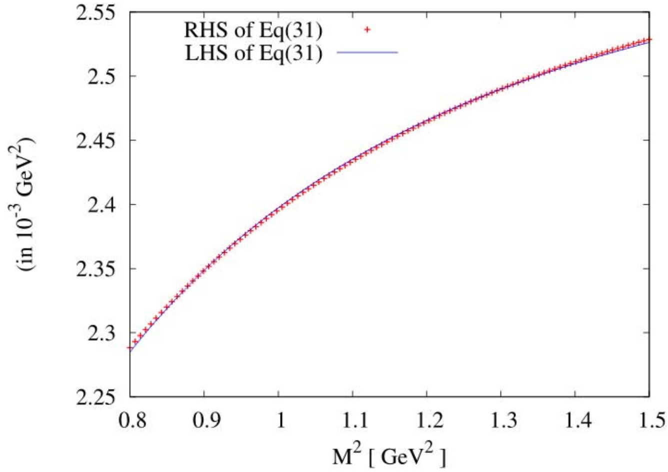

Fig. 2. Estimate of η' mass and coupling in the chiral limit, see Eq. (31). The continuous curve corresponds to $m_\chi = 723$ MeV.

which itself is much smaller than the perturbative term. In the chiral limit $\langle 0|Q|\pi\rangle = \langle 0|Q|\eta\rangle = 0$. If we assume that the quark mass dependence of $\chi'(0)$ is negligible then $\chi'(0)$ in Eq. (20) can also be expressed in term of f_{η_χ} and m_χ as

$$\chi'(0) = \frac{1}{12} f_{\eta_\chi}^2 \left(1 + \frac{m_\chi^2}{M^2}\right) e^{-m_\chi^2/M^2} - \hat{B}\left[\frac{\chi'_{\mathrm{OPE}}(q^2)}{q^2}\right]. \tag{30}$$

From Eqs. (20) and (30) we may then write

$$\begin{aligned} \frac{1}{12} f_{\eta_\chi}^2 \left(1 + \frac{m_\chi^2}{M^2}\right) e^{-m_\chi^2/M^2} \approx{}& \frac{1}{24} f_\pi^2 \left(1 + \frac{m_\pi^2}{M^2}\right) e^{-m_\pi^2/M^2} \\ &+ \frac{1}{24}\left(f_8 \cos\theta_8 - \sqrt{2} f_0 \sin\theta_0\right)^2 \left(1 + \frac{m_\eta^2}{M^2}\right) e^{-m_\eta^2/M^2} \\ &+ \frac{1}{24}\left(f_8 \sin\theta_8 + \sqrt{2} f_0 \cos\theta_0\right)^2 \left(1 + \frac{m_{\eta'}^2}{M^2}\right) e^{-m_{\eta'}^2/M^2}. \end{aligned} \tag{31}$$

We can find f_{η_χ} and m_χ from Eq. (31) using the least "chi-squared" criterion in the range $0.8\ \mathrm{GeV}^2 < M^2 < 1.5\ \mathrm{GeV}^2$. We find $m_\chi \approx 723$ MeV and corresponding $f_{\eta_\chi} = 178$ MeV. In Fig. 2 we have plotted the l.h.s. and r.h.s. of Eq. (31) as a function of M^2 for best-fit values of m_χ and f_{η_χ}. The decay constant f_{η_χ} is of the same order as physical decay constants f_8 and f_0. We would like to stress that Eq. (31) is robust in that even if the coefficients of $\chi'_{\mathrm{OPE}}(q^2)$ change and direct instantons are included in Eq. (20) and (30), Eq. (31) remains unchanged and we are using experimental numbers for decay constants, f_0 and f_8, and mixing angles, θ_0 and θ_8.

We now compare our result for $\chi'(0)$ with some earlier results. In Ref. [1], Narison et al. obtained a value for $\chi'(0) \approx 0.7 \times 10^{-3}\ \mathrm{GeV}^2$, substantially different from the value derived here. Since the expression for χ_{OPE} used by us is identical to theirs, albeit the estimate used for the gluon condensates are slightly different, we need to explain the difference in the end result for $\chi'(0)$. The most important difference is in the expression of $\chi(q^2)$ in terms of physical intermediate states. We have seen that both η and η' contribute, and in fact η makes a larger contribution than η'. In Ref. [1] only $\eta'(958)$ state is taken into account. We have also seen that if we were to take the chiral limit then η and η' contribution to $\chi(q^2)$ is representable by η_χ with mass $m_\chi = 723$ MeV, which is substantially different from the physical η' mass. This also explains why Narison et al. find stability in the sum rule only for rather larger W^2 ($= 6\ \mathrm{GeV}^2$) instead of our $W^2 = 2.3\ \mathrm{GeV}^2$. We must also add that while our Eq. (6) involves only $[\chi'(q^2)/q^2]$, Narison et al. use the linear combination of two sum rules (cf. Eq. (6.22) of Ref. [1]). Comparing with Ref. [5], we note the following: the radiative corrections to the perturbative loop given in Eq. (20), viz., $\frac{\alpha_s}{\pi}\frac{74}{4}$, which is large, is ignored in Ref. [5]. We also note that the coefficient of the $\langle\frac{\alpha_s}{\pi}G^2\rangle$ in Eq. (20) arises from radiative corrections, which is also ignored in Ref. [5]. As already remarked, they use the physical η' mass even when $m_s = 0$, the chiral limit. Since in the sum rules squares of the masses $\exp[-(723)^2/M^2]$ as against $\exp[-(958)^2/M^2]$ occur, this is a serious error both in Ref. [5] and [1]. Even disregarding all the above drawbacks, the sum rule in Ref. [5] for $\tilde{f}_{\eta'}^2$ works rather poorly. It is easy to read off from Fig. 1 of Ref. [5] that $\tilde{f}_{\eta'}^2 = 12\chi'(0)$ varies from $0.019\ \mathrm{GeV}^2$ at $M^2 = 1.5\ \mathrm{GeV}^2$ to $0.034\ \mathrm{GeV}^2$ at $M^2 = 1.1\ \mathrm{GeV}^2$, and grows even faster at lower M^2, hardly a constant. This is to be contrasted with $\chi'(0)$ as computed here and shown in our Fig. 1, where it changes barely by 2% within the same range of M^2.

In Ref. [2], Ioffe and Khodzhamiryan's claim that the OPE for $\chi(q^2)$ does not converge is based on the following. They computed the correlators

$$iq_\mu q_\nu \int d^4x\, e^{iq\cdot x} \langle 0|T\left\{J^0_{\mu 5}(x), J^q_{\nu 5}(0)\right\}|0\rangle,$$

where $J^q_{\mu 5} = \bar{q}\gamma_\mu\gamma_5 q$ $(q = u, d, s)$ with $m_u = m_d = 0$ but $m_s \neq 0$ and $J^0_{\mu 5}$ is the flavor singlet axial current. Introducing the definition

$$\langle 0|J^q_{\mu 5}(x)|\eta'(p)\rangle = ip_\mu g^q_{\eta'}$$

they estimated

$$g^s_{\eta'}/g^u_{\eta'} \approx 2.5. \tag{32}$$

If $SU(3)$ symmetry were to be exact, this ratio would be unity. Insisting that the ratio in Eq. (32) should be close to unity even when $m_s \neq 0$, they concluded that their result signals a breakdown of OPE [2]. As discussed earlier, $\langle 0|J^8_{\mu 5}|\eta'\rangle \neq 0$. In fact, using the phenomenological values given in Eqs. (12) and (13), it is easy to obtain

$$\frac{g^s_{\eta'}}{g^u_{\eta'}} = \frac{\sqrt{2}(f_0 \cos_0 - \sqrt{2} f_8 \sin\theta_8)}{f_8 \sin_8 + \sqrt{2} f_0 \cos\theta_0} \approx 2.24, \tag{33}$$

which is close enough to the estimate of Ref. [2]. In Ref. [5] θ_8 was estimated to be -18.8° assuming $f_8/f_0 = 1.12$ and $\theta_0 = -2.7^\circ$ using QCD sum rules. With these values one will still find that the ratio $g^s_{\eta'}/g^u_{\eta'} = 1.96$, far different from unity as may be naively expected. As in the case of Narison et al. [1], Ioffe and Samsonov [5] and Forkel [13] also do not take into account the π, η matrix element of the anomaly in their sum rules involving $\chi(q^2)$. We also note that $\chi'(0)$ was estimated in Refs. [3,4] to be $\chi'(0) = (2.3 \pm 0.6) \times 10^{-3}$ by fitting the QCD sum rule for singlet axial vector matrix element of the proton. We must add, $\chi'(0)$ coincides with the longitudinal part of the $SU(3)$ singlet axial vector current correlator only in the limit of zero strange quark mass.

In conclusion, we find a value of $\chi'(0) \approx 1.82 \times 10^{-3}$ GeV^2 without incorporating direct instantons. Screening corrections to the latter appears to be significant. We also obtain an estimate $m_\chi = 723$ MeV and $f_{\eta_\chi} = 178$ MeV.

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