The optimal portfolio size of venture capital under staged financing

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Abstract

Traditional theory thinks that portfolio is not a fundamental characteristic of venture capital (VC), the literatures research on venture enterprise portfolio is relatively rare at home and abroad. Venture capital tends to hold a certain number of start-up firms to form a portfolio and at the same time to provide funds and value-added services for more than one start-up firm. Under the scarcity of resources such as attention, venture capitalists should consider how to determine the optimal portfolio size of start-up firms in venture capital finance. The previous studies generally neglected the characteristic that staged financing is the common method used by venture capitalists in most cases. Under the staged financing mechanism, based on double-size moral hazard, using optimization theory we get the expression about the optimal portfolio size of venture capital, and then we find that the optimal portfolio size decreases in start-up capital and following capital, and increases in earlier stage output and total output.

Key words: venture capital; start-up firms; staged financing; portfolio size

1. Introduction

Venture capital (VC) is characterized as providing start-up firms which have high potential growth and entrepreneurial talent with finance and business skills to exploit market opportunities. It is a specific type of financial intermediaries that provide funds, but also expertise to innovative projects. Traditional theory thinks that portfolio is not a fundamental characteristic of venture capital, however, portfolio investment is superior to single investment in performance [1], and the suitable portfolio size helps investment diversification effectively [2]. Venture capital investment portfolio theory should be unique and be different from the traditional portfolio theory on account of its characteristics compared with the securities investment and the investment of financial derivatives and the industrial investment. This is worthy of our in-depth study of the subject.

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Kanniainen and Keuschnigg(2003, 2004)[3-4] first put forward the theory about venture capital portfolio, they pointed out that the existence of the optimal portfolio size is because: venture capitalists invest time and effort to manage and help the start-up firms to realize innovation, then the resulting increase in the number of income portfolio firms also accordingly "diluted" the input to each enterprise's help and the value-added service quality meanwhile, which will reduce the probability of success of each project and the entrepreneur(EN) incentive. This will force it to give entrepreneurs more shareholding ratio to maintain sufficient incentive, thus there must exist trade-off between value-added service strength and the number of portfolio firms. Based on double-size moral hazard, Bernile, Cumming and Lyandres(2007)[5] set up a model of the optimal portfolio size of venture capital based on double-size moral hazard and discussed the influence mechanism of some factors on the optimal portfolio size. Fulghieri and Sevilir(2009)[6] make a portfolio size selecting model based on the focus of investment. They pointed out that the more concentrated investment in the field of investment, the more conducive to the VC to play professional knowledge to provide better value-added services for the investment business to get better benefits. They find that the scarcity of venture capital will make venture capital institutions have a higher bargaining power. The model illustrates the impact of bargaining power and professional knowledge on the portfolio size of venture capital. But all of above studies have not considered the case of staged financing.

Cumming(2006)[7]indicated that staging finance is an important factor affecting the size of venture capital portfolio. In this paper, we will introduce the staged financing mechanism, and analyse the decision of the optimal portfolio size of venture capital under staged financing.

2. The model

We assumed that all agents are risk-neutral, and that entrepreneurs start one firm each, have no own funds, and are commercially inexperienced. Consequently, they need not only finance, but also managerial advice [8-9]. VC finances and advises a portfolio of \( i=1,...,n \) start-up firms. Generally speaking, venture capitalists are mainly to give capital and management support, and technological innovation is mainly dependent on entrepreneurs [10-11]. Neither the effort of entrepreneurs nor the extent of VC service is verifiable and contractible. Hence, in the different stages of the growth of the start-up firms, the role of venture capitalists and entrepreneurs is different [12]. We roughly divided the development phase into two stages, respectively called technological innovation stage (stage 1) and management innovation stage (stage 2). The phase is shown below in table 1:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Deal</th>
<th>EN Efforts</th>
<th>Results</th>
<th>VC Efforts</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( I_{t,j}, s_{t,j} )</td>
<td>( e_i, cEN )</td>
<td>( R_{t,j} )</td>
<td>( I_{t,j}, s_{t,j} )</td>
<td>( a_i, A, cVC )</td>
</tr>
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</table>

Stage 1: In the first stage, the main activity of the start-up firm is technological innovation, and the main criterion of success or failure is the success or failure of technological innovation. In the stage, VC only provides to EN for start-up capital, to support entrepreneurial enterprises to carry out technological innovation activities.

\( t=0 \): Venture capitalists choose to the enterprise portfolio \( \{EN_1, EN_2, ..., EN_n\} \), determine the size of the portfolio.
t=1: VC and EN_i sign the first contract, VC need to give EN_i start-up capital $I_{1,i}$ in exchange for equity share $s_{1,i}$. During this period, the technological innovation is mainly dependent on the entrepreneur, so we suppose EN has the full bargaining power for simplicity [13-14].

$t=2$: EN_i choose his effort level $e_i$ based on the contract in last time. Refer to the assumption proposed by Kanniainen and Keuschnigg(2004)[4], we suppose that the expression of his effort cost is: $c_{EN_i}(e_i) = \alpha \frac{e_i^{1+\delta}}{1+\delta}$, where $\alpha, \delta > 0$.

$t=3$: VC and EN_i realize the technological innovation with probability $p_{1,i}(e_i) = \bar{p} \frac{e_i^{1-\theta}}{1-\theta}$, where $0 < \theta < 1$. They will get $R_{1,i}$ if succeed or 0 if fail.

Stage 2: The capitalist decide whether or not to give following capital and management support by judging the technological innovation effect in the first stage. If the technological innovation success, then venture capitalists will continue to invest. In the second stage, the start-up firm’s activities will focus on management innovation, market development and so on. The smooth development of these activities is the advantage of venture capitalists, the venture capitalist's efforts to determine the final output of the project.

$t=4$: VC and EN_i sign the second contract, VC need to give EN_i following capital $I_{2,i}$ in exchange for equity share $s_{2,i}$. During this period, the management innovation is mainly dependent on the capitalist, so we suppose VC has the full bargaining power for simplicity [13-14].

$t=5$: VC choose the effort level $a_i$ on the project, the total effort $\sigma = \sigma_i + \sigma_2$, Refer to Keuschnigg (2004) [15], we still suppose that: $p_{2,i}(a_i) = \beta \frac{a_i^{1-\rho}}{1+\rho}$, where $0 < \rho < 1$.

They will get the total output $R_{2,i}$ if succeed or 0 if fail. We suppose that $R_{2,i} \gg R_{1,i}$, which depict an important feature of venture capital: If venture capitalists do not give the necessary support, then the start-up firms will be loosen in the management of innovation, market development and other activities, and cannot be further developed.

For simplicity, we treat all start-up firms as identical. This assumption has the following realistic basis: ex ante information asymmetry makes VC cannot predict the operating conditions of EN, only can regard all enterprises as identical, and then put the same capital and management support in each project. Based on the homogeneity assumption, we get: $s_1 \equiv s_{1,i}; s_2 \equiv s_{2,i}; p_1 \equiv p_{1,i}; p_2 \equiv p_{2,i}; R_1 \equiv R_{1,i}; R_2 \equiv R_{2,i}; I_1 \equiv I_{1,i}; I_2 \equiv I_{2,i}; e \equiv e_i; a \equiv a_i; c_{VC}(A) = \beta \frac{(na)^{1+\varepsilon}}{1+\varepsilon}$.

VC and EN_i respectively maximize the expected values of their respective shares of the projects net of effort costs:

\[
\pi_{VC} = \sum_{i=1}^{n} \left\{ p_{1,i} \left[ (s_{1,i}R_{1,i} + s_{2,i}(R_{2,i} - R_{1,i}) - I_{2,i} - I_{1,i} \right] - c_{VC} \right\} = np_1s_1R_1 + np_2s_2(R_2 - R_1) - np_1I_2 - nI_1 - c_{VC} \tag{1}
\]

\[
\pi_{EN_i} = p_{1,i} \left[ (1-s_{1,i})R_{1,i} + (1-s_{2,i})p_{2,i}(R_{2,i} - R_{1,i}) \right] - c_{EN_i} = p_1(1-s_1)R_1 + p_1p_2(1-s_2)(R_2 - R_1) - c_{EN_i} \tag{2}
\]

Our analysis importantly draws on Kanniainen and Keuschnigg(2003)[3]and Bernile, Cumming and Lyandres(2007)[5]. We will solve the problem by backward induction.
3. Optimal portfolio size of venture capital

3.1. VC’s optimal effort level at t=5

In this section we analyse the optimal effort of VC. Substituted the related expression into (1), the partial derivative of (1) on \( a \) is:

\[
\frac{\partial \pi_{VC}}{\partial a} = np_1 s_2 (R_2 - R_1) \frac{\partial p_2}{\partial a} = \frac{\partial c_{VC}}{\partial a}
\]  

Make (3) equal to 0, we get the expression about VC’s optimal effort level:

\[
a^{*} = \left[ \frac{p_2 p_1 s_2 (R_2 - R_1)}{\beta n^e} \right]^{\frac{1}{\alpha \tau \beta}}
\]  

3.2. Optimal profits share contract at t=4

Substituted the related expression into (1), using envelope theorem, after a series of calculations we get the partial derivative of (1) on \( s_2 \) is:

\[
\frac{\partial \pi_{VC}}{\partial s_2} = np_1 p_2 (R_2 - R_1) > 0
\]  

From (5) we know that \( \pi_{VC} \) increases with \( s_2 \). As we mentioned above, VC has the full bargaining power during this period, therefore, the optimal profits share \( s_2^{*} = 1 \). This shows that, for the value increased in the second stage, the share of VC obtained is 1, while the share of the entrepreneur to get 0. To be noted that the result depends on our bargaining power assumption, this does not affect our conclusions because the purpose of this paper is to discuss the optimal portfolio size of venture capital.

3.3. EN’s optimal effort level at t=2

Substituted the related result at t=5 and t=4 into (2), we get:

\[
\pi_{EN_1} = p_1 (1 - s_1) R_1 - c_{EN_1}
\]  

The partial derivative of (6) on \( e \) is:

\[
\frac{\partial \pi_{EN_1}}{\partial e} = (1 - s_1) R_1 \frac{\partial p_1}{\partial e} - \frac{\partial c_{EN_1}}{\partial e}
\]  

Make (7) equal to 0, we get the expression about EN’s optimal effort level:

\[
e^{*} = \left[ \frac{p_1}{\alpha} (1 - s_1) R_1 \right]^{\frac{1}{1 + \beta}}
\]  

3.4. Optimal profits share contract at t=1

From (6) and (8), using envelope theorem, after a series of calculations we can easily get:
\[
\frac{\partial \pi_{EN}}{\partial s_1} = \left[ -p_1 + (1 - s_1) \frac{\partial p_1}{\partial e^*} \right] R_1 - \frac{\partial c_{EN}}{\partial e^*} \frac{\partial e^*}{\partial s_1}
\]

\[
= -p_1 R_1 < 0 \tag{9}
\]

From (9) we know that \( \pi_{EN} \) decreases with \( e^* \). As we mentioned above, EN has the full bargaining power during this period, therefore, the optimal profits share \( s_1^* = 0 \). This shows that, for the value realized in the first stage, the share of EN obtained is 1, while the share of the VC to be 0.

### 3.5. Optimal portfolio size at \( t=0 \)

Substituted \( s_1^* = 0 \) and \( s_2^* = 1 \) into (1), combined with (3), (4), we obtain:

\[
\frac{d \pi_{VC}}{dn} = (p_2 + n \frac{\partial p_2}{\partial e^*} \frac{\partial e^*}{\partial n}) p_1 (R_2 - R_1) - p_1 I_2 - I_1 - \frac{\partial c_{VC}}{\partial e^*} \frac{\partial e^*}{\partial n}
\]

\[
= p_1 p_2 (R_2 - R_1) - p_1 I_2 - I_1 \tag{10}
\]

\[
\frac{d^2 \pi_{VC}}{dn^2} = -\frac{\varepsilon(1 - \rho)}{\varepsilon + \rho} \frac{p_1 p_2 (R_2 - R_1)}{n} < 0 \tag{11}
\]

From (11) we see \( \pi_{VC} \) is a concave function on parameter \( n \), which means there exist only \( n^* \) satisfied \( \frac{d \pi_{VC}}{dn} = 0 \). In other words, a unique optimal number of portfolio start-up firms exist. Substituted the expression of \( p_2 \) into (10), and make (10) equal to 0, we obtain:

\[
n^* = \left( \frac{[p_1 p_2 (R_2 - R_1)]^{1+\varepsilon}}{\beta [(1 - \rho)(p_1 I_2 + I_1)]^{1+\varepsilon}} \right)^{\frac{1}{\varepsilon}} \tag{12}
\]

Here we get the optimal portfolio size of venture capital under staged financing. Using (12), giving the parameter value, the venture capital institutions can calculate their optimal number of portfolio start-up firms specifically.

### 4. Comparative static analysis

Consider the relationship between parameter \( \beta \) and \( n^* \), \( n^* \) decreases with \( \beta \) obviously. It means that if the effort costly the venture capital will invest a small number of start-up firms. The result is consistent with the viewpoint of Kanniainen and Keuschnigg(2003)[3]and Bernile, Cumming and Lyandres(2007)[5]. We will analyse how \( n^* \) effected by \( I_1, I_2, R_1, R_2 \) particularly. Now we give the two propositions as below:

**Proposition 1**: The optimal portfolio size of venture capital under staged financing \( n^* \) decreases in start-up capital \( I_1 \) and following capital \( I_2 \).

**Proposition 2**: The optimal portfolio size of venture capital under staged financing \( n^* \) increases in earlier stage output \( R_1 \) and total output \( R_2 \).

Proofs see Appendix.

Intuitively, the optimal portfolio size is positively related to the profitability of each venture. Thus, a higher value of a successful project, and lower required initial investment and following investment result in a higher value of each venture and in a larger optimal portfolio size. The crucial assumption in Bernile, Cumming and Lyandres(2007)[5], however, is that the profit sharing rule is determined exogenously. In our paper, we assume
that VC and EN have full bargaining power in the different stages, so the profit sharing rule is certain. The assumption does not change the result about the analysis.

5. Conclusions

This paper develops a theoretical model of venture capital portfolio by introducing the staged financing mechanism. Venture capitalists have to consider whether the benefits brought by portfolio can offset the cost of building a portfolio under the scarcity of resources. We obtain the specific expression of the optimal portfolio size of venture capital under staged financing based on double-size moral hazard and make a comparative static analysis, and then we find that the portfolio enterprise number is decreasing in initial capital and following capital, and is increasing in earlier stage output and total output, which to be consistent with the empirical results in [16-17].

The theory and application of the traditional investment portfolio has been mature, however, the study of investment portfolio theory of venture capital has just begun, and has not yet formed a mature theoretical system. In this paper, the staged financing mechanism has been introduced which provides a method for venture capital institutions to make a scientific investment portfolio decision. We must see that our analysis having some limitation. For example, we divide the investment phase into two stages roughly. In reality, VC may invest to a project more than two times. Generalized the model to more than two stages is necessary, which is our further research direction.

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References

Appendix A.

A.1. Some useful expressions

\[
\frac{\partial c_{EN,i}(e_i)}{\partial e_i} = \frac{1 + \delta}{e_i} c_{EN,i}(e_i)
\]

\[
\frac{\partial c_{VC}(A)}{\partial a} = \frac{1 + \epsilon}{a_i} c_{VC}(A)
\]

\[
\frac{\partial p_{1,i}(e_i)}{\partial e_i} = \frac{1 - \theta}{e_i} p_{1,i}(e_i)
\]

\[
\frac{\partial p_{2,i}(a_i)}{\partial a_i} = \frac{1 - \rho}{a_i} p_{2,i}(a_i)
\]

\[
\frac{\partial e^*}{\partial s_1} = -\frac{1}{\theta + \delta} \frac{e^*}{1 - s_1} \frac{\partial e^*}{\partial R_1} = \frac{1}{\theta + \delta} \frac{e^*}{R_1}
\]

\[
\frac{\partial a^*}{\partial s_2} = \frac{1}{\epsilon + \rho} \frac{a^*}{s_2} \frac{\partial a^*}{\partial n} = -\frac{\epsilon}{\epsilon + \rho} \frac{a^*}{n}
\]

A.2. Proof of Proposition 1

\[
\frac{d\pi_{VC}}{dn} = p_1 p_2 (R_2 - R_1) - p_1 l_2 - l_1 = 0
\]

(A.1)

The partial derivative of (A.1) on \(l_1\) is:

\[ p_1 (R_2 - R_1) \frac{\partial p_2 \partial a^* \partial n}{\partial a^* \partial n \partial l_1} = 1 \]

Substituted the related expressions, so we get:

\[ \frac{\partial n}{\partial l_1} = -\frac{\epsilon + \rho}{\epsilon (1 - \rho)} \frac{n}{p_1 p_2 (R_2 - R_1)} < 0 \]

Similarly, the partial derivative on \(l_2\) is:

\[ p_1 (R_2 - R_1) \frac{\partial p_2 \partial a^* \partial n}{\partial a^* \partial n \partial l_2} = p_1 \]

Substituted the related expressions, so we get:

\[ \frac{\partial n}{\partial l_2} = -\frac{\epsilon + \rho}{\epsilon (1 - \rho)} \frac{n}{p_2 (R_2 - R_1)} < 0 \]
A.3. Proof of Proposition 2

\[ \frac{\partial p_1}{\partial R_1} = \frac{\partial p_1 \partial e^*}{\partial e^* \partial R_1} = \frac{1 - \theta p_1}{\theta + \delta R_1} \]  

(A.2)

Substituted (A.1) into (4), we get:

\[ (a^*)^{1+\varepsilon} = \frac{1 - \rho}{\beta} (p_1 l_2 + l_1)(n^*)^{-\varepsilon} \]

The partial derivative of above formula on \( R_1 \) is:

\[ \frac{\partial a^*}{\partial R_1} = \frac{a^* \left[ 1 - \theta \frac{1}{\theta + \delta p_1 p_2 (R_2 - R_1)} - \varepsilon \frac{\partial n^*}{n^* \partial R_1} \right]}{1 + \varepsilon} \]

So:

\[ \frac{\partial p_2}{\partial R_1} = \frac{\partial p_2}{\partial a^*} \frac{\partial a^*}{\partial R_1} = \frac{1 - \rho}{1 + \varepsilon} \ [\frac{1 - \theta}{\theta + \delta p_1 p_2 (R_2 - R_1)} - \varepsilon \frac{\partial n^*}{n^* \partial R_1}] \]

The partial derivative of (A.1) on \( R_1 \) is:

\[ p_1(R_2 - R_1) \frac{\partial p_2}{\partial R_1} + p_2(R_2 - R_1) \frac{\partial p_1}{\partial R_1} - l_2 \frac{\partial p_1}{\partial R_1} - p_1 p_2 = 0 \]  

(A.4)

Substituted (A.2) and (A.3) into (A.4), we obtain:

\[ \frac{\partial n^*}{\partial R_1} = \frac{1 - \theta}{R_1 (\theta + \delta)} (l_1 + \frac{1 - \rho}{1 + \varepsilon} p_1) \]

Similarly, notice that \( \frac{\partial p_1}{\partial R_2} = 0 \), we can get:

\[ \frac{\partial a^*}{\partial R_2} = -\frac{a^*}{1 + \varepsilon n^*} \frac{\partial n^*}{\partial R_2} \]

So:

\[ \frac{\partial p_2}{\partial R_2} = \frac{\partial p_2}{\partial a^*} \frac{\partial a^*}{\partial R_2} = -\frac{\varepsilon (1 - \rho) p_2 \partial n^*}{1 + \varepsilon n^* \partial R_2} \]  

(A.5)

The partial derivative of (A.1) on \( R_2 \) is:

\[ p_1(R_2 - R_1) \frac{\partial p_2}{\partial R_2} + p_2(R_2 - R_1) \frac{\partial p_1}{\partial R_2} - l_2 \frac{\partial p_1}{\partial R_2} + p_1 p_2 = 0 \]  

(A.6)

Substituted (A.5) and \( \frac{\partial p_1}{\partial R_2} = 0 \) into (A.6), we obtain:
\[
\frac{\partial n^*}{\partial R_2} = \frac{1 + \epsilon}{\epsilon (1 - \rho)} \frac{n^*}{R_2 - R_1} > 0
\]