Investigation on Spanwise Wall Oscillation in Turbulent Channel Flow Based on Reynolds Stress Model (RSM)

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Abstract

A variety of studies have showed that wall oscillation contributes much to skin-friction drag reduction, however, the mechanism of drag reduction has not been fully understood. In this paper, one of the common wall-bounded turbulent flows to simulate is the fully developed turbulent flow in a plane channel. Based on the RSM, the numerical simulation results of the distribution among different components of Reynolds stresses are obtained under the action of oscillation or not, respectively. It is found that wall oscillation suppresses turbulent intensity and leads to skin-friction drag reduction, because of the significant role played by the pressure-strain term.

Keywords: Reynolds stress model (RSM); wall oscillation; turbulent channel flow; Reynolds stress; turbulent drag reduction

Introduction

Recently in order to develop drag reduction on aerocrafts, the active flow control methods have been a focus of attention because of their widely applied in future. Wall oscillation becomes an effective active flow control methods. Compared to common turbulence models, Reynolds-Stress-Model (RSM) shows much outstanding advantages in varieties of complex inhomogeneous flows, especially in intense rotation flow, and so forth. Moreover, RSM model has much less limits to computational resource than the direct numerical simulation (DNS) and Lager eddy simulation (LES). Therefore the RSM model has huge potential in future engineering applications[1].

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An attempt was made to study the distribution of Reynolds-stresses in channel flow in this paper, which is under the action of wall oscillation by the numerical simulation based on the developed RSM model. The intrinsic factors of the skin-friction drag reduction in turbulence was analyzed through the relation between the distribution of Reynolds-stresses and the corresponding effect of skin-friction drag reduction, as well as the contribution from the balance of the transport terms to the Reynolds-stresses.

1. Governing Equations of the RSM Model

Fully developed turbulent channel flow was numerically simulated by the RSM model in this paper. The computational domain and coordinates reference frame, the direction x, y, z denote the streamwise, spanwise, and normal to the wall, directions respectively. And the \((U, V, W)\) corresponds to the average velocity component in three directions above. For incompressible turbulent flow, all transport equations are non-dimensionalized with \(u_T\) (the wall-shear velocity, \(u_T = \sqrt{\tau_w/\rho}\)), and the channel half-width \(h\). Thus the simulation flow fields depend only on the friction Reynolds number \(Re_T = u_T h/\nu\) based on the wall-shear velocity \(u_T\) and the channel half-width \(h\).

1.1. Reynolds-Stress Transport Equations

In Reynolds averaging, the instantaneous flow variables are decomposed into an average quantity and fluctuation, where capital letters \(U_i\) denote averaged quantities, and small letters \(u_i\) denote purely fluctuating quantities. Considering the periodicity in streamwise and spanwise directions and the no-slip wall condition, it can deduce that only four Reynolds stress components are not zero, such as \(u u, v v, w w\) and \(u v\). But the average velocity component is non-zero only in streamwise. As is well known, all variations of turbulent variables only depend on normal direction \(y\). The exact Reynolds-stress transport equations for incompressible flow can be written as

\[
\frac{\partial}{\partial t} u_i u_j = \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} - \frac{2}{3} \frac{\partial U_k}{\partial x_k} \right) + \frac{\partial}{\partial x_j} \left[ \frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i} - \frac{2}{3} \frac{\partial U_j}{\partial x_k} \right] - \frac{\partial}{\partial x_k} \left[ \frac{\partial U_k}{\partial x_j} + \frac{\partial U_j}{\partial x_k} - \frac{2}{3} \frac{\partial U_i}{\partial x_k} \right] - \frac{\partial^2 U_i}{\partial x_k \partial x_k} + \frac{1}{Re} \frac{\partial^2 U_i}{\partial x_k \partial x_k}
\]

Which symbolically can be written as

\[
C_4 = P + \phi_4 + D^{(p+\omega)}(\nu_T) - \epsilon_4
\]

On the right hand side above are the production term, pressure-strain term, diffusion term, and dissipation term, respectively. As is seen from all terms, except for the production term, remaining terms require modeling.

Recently varieties of closure forms have been proposed, but the main difference is in modeling pressure-strain term, such as the Launder–Reece–Rodi isotropization of production models (LRR-IP), the Hanjalic–Jakirlic low-Re model (LowRe), and so forth[2]. The modeling in pressure-strain term in this paper selected a modified model, which handled Low-Re and near-wall effects little, to simulate the Reynolds stress transportation.

1.1.1. Diffusion term

Using the generalised gradient diffusion hypothesis (GGDH) of Daly & Harlow (1970), Launder (1990) proposed the model[2].

\[
D_q = \frac{\partial}{\partial x_i} \left[ (C_s k u_i u_i + \frac{1}{Re} \frac{\partial u_i u_j}{\partial x_k}) C_s \right] C_s = 0.22
\]
1.1.2. Pressure strain term

From continuity this term is traceless, so it does not contribute directly to the kinetic energy of the turbulence. Its effect is to redistribute the energy between the stress components. It is difficult in modeling the pressure strain term, often composed of rapid pressure term, slow pressure term, and wall effect term. Launder (1996) suggested the model[3-4].

\[ \phi_{ij} = p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \phi_{ij} + \phi_{ij}' + \phi_{ij}'' \]  

(D 3)

Detailed expressions follow

\[ \phi_{ij}' = -C_e \frac{\epsilon}{k} (u_i u_j - \frac{2}{3} k \delta_{ij}), \quad \phi_{ij}'' = \alpha (P_{ij} - \frac{2}{3} P_{ij} \delta_{ij}) + \eta D_{ij} + \frac{\eta}{5} \phi_{ij} + \gamma k \left( S_{ij} - \frac{2}{3} S_{ij} \delta_{ij} \right) \]

where, \( C_e = 1.5, \alpha = -\frac{C_s}{11}, \eta = -\frac{8 C_s}{11}, \gamma = -\frac{60 C_s}{55}, \phi_{ij} = \frac{1}{2} \phi_{ij}, 0.4 < C_s < 0.6, \quad C_s = 0.4 \)

\[ \phi_{ij}'' = C_4' \left( u_i (u_i u_j - \frac{2}{3} k \delta_{ij}) - \frac{3}{2} u_j u_i n_j - \frac{3}{2} u_i u_j n_i \right) f_w + C_4'' \left( \phi_{ij} - \frac{3}{2} \phi_{ij} - \frac{3}{2} \phi_{ij} - \frac{3}{2} \phi_{ij} \right) f_w \]

where: \( C_4' = 0.3, C_4'' = 0.3, \text{and } f_w = 0.4 k^{3/2} / \epsilon x_a \)

Some other terms were expressed as

\[ P_{ij} = -(u_i u_j - \frac{2}{3} k \delta_{ij}), S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), D_{ij} = -(u_i u_j - \frac{2}{3} k \delta_{ij}) \]

(4)

1.1.3. Dissipation term

Because of asymptotic near-wall stress dissipation rates in turbulent channel flow, Launder and Reynolds (1983) gave the heterogeneous closure [5-6], the form is

\[ \epsilon_{ij} = f_i \frac{\epsilon}{k} u_i u_j + \left( 1 - f_i \right) \frac{2}{3} \epsilon \delta_{ij} \]  

(D 5)

Where the function \( f_i = 0.1/(1 + 0.1 \Re), \Re = k^{2/3} / \epsilon v \)

1.1.4. The dissipation rate of turbulent kinetic energy

Modifications for low Reynolds number and wall effects in standard dissipation equation were made in transport equation of turbulent kinetic energy.

\[ \frac{D \epsilon}{D t} = \left( C_s f_i + \frac{k}{\epsilon} \frac{\epsilon}{k} u_i u_j + \frac{1}{\Re} \right) \frac{\partial \epsilon}{\partial x_i} + C_s \frac{\epsilon}{k} P_{ij} f_i - C_s \frac{\epsilon^2}{k} f_2 \]

(6)

where, \( \Re = \frac{k^2}{\epsilon v}, \gamma' = \frac{u_v}{v}, f_n = \left( \frac{1 + 3.45}{\Re} \right) \frac{1 - \exp(-y' / 35)}{1 + \exp(-y' / 35)}, f_1 = 1, f_2 = \left[ 1 - \exp(-y' / 5.2) \right]^2 \)

2. Numerical Method

Based on the periodicity in spanwise and streamwise direction of channel flow, the governing equations of the RSM model can be simplified to be suitable for plane channel flow. The non-dimensional equations for above RSM model in terms of uniform vector can be expressed as

\[ \frac{\partial \Phi}{\partial t} - \frac{d}{dy} \left( \Gamma \frac{d \Phi}{dy} \right) = S_{\Phi} = \Phi + \epsilon + S_{\text{wind}} \]  

(7)
Where $\Phi$ stands for dependent variable. The transform factor $\Gamma_\Phi$ is given for each $\Phi$ in the diffusion term, and $S_\Phi$ is the source terms at right of the equation, composed of production term $P$, pressure-strain term $\phi$, dissipation term $\varepsilon$, and user-defined source term $S_{user}$.

A half implicit time approach is performed for numerical solution for fully developed channel flow by the finite difference method in this paper [7]. For the diffusion terms, the transform factor $\Gamma_\Phi$ is evaluated at the current time step $n$, while the spatial-derivative is evaluated at the next time $n+1$, the derivatives being replaced by the central difference. The grid is discretized with an equal-spacing in normal direction. The symbols $\Delta y$ and $\Delta t$ are the increments in $y$ and $t$ respectively. The discretized equation thus obtained is

$$\frac{\Phi_{i}^{n+1} - \Phi_{i}^{n}}{\Delta t} - \frac{\Gamma_{\Phi,i}^{n+1} - \Gamma_{\Phi,i-1}^{n}}{2\Delta y} - \frac{\Phi_{i+1}^{n+1} - \Phi_{i+1}^{n+1}}{2\Delta y} - \frac{\Gamma_{\Phi,i}^{n} \left( \Phi_{i+1}^{n+1} - 2\Phi_{i}^{n+1} + \Phi_{i-1}^{n+1} \right)}{\Delta y^2} = S_{\Phi,i}$$

It is significant to obtain exact boundary condition, especially the wall condition for dissipation $\varepsilon$. Reynolds stress components is zero for no-slip condition at the wall, so turbulent kinetic energy $k$ is clear, which is summation of the normal stresses. Recently most researchers suggest a condition that is the balance between the dissipation rate of turbulent kinetic energy and the production rate of turbulent kinetic energy near wall [5], the form is $\varepsilon = 2\nu \left( \frac{\partial \sqrt{k}}{\partial y} \right)^2$.

In the solution procedure, the discretized equations are solved with the selected boundary conditions by using a standard tri-diagonal matrix algorithm. The time step length $\Delta t$ and cell space $h$ were adaptive to characteristic Reynolds number in the computational cases. Grid independence and the first normalized grid space within 3 are ensured, thus the solution procedure decides 200 grid points and integrates from the wall finally.

The developed RSM procedure has been tested by many cases and verified its reliability, but it is difficult to agree with the DNS results except for in trend through all regions. Therefore, given the less computation, it is much suitable to qualitatively analyze the distribution of Reynolds stresses in turbulent channel flow to some degree.

### 3. Control Effect of Spanwise Oscillation

Theoretically, the effects of spanwise oscillation in turbulent channel flow can be seen as an effective internal fluctuation disturber. To put it, the effect of oscillation occurs in boundary condition, and then changes the distribution of Reynolds stress components in turbulent channel flow through the transport equations. Recently it has been showed that spanwise oscillation can change the variation of Reynolds stresses to some extent, and the range of stress also reflects the oscillation intensity.

The distribution of Reynolds stresses in turbulent channel flow was simulated by the developed RSM procedure at the Reynolds number 180 under the effect of spanwise oscillation. The control effects were investigated with different oscillation intensity. The label value in figures shows the value of stress component $ww$ in wall condition. If the label value is zero, there is no oscillation effect in the channel flow. However, if the value is none-zero, there is oscillation effect in the channel flow, and the extent of oscillation effect depends on the quantity of label value.
3.1. The effects for turbulent statistics

Fig. 2. the control effect of $uu$.  
Fig. 3. the control effect of $vv$.  
Fig. 4. the control effect of $ww$.  
Fig. 5. the control effect of turbulent energy $k$.  
Fig. 6. the control effect of $uv$.  
Fig. 7. the control effect of velocity $U$.  

3.2. The effects for transport terms

Fig. 8. the control effect of pressure strain term.  
Fig. 9. the control effect of production.

As we can see from figure 2 to figure 9,  
(1) For normal stress, spanwise oscillation reduces the peak value of stress components $uu$ and $vv$ near wall, and peak position is a little close to the wall. When the oscillation intensity is higher, the decrease in stress components is bigger. Fig 4 shows spanwise oscillation directly affects the component $ww$, and fig 8 and fig 9 show the range of
the pressure strain term is biggest among all turbulent transportation terms, and the production and pressure strain term are main terms for stress transportation.

(2) For the shear stress $uv$ in fig 6, the peak position keep fixed, but the peak value decreases as the oscillation intensity increase. As the component $uv$ decreases, the peak value of velocity profile decreases relatively.

(3) From the figures above, spanwise oscillation affects most in the near wall region, as the normal distance is bigger than 80, the effects become smaller and smaller gradually.

In conclusion, spanwise oscillation directly affects the component $ww$, and then the effects are transported to other stress components $uu$ and $vv$ through the contribution of the pressure strain term. The decrease in $uu$ and $vv$ causes the reduction on the production of $uv$, and the component $uv$ decrease. Finally, the peak value of velocity profile in Fig 7 also decreases relatively. Thus spanwise oscillation can suppress the turbulent intensity, and reduce the peak value in Reynolds stress to a certain extent. Most important, the skin friction reduction decreases effectively.

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**References**