# Fusion of Pedigreed Preferential Relations

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**Abstract**

Belief fusion, instead of AGM belief revision, was first proposed to solve the problem of inconsistency, that arise from repetitive application of the operation when agents' knowledge were amalgamated. However in the theory, all the sources must be totally ordered and thus applicable area is quite restrictive. In this paper, we realize the belief fusion of multiple agents for partially ordered sources. When we consider such a partial ranking over sources, there is no need to restrict that each agent has total preorders over possible worlds. The preferential model allows each agent to have strict partial orders over possible worlds. Especially, such an order is called a preferential relation, that prescribes a world is more plausible than the other. We introduce an operation which combines multiple preferential relations of agents. In addition, we show that our operation can properly include the ordinary belief fusion.

## 1 Introduction

In amalgamation of beliefs in multiple agents, the problem is the complicated order of reliability. Belief *fusion*, instead of AGM belief revision [1,2], was first proposed by Maynard-Reid II and Shoham [6], to ensure the consistency, as follows. Suppose each agent has a total preorder on possible worlds, based on the semantic work (cf. [3,4]). This order can be *refined* with the order of the other agent, as '$\preceq_A \oslash \preceq_B$'; where $\preceq_A$ is the order of possible worlds of Agent $A$, that is more reliable than that of Agent $B$, and the result of '$\oslash$' is the refined order. However, the iterated aggregation $(\preceq_A \oslash \preceq_C) \oslash \preceq_B$ is

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spurious in case $\preceq_C$ is more reliable than both of $\preceq_A$ and $\preceq_B$. In order to solve this problem, they regarded agents as *sources*, between which there is a total credibility ranking, and considered that each source has a *belief state*, a total preorder over worlds.

Let us consider the following piece of detective story.

> "A criminal is said to be one of the four: $P$, $Q$, $R$, and $S$. Two inspectors $A$ and $B$ had information that there are fingerprints of them except for $S$ at the crime scene ($s_1$). Moreover, $A$ knew that an old man uttered that $Q$ remained at the neighborhood of the scene, but he had no opinion about $P$ and $R$ ($s_2$). $B$ heard a story that a child witnessed that $P$ bought the weapon ($s_3$). The investigation headquarter wants to amalgamate all these information, considering the reliability of each source. $s_1$ is more credible than $s_2$ and $s_3$, but $s_2$ is incomparable with $s_3$. Who should the police investigate first?"

The problem is how we put the plausibility order in possible worlds, in each of which the criminal is different. The credibility order in two agents $A$ and $B$ is not worth considering, because the situation is already regarded as the result of amalgamation of three primitive agents, called sources $s_i$ $(i = 1, 2, 3)$. However, we cannot directly apply the belief fusion to this case because sources are only *partially* ordered.

We give another example to show that the *totality* of the ranking of sources is too strong to hypothesize. Suppose that two TV directors have different opinions about a program, and their ranking is the same. Each opinion cannot be regarded as only an individual opinion, because it affects behaviors of assistants, cameramans, performers, and so on. That is, the opposition of two directors changes relations of commitments in their personnel. As we will discuss, we simply consider these various relations as *chains*. So, we will deal with the *partiality* of the ranking of sources.

When we consider such a partial ranking over sources, there is no need to restrict that each agent has *total preorders* over possible worlds. The *preferential model* allows each agent to have *strict partial orders* over possible worlds, known as [8,5,7], and so on. Especially, such an order is called a *preferential relation*, that prescribes a world is more plausible than the other. When we declare an inference relation $\alpha \mid\!\sim \beta$, meaning "If $\alpha$ then naturally $\beta$," for any minimal world which satisfies $\alpha$, it also satisfies $\beta$.

We introduce an operation which combines multiple preferential relations of agents, in the similar way of the fusion of belief state, naming *fusion of preferential relations*. Note again that preferential relations are strict partial orders whereas belief states are total preorders. Naturally, the operation would become more complicated than the belief fusion. However, by the proper translation from belief states to preferential relations, we will show that our framework can simulate the belief fusion.

In this paper, we introduce the refinement operator for preferential relations in Section 2; although the same operator was mentioned rather easily in [6], our definition of the refinement includes various problems, and thus we spare one section for the explanation. In Section 3, we introduce the pedigreed preferential relation and construct the fusion of pedigreed preferential models. In Section 4, we show that our formalism can properly include the ordinary belief fusion. Finally in Section 5, we summarize our contribution and discuss various issues of our formalization.

## 2 Refinement

We assume a language $\mathcal{L}$. A world $w$ gives an interpretation over $\mathcal{L}$. We denote the set of worlds as $\mathcal{W}$. We use $r$ as an arbitrary relation over $\mathcal{W}$, but it usually means an ordering. If $(w_a, w_b)$ satisfies $r$, we denote $w_a r w_b$ or $(w_a, w_b) \in r$, interchangeably. $Tr(r)$ means the transitive closure of $r$. Let us define *anonymous* preferential relations.

**Definition 2.1** An (anonymous) preferential relation $r$ (over $\mathcal{W}$) is a strict partial order over $\mathcal{W}$.

That is to say, a preferential relation is an anti-reflexive and transitive relation over $\mathcal{W}$. When $(w_a, w_b)$ is neither $w_a r w_b$ nor $w_b r w_a$, we denote it by $w_a \asymp w_b$. We also denote the set of preferential relations by $\mathcal{P}$.

We will define the fusion operator which accepts two preferential relations and produces another, but in order for the operator to be meaningful, it will require an additional input which concerns the reliability. At first, we resolve conflicts by declaring that one agent (A) is more credible than the other (B) and A's judgment dominates B's. We consider the following tentative definition of *refinement*.

Suppose $r_A, r_B \in \mathcal{P}$. The refinement of $r_A$ by $r_B$ is
$r_A \hat{\circledcirc} r_B = \{(w_a, w_b) : w_a r_A w_b \vee (w_a \asymp_A w_b \wedge w_a r_B w_b)\}$.

In other words, to construct the fused relation, whenever the more credible agent prefers one world to another, we side with this preference. In case the most credible agent has no preference, we follow the ranking of the less credible agent. However, this definition has problems, because the produced relation may not be a preferential relation. At first, a produced relation may not be transitive. In the left of Figure 1, $w_3 r_A w_2$ and $w_2 r_B w_1$, but $(w_3, w_1) \notin r_A \hat{\circledcirc} r_B$ by the definition. Secondly, even if a produced relation would be transitively closed, the relation might not be anti-reflexive. See the right of Figure 1 where $(w_1, w_1) \in Tr(r_A \hat{\circledcirc} r_B)$ is not anti-reflexive.

Thus, we revise the definition of the refinement so as to satisfy the transitivity and the anti-reflexivity, with a fixed-point equation. We write a finite number of repetitions of the relation $r$ as $r^*$.

**Definition 2.2** Suppose $r_A, r_B \in \mathcal{P}$. A relation over possible worlds, denoted

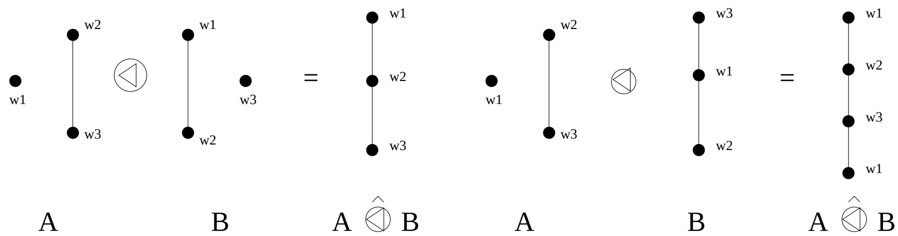


Fig. 1. Example: the produced relation is neither transitive nor anti-reflexive.

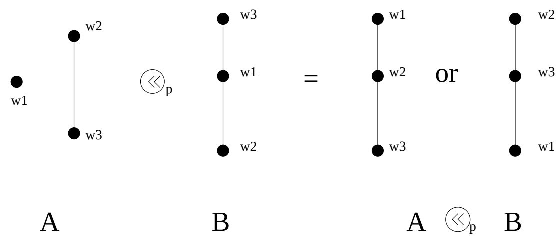


Fig. 2. Example : there are two refinements of $r_A$ by $r_B$.

by $r_A \textcircled{\ll}_p r_B$, is a primitive refinement of $r_A$ by $r_B$ iff

$$r_A \textcircled{\ll}_p r_B = r_A \cup \{(w_a, w_b) : w_a \asymp_A w_b \wedge w_a r_B w_b \\ \wedge \forall w (w[r_A \textcircled{\ll}_p r_B]^* w_b \rightarrow w \neq w_b)\}.$$

**Definition 2.3** Suppose $r_A, r_B \in \mathcal{P}$. A relation over possible worlds, denoted by $r_A \textcircled{\ll}\ r_B$, is a refinement of $r_A$ by $r_B$ iff

$$Tr(r_A \textcircled{\ll}_p r_B) = r_A \textcircled{\ll}\ r_B.$$

Note that there can be multiple refinements of $r_A$ by $r_B$. For example in Figure 2, two relations satisfy the condition of 2.2 and thus there are two refinements of $r_A$ by $r_B$ by 2.3. Therefore, we need a rationale to decide a unique result of fusion. We propose that the result is the common relations in those multiple candidates.

**Definition 2.4** Suppose $r_A, r_B \in \mathcal{P}$ and $RF(r_A, r_B)$ is the set of all of refinements of $r_A$ by $r_B$. The cautious refinement of $r_A$ by $r_B$ is

$$r_A \olessthan\ r_B = \cap RF(r_A, r_B).$$

The cautious refinement is well-defined.

**Proposition 2.5** *If* $r_A, r_B \in \mathcal{P}$*, then* $r_A \olessthan\ r_B \in \mathcal{P}$*.*

**Proof.** It suffices that we show $r_A \textcircled{\ll}\ r_B$ is a strict partial order over $\mathcal{W}$. Transitivity is straightforward. Anti-reflexivity is shown by contradiction. If we suppose there is $w \in \mathcal{W}$ such that $w[r_A \textcircled{\ll}\ r_B]w$, then there exists $w[r_A \textcircled{\ll}_p r_B]^* w$, and it contradicts 2.1. □

Obviously, $r_A \subseteq r_A \olessthan\ r_B$.

## 3 Fusion of Preferential Relations

Because '$\circledtriangleleft$' is not a symmetric operator, we encounter the same problem as we mentioned at the top of Section 1 when we iteratively apply the operator. Consider $r_A$, $r_B$, and $r_C$ with increasing order of dominance ($r_A$, dominated by $r_B$, both by $r_C$). Presumably, the above definition would give meaningful interpretation to $(r_A \circledtriangleleft r_B) \circledtriangleleft r_C$, since all the information in $r_C$ dominates all the information in $r_A \circledtriangleleft r_B$. However in case $(r_A \circledtriangleleft r_C) \circledtriangleleft r_B$, it would seem that some of the information in $r_A \circledtriangleleft r_C$ dominates the information in $r_B$ (because it originated from $r_C$) and some is dominated by it (because it originated from $r_A$).

In the similar way to [6], we introduce *pedigree.* The sources can be thought of as primitive agents with fixed preferential relations, and an agent's *pedigreed preferential relation* is simply the amalgamation of all these opinions, each of which is annotated by its origin or pedigree. In this paper, we assume that the agent places a strict credibility ranking on the sources, but we do not assume that the ranking is total as the previous study.

We will use $\mathfrak{S}$ to denote the finite set of all of sources. Each source has a preferential model from $\mathcal{P}$. To distinguish between agent and source preferential relations, we will use $r_s$ to denote the preferential relation of source $s \in \mathfrak{S}$.

We now define *pedigreed preferential relations.*

**Definition 3.1** Given a set of sources $S \subseteq \mathfrak{S}$ the pedigreed preferential relation induced by $S$ is a function $\Phi : \mathcal{W} \times \mathcal{W} \rightarrow 2^S$ such that $\Phi(w_a, w_b) = \{s \in S : w_a r_s w_b\}$.

We assume a strict partial ranking $\sqsupseteq$ on $\mathfrak{S}$ and thus, a set of sources induces a unique $\Phi$. We interpret $s_1 \sqsupseteq s_2$ as "$s_1$ is at least as credible as $s_2$."

Before we define the ordering induced by $\Phi$, we define the set of maximal chains of $S$.

**Definition 3.2** The set of the maximal chains of $S$ is

$$MC(S) = \{S_{mc} \subseteq S : S_{mc} \text{ is a chain of } S \text{ and}$$
$$\text{for any chain } S_c \text{ of } S, S_{mc} \subseteq S_c \text{ implies } S_c = S_{mc}\}.$$

Our interest in the section is to show that a standard (anonymous) preferential relation is induced by a pedigreed preferential relation. However, the partiality in the order complicates the proof. Therefore, we plan the following strategy; (i) we divide the partial ranking over resources to maximal chains, (ii) construct an order over possible worlds on each maximal chain, and (iii) calculate the intersection of all of the orders over possible worlds. Using this strategy, we deal with an ranking over maximal chains. Because of the partiality of preferential model of each source, the ordering process consists of the following multiple steps.

**Definition 3.3** Let $S \subseteq \mathfrak{S}$, and $S_{mc} = \{s_1, ..., s_N\} \in MC(S)$ such that $s_i \sqsupseteq s_{i+1}$ for all $1 \leq i \leq N$. The ordering induced by $\Phi$ and $S_{mc}$ is

$$r_{\Phi,S_{mc}} = Order^N(\Phi, S_{mc}).$$

At this point,

(i) $Order^n(\Phi, S_{mc}) = \cap GO^n(\Phi, S_{mc})$,

(ii) $GO^n(\Phi, S_{mc})$ is the set of all of $Tr(GenOrder^n(\Phi, S_{mc}))$, and

(iii) $GenOrder^n(\Phi, S_{mc})$

$$= \begin{cases} \text{if } n = 1, \\ \{(w_a, w_b) : s_1 \in \Phi(w_a, w_b)\} \\ \\ \text{otherwise}(n > 1), \\ Order^{n-1}(\Phi, S_{mc}) \cup \\ \{(w_a, w_b) : (w_a, w_b) \notin Order^{n-1}(\Phi, S_{mc}) \wedge \\ \qquad (w_b, w_a) \notin Order^{n-1}(\Phi, S_{mc}) \wedge \\ \qquad s_n \in \Phi(w_a, w_b) \wedge \\ \forall w((w, w_b) \in GenOrder^n(\Phi, S_{mc})^* \rightarrow w \neq w_b)\}. \end{cases}$$

Finally, we define the ordering induced by a pedigreed preferential relation.

**Definition 3.4** The ordering induced by $\Phi$ is

$$r_\Phi = \{(w_a, w_b) : \forall S_{mc} \in MC(S), w_a r_{\Phi,S_{mc}} w_b\}.$$

With the above definitions, we can show $\Phi$ induces a standard (anonymous) preferential relation.

**Proposition 3.5** *The ordering induced by a pedigreed preferential relation is in $\mathcal{P}$.*

**Proof.** Given a pedigreed preferential relation $\Phi$, it suffices that we show that for all $S_{mc} \in MC(S)$, $r_{\Phi,S_{mc}}$ is a strict partial order over $\mathcal{W}$. The proof is similar to Proposition 2.5. □

For the proof of Proposition 4.14, we show the following lemma.

**Lemma 3.6** *Let $S \subseteq \mathfrak{S}$, and $S_{mc} = \{s_1, ..., s_N\} \in MC(S)$ such that $s_i \sqsupseteq s_{i+1}$ for all $1 \leq i < N$. If $r_{\Phi,S_{mc}}$ is the ordering induced by the pedigreed preferential relations induced by $S$, and $S_{mc}$, then*

$$r_{\Phi,S_{mc}} = \begin{cases} r_{s_1} & \text{if } N = 1 \\ ((r_{s_1} \circledtriangleleft r_{s_2}) \circledtriangleleft \dots \, r_{s_N}) & \text{otherwise(i.e., } N > 1) \end{cases}.$$

**Proof.** We show this by induction. If $N = 1$, then $Order^1(\Phi, S_{mc}) = r_{s_1}$ is obvious. Suppose $Order^{n-1}(\Phi, S_{mc}) = ((r_{s_1} \circledcirc r_{s_2}) \circledcirc \ldots\ r_{s_n-1})$. If we want to show $Order^n(\Phi, S_{mc}) = ((r_{s_1} \circledcirc r_{s_2}) \circledcirc \ldots\ r_{s_n})$, then it suffices to show that $r$ is a primitive refinement of $((r_{s_1} \circledcirc r_{s_2}) \circledcirc \ldots\ r_{s_n-1})$ by $r_{s_n}$ iff $r$ is a $GenOrder^n(\Phi, S_{mc})$. That is,

$$
\begin{aligned}
r = {} & ((r_{s_1} \circledcirc r_{s_2}) \circledcirc \ldots\ r_{s_n-1}) \cup \\
& \{(w_a, w_b) : (w_a, w_b) \notin ((r_{s_1} \circledcirc r_{s_2}) \circledcirc \ldots\ r_{s_n-1}) \\
& \quad \wedge (w_b, w_a) \notin ((r_{s_1} \circledcirc r_{s_2}) \circledcirc \ldots\ r_{s_n-1}) \wedge \\
& \quad w_a r_{s_n} w_b \wedge \forall w(w r^* w_b) \to w \neq w_b)\} \\
& (\text{By Definition 2.2.} \\
& r \text{ is a primitive refinement of } ((r_{s_1} \circledcirc r_{s_2}) \circledcirc \ldots\ r_{s_n-1}) \text{ by } r_{s_n}.) \\
= {} & Order^{n-1}(\Phi, S_{mc}) \cup \\
& \{(w_a, w_b) : (w_a, w_b) \notin Order^{n-1}(\Phi, S_{mc}) \wedge (w_b, w_a) \notin Order^{n-1}(\Phi, S_{mc}) \wedge \\
& \quad s_n \in \Phi(w_a, w_b) \wedge \forall w(w r^* w_b) \to w \neq w_b)\}. \\
& (\text{By induction, } Order^{n-1}(\Phi, S_{mc}) = ((r_{s_1} \circledcirc r_{s_2}) \circledcirc \ldots\ r_{s_n-1}). \\
& \text{It follows that } r \text{ is a } GenOrder^n(\Phi, S_{mc}).)
\end{aligned}
$$

□

If we can compute a fused pedigreed preferential relation, we can also compute a fused standard preferential relation with it.

**Definition 3.7** Let $\Phi_1$ and $\Phi_2$ be the pedigreed preferential relation induced by sets of sources $S_1$ and $S_2$, respectively. The fusion of $\Phi_1$ and $\Phi_2$, denoted $\Phi_1 \bigtriangledown \Phi_2$, is the pedigreed preferential relation induced by $S_1 \cup S_2$.

Obviously, the set of pedigreed preferential relations is closed under $\bigtriangledown$. The following property is also immediate.

**Proposition 3.8** *If $\Phi_1$ and $\Phi_2$ be the pedigreed preferential relation induced by sets of sources $S_1$ and $S_2$, respectively, then*

$$(\Phi_1 \bigtriangledown \Phi_2)(w_a, w_b) = \Phi_1(w_a, w_b) \cup \Phi_2(w_a, w_b).$$

**Proof.** Straightforward. □

Obviously, a set of pedigreed preferential relations that is closed under $\bigtriangledown$, and they form a semi-lattice, i.e., $\bigtriangledown$ is idempotent, commutative, and associative. Note that the ordering induced by a pedigreed preferential relation may be empty. For example in Figure 3, let $s_1 \sqsupseteq s_2$, but $s_1 \not\sqsupseteq s_3$, $s_3 \not\sqsupseteq s_1$,

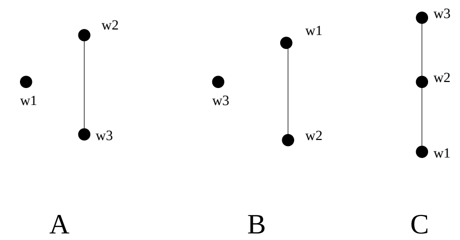


Fig. 3. Example : the ordering induced by the pedigreed preferential relation is empty.

$s_2 \not\trianglelefteq s_3$, and $s_3 \not\trianglelefteq s_2$, where agent A is assigned to $s_1$, B is assigned to $s_2$, and C is assigned to $s_3$. Then the ordering induced by a pedigreed preferential relation induced by the set of sources is empty.

We need to show what kind of pedigreed preferential relation induces the nonempty ordering. We define the maximally-ordered set of sources.

**Definition 3.9** Let $S \subseteq \mathfrak{S}$. $S$ is maximally-ordered iff $S$ has a maximal source $s_m$.

**Proposition 3.10** *If $S$ is maximally-ordered, and the ordering induced by the pedigreed preferential model induced by $S$ is $r_\Phi$, then $r_{s_m} \subseteq r_\Phi$.*

**Proof.** Straightforward. □

Therefore, an ordering which is induced by a pedigreed preferential relation is nonempty, when its maximal source's ordering is nonempty.

## 4 Mapping from Belief Fusion to Fusion of Preferential Relations

In this section, we show the connection between fusion of preferential relations, and the belief fusion.

**Definition 4.1** An (anonymous) belief state $\preceq$ (over $\mathcal{W}$) is a total pre-order over $\mathcal{W}$.

Suppose $\mathcal{B}$ is the set of belief states, $\prec$ is the asymmetric restriction of a belief state $\preceq$, and $\sim$ is the symmetric restriction of it.

**Definition 4.2** Suppose $\preceq_A, \preceq_B \in \mathcal{B}$. The refinement for belief states of $\preceq_A$ by $\preceq_B$ is $\preceq_A \oslash \preceq_B = \{(w_a, w_b) : w_a \preceq_A w_b \vee (w_a \sim_A w_b \wedge w_a \not\prec_B w_b)\}$.

Then the refinement for belief states is a well-defined operation:

**Proposition 4.3** *If $\preceq_A, \preceq_B \in \mathcal{B}$, then $\preceq_A \oslash \preceq_B \in \mathcal{B}$.*

Can we show any relation between our refinement and the refinement for belief states? Before we discuss it, we show a proper translation from a belief state to a preferential relation. The translation is defined as follows:

**Definition 4.4** Suppose $\preceq \in \mathcal{B}$. The translated relation of $\preceq$ is $trans(\preceq) = \{(w_a, w_b) : w_a \prec w_b\}$.

**Proposition 4.5** *If $\preceq \in \mathcal{B}$, then $trans(\preceq) \in \mathcal{P}$.*

**Proof.** Anti-reflexivity is trivial. Transitivity is shown by contradiction. □

It is easy to show that $trans(\preceq)$ is an injection from $\mathcal{B}$ to $\mathcal{P}$, and then, if $\mathcal{T}$ is the set of translated relations of belief states, then $\mathcal{T} \subseteq \mathcal{P}$ and $trans(\preceq)$ is a bijection from $\mathcal{B}$ to $\mathcal{T}$.

We can use the cautious refinement operator for translated relations of belief states as the refinement operator for belief states as follows:

**Proposition 4.6** *Suppose $\preceq_A$ and $\preceq_B \in \mathcal{B}$. Then $trans(\preceq_A \olessthan \preceq_B) = trans(\preceq_A) \olessthan trans(\preceq_B)$.*

**Proof.** Suppose $\preceq = trans(\preceq_A \olessthan \preceq_B)$. It suffices to show that (i) $trans(\preceq) = trans(\preceq_A) \circledll_p trans(\preceq_B)$, and (ii) $trans(\preceq)$ is the only refinement of $trans(\preceq_A)$ by $trans(\preceq_B)$. At first, we use

$$\begin{aligned} trans(\preceq) = trans(\preceq_A) \cup & \\ & \{(w_a, w_b) : (w_a, w_b) \notin trans(\preceq_A) \wedge \\ & \quad (w_b, w_a) \notin trans(\preceq_A) \wedge \\ & \quad (w_a, w_b) \in trans(\preceq_B)\} \end{aligned}$$

by Definition 4.2, and

$$\begin{aligned} & trans(\preceq_A) \circledll_p trans(\preceq_B) \\ & = trans(\preceq_A) \cup \\ & \quad \{(w_a, w_b) : (w_a, w_b) \notin trans(\preceq_A) \wedge \\ & \qquad (w_b, w_a) \notin trans(\preceq_A) \wedge \\ & \qquad (w_a, w_b) \in trans(\preceq_B) \wedge \\ & \quad \forall w((w, w_b) \in [trans(\preceq_A) \circledll trans(\preceq_B)]^* \rightarrow w \neq w_b)\}, \end{aligned}$$

by Definition 2.2, and show $trans(\preceq) \subseteq trans(\preceq_A) \circledll_p trans(\preceq_B)$ and $trans(\preceq) \supseteq trans(\preceq_A) \circledll_p trans(\preceq_B)$. '$\supseteq$' is trivial. '$\subseteq$' is shown by contradiction. Finally, we assume that $r$ is a refinement of $trans(\preceq_A)$ by $trans(\preceq_B)$, and also show $r \subseteq trans(\preceq)$ and $r \supseteq trans(\preceq)$. □

Therefore, $trans$ is isomorphism w.r.t the refinement operator from $\mathcal{B}$ to $\mathcal{T}$.

We can translate not only refinement; a *pedigreed belief state* can also be translated to a pedigreed preferential relation. In the following definition, $\leq_s$ denote the belief state of source $s \in \mathcal{S}$, and $<_s$ denote its asymmetric restriction.

**Definition 4.7** Given a set of sources $S \subseteq \mathfrak{S}$, the pedigreed belief state induced by $S$ is a function $\Psi : \mathcal{W} \times \mathcal{W} \to 2^S$ such that $\Psi(w_a, w_b) = \{s \in S : w_a <_s w_b\}$.

**Definition 4.8** Given a set of sources $S \subseteq \mathfrak{S}$, the pedigreed translated relation of $\Psi$ is a function $\Psi_{trans} : \mathcal{W} \times \mathcal{W} \to 2^S$ such that $\Psi_{trans}(w_a, w_b) = \{s \in S : w_a trans(\leq_s) w_b\}$.

**Proposition 4.9** *Given a set of sources $S \subseteq \mathfrak{S}$ and $w_a, w_b \in \mathcal{W}$,*

$$\Psi_{trans}(w_a, w_b) = \Psi(w_a, w_b)$$

**Proof.** Straightforward. □

Obviously, a pedigreed translated relation is a pedigreed preferential relation. We can also show the connection between the ordering induced by the dominating belief state of a pedigreed belief state and the ordering induced by a pedigreed translated relation; however, in which case, we need to restrict a ranking $\sqsupseteq$ on $\mathfrak{S}$ to be total.

We denote by $s_0$ the "agnostic" source, that is, a source such that $\preceq_{s_0}$ is a complete relation, and we will assume that $\{s_0\}$ is the least credible source.

**Definition 4.10** Suppose that $\sqsupseteq$ on $\mathfrak{S}$ is restricted to be total. Given a pedigreed belief state $\Psi$, the dominating belief state of $\Psi$ is the function $\Psi_{\sqsupseteq} : \mathcal{W} \times \mathcal{W} \to S$ such that $\Psi_{\sqsupseteq}(w_a, w_b) = max(\Psi(w_a, w_b) \cup \{s_0\})$.

**Definition 4.11** Suppose that $\sqsupseteq$ on $\mathfrak{S}$ is restricted to be total. The ordering induced $\Psi_{\sqsupseteq}$ is the relation $\preceq \subseteq \mathcal{W} \times \mathcal{W}$ such that $w_a \preceq w_b$ iff $\Psi_{\sqsupseteq}(w_a, w_b) \sqsubseteq \Psi_{\sqsupseteq}(w_b, w_a)$.

Then, $\Psi_{\sqsupseteq}$ induces a standard (anonymous) belief state:

**Proposition 4.12** *The ordering induced by a dominating belief state is in $\mathcal{B}$.*

Proposition 4.12 is deduced by the following lemma:

**Lemma 4.13** *Suppose that $\sqsupseteq$ on $\mathfrak{S}$ is restricted to be total. Let $S = \{s_1, ..., s_N\} \subseteq \mathfrak{S}$ such that $s_i \sqsupseteq s_{i+1}$ for all $1 \leq i < N$. If $\preceq$ is the ordering induced by the dominating belief state of the pedigreed belief states induced by $S$, then*

$$\preceq = \begin{cases} \leq_{s_1} & \text{if } N = 1 \\ ((\leq_{s_1} \oslash \leq_{s_2}) \oslash \ ... \ \leq_{s_N}) & \text{otherwise(i.e., } N > 1) \end{cases}.$$

We show the following proposition:

**Proposition 4.14** *Suppose that $\sqsupseteq$ on $\mathfrak{S}$ is restricted to be total. If $\preceq$ is the ordering induced by the dominating belief state of the pedigreed belief states induced by $S$, $trans(\preceq)$ is the ordering induced by the pedigreed preferential relations induced by $S$.*

**Proof.** It is obvious by the Definition 3.2 and 3.4, Proposition 4.6, and Lemma 3.6 and 4.13. □

Finally, we show the relation between our fusion operator and the ordinary belief fusion.

**Definition 4.15** Let $\Psi_1$ and $\Psi_2$ be the pedigreed belief states induced by sets of sources $S_1$ and $S_2$, respectively. The fusion of $\Psi_1$ and $\Psi_2$ denoted $\Psi_1 \circledvee \Psi_2$, is the pedigreed belief state induced by $S_1 \cup S_2$.

We simply show the following proposition:

**Proposition 4.16** *Suppose that $\sqsupseteq$ on $\mathfrak{S}$ is restricted to be total and let $\Psi_1$ and $\Psi_2$ be the pedigreed belief states induced by sets of sources $S_1$ and $S_2$, respectively. Then $(\Psi_1 \circledvee \Psi_2)_{trans} = \Psi_{1trans} \bigtriangledown \Psi_{2trans}$.*

**Proof.** Straightforward. □

As the above discussion, we can substitute our fusion of preferential relations for the belief fusion.

## 5 Conclusion and Discussion

This paper's contribution is the following. Maynard-Reid II et al. proposed the idea of pedigreed sources to solve the inconsistency of knowledge amalgamation when the refinement operator is iteratively applied. However in the theory, all the sources must be totally ordered and this fact restricts the area of application. In this paper, we realized the partiality of preferential relations, and showed the procedure of fusion of them. In addition, we showed that our operation can properly include the ordinary belief fusion.

In our method of fusion of preferential relations, there were still several issues. First, we defined the primitive refinement by a fixed-point equation. Although the definition was mathematically sound, we need to argue the efficient procedure to compute $Tr(Pr(r_a \lessgtr r_b))$ independently. Secondly, we defined the cautious refinement by the intersection of all the possible refinements. Of course, this is not an only method to uniquely decide the refinement, and we can consider other ways to select one refinement among other refinements.

In future, we are to study all these branches and to evaluate the adequateness, considering the applicability of practical problems.

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