



An ultra-weak sector, the strong CP problem and the pseudo-Goldstone dilaton

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Abstract

In the context of a Coleman–Weinberg mechanism for the Higgs boson mass, we address the strong CP problem. We show that a DFSZ-like invisible axion model with a gauge-singlet complex scalar field S , whose couplings to the Standard Model are naturally ultra-weak, can solve the strong CP problem and simultaneously generate acceptable electroweak symmetry breaking. The ultra-weak couplings of the singlet S are associated with underlying approximate shift symmetries that act as custodial symmetries and maintain technical naturalness. The model also contains a very light pseudo-Goldstone dilaton that is consistent with cosmological Polonyi bounds, and the axion can be the dark matter of the universe. We further outline how a SUSY version of this model, which may be required in the context of Grand Unification, can avoid introducing a hierarchy problem.

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1. Introduction

In a recent paper [1] we discussed the possibility that new gauge singlet fields can have natural ultra-weak couplings amongst themselves and to Standard Model (SM) fields. The ultra-weak

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couplings, ζ_i , are protected by shift symmetries that are exact in the limit $\zeta_i \rightarrow 0$ and act as a custodial symmetry. This ensures that the couplings are technically natural (a similar proposal has also appeared in [2]). In the context of a very simple extension of the SM involving an ultra-weakly coupled real scalar field, we showed that a very large vacuum expectation value (VEV) of the scalar field can be generated. This, in turn, induces electroweak (EW) symmetry breaking through the Higgs portal coupling. The large VEV of the scalar field is generated through Coleman–Weinberg (CW) symmetry breaking [3] under the assumption that the renormalised mass terms of the Higgs and ultra-weak field are zero.¹

This “classical scale invariance” approach to the Higgs mass is essentially empirical, following from the experimental observation of the low mass Higgs boson. Scale invariance can be viewed as a symmetry of a pure $SU(3) \times SU(2) \times U(1)$ SM [4], but it would be expected to be broken in the real world when including GUT, gravitational, or any new threshold effects below the scale at which the SM couplings are defined (e.g., [5,6]). However, the existence of the fundamental spin-0 Higgs boson makes it interesting to examine the possibility that the lower-dimension operators, *i.e.*, the $d = 2$ renormalised boson mass terms (and $d = 0$ cosmological constant), are absent in the Lagrangian — perhaps as the result of a deeper classical scale invariance of the underlying theory.² The physical Higgs mass can then be generated by an infrared instability involving new physics.

Whether or not the CW mechanism applies to the Higgs boson is a phenomenological question that has been explored in a large number of recent papers [7,8]. However, none of the CW-Higgs models to date have addressed the strong CP problem. We consider this to be an important issue. The usual “invisible” axion solution involves a new SM singlet scalar field S that carries a global charge under the Peccei–Quinn (PQ) symmetry [9] and develops a very large VEV. Clearly it is important that the coupling of this field to the Higgs boson does not generate an unacceptably large contribution to the Higgs mass. In this paper, we show that the spontaneous breaking of the PQ symmetry in an ultra-weak sector via the CW mechanism can lead to an acceptable Higgs boson mass while solving the strong CP problem.

There have been two main suggestions for the nature of the PQ symmetry and the origin of the axion. The DFSZ axion [10] extends the Higgs sector to include a second Higgs doublet as well as the complex SM singlet scalar field S . The Higgs doublets and the singlet field are charged under the PQ symmetry. The KSVZ axion [11] postulates that the Standard Model fields are singlets under the PQ symmetry and requires the addition of a “heavy” quark that carries non-zero PQ charge and couples to S . In both cases the axion is identified with the phase of S while its modulus is identified with a light pseudo-dilaton.³

The origin of a light pseudo-dilaton state can be traced to the ultra-weak couplings of the S field, which are needed to avoid generating an unacceptably large mass for the Higgs and to enable CW breaking to generate the EW scale. Such small couplings are natural due to the underlying shift symmetry of S in the limit its couplings are zero. As a result, these couplings are

¹ In a field theoretic context, the radiative corrections to the Higgs mass that are quadratically dependent on the loop integral cut-off scale are not physically meaningful as only the renormalised m^2 , the sum of the radiative corrections and the mass counter term, is measurable.

² The $d = 2$ and $d = 0$ terms are special in the sense that if set to zero at a high scale they remain zero in the absence of spontaneous symmetry breaking, raising the possibility that classical scale invariance is an emergent symmetry at a high scale.

³ The large S VEV provides the dominant source of scale breaking, hence the identification of the modulus of S with the pseudo-dilaton.

multiplicatively renormalised in the absence of gravity and there is no underlying expectation for their magnitude. Gravitational effects will generate S couplings, but these may also be small due to the shift symmetry. Phenomenologically, the axion acquires its mass via the usual QCD effects $m_a \sim \Lambda_{\text{QCD}}^2/f_a$, where $f_a \equiv v_s/N_{\text{DW}}$ is the axion decay constant for a domain wall number N_{DW} , while the dilaton acquires a mass through mixing with the Higgs of order $m_s \sim m_h^2/v_s$. Indeed, the observation of the pseudo-dilaton together with the axion would provide a smoking gun for this kind of ultra-weak mechanism.

2. Electroweak breaking via the Coleman–Weinberg mechanism

2.1. The DFSZ model

We consider the DFSZ model, which has two Higgs doublets, $H_{1,2}$, whose neutral components couple to the up and down quarks respectively and generate their masses. We also include the complex singlet, S , which carries only the global PQ charge. The most general classically scale invariant potential for $H_{1,2}$ and S , consistent with the PQ symmetry, has the form:

$$\begin{aligned}
 V(H_1, H_2, S) = & \frac{\lambda_1}{2}|H_1|^4 + \frac{\lambda_2}{2}|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 \\
 & + \lambda_4|H_1^\dagger H_2|^2 + \zeta_1|S|^2|H_1|^2 + \zeta_2|S|^2|H_2|^2 \\
 & + \zeta_3 S^2 H_1^\dagger H_2 + h.c. + \zeta_4|S|^4,
 \end{aligned} \tag{1}$$

where the fields $H_{1,2}$ and S are parametrised as

$$\begin{aligned}
 H_1 = & \begin{pmatrix} \phi_1^+ \\ \frac{\phi_1}{\sqrt{2}} e^{i\theta_1/v_1} \end{pmatrix}, & H_2 = & \begin{pmatrix} \phi_2^+ \\ \frac{\phi_2}{\sqrt{2}} e^{i\theta_2/v_2} \end{pmatrix}, \\
 S = & \frac{\phi_s}{\sqrt{2}} e^{i\theta_s/v_s},
 \end{aligned} \tag{2}$$

with real moduli, ϕ_1 , ϕ_2 , and ϕ_s , where $\langle \phi_1 \rangle \equiv v_1$, $\langle \phi_2 \rangle \equiv v_2$ and $\langle \phi_s \rangle \equiv v_s$. We simplify the model by taking $\lambda_4 = 0$; λ_4 will be generated by gauge interactions, but it remains negligibly small [8]. We also consider the parameter range for which v_2 is small and can be treated as a perturbation, thereby allowing for an analytic solution to the minimisation conditions. A more complete study will require a numerical analysis.

There are two ways in which CW breaking can proceed. In the first, the dominant CW potential term is proportional to λ_3^2 and the interaction with the second Higgs field drives the quartic coefficient of the first Higgs field negative at some scale. This limit is equivalent to that studied in [8]. This requires such a large λ_3 that there is a Landau pole in the $\sim 10\text{--}100$ TeV range. To avoid the appearance of a low-lying Landau pole, we therefore turn to the second possibility in which λ_3 is negligible and EW breaking is triggered by the VEV of ϕ_s . The H_1 mass squared is then $\zeta_1 v_s^2$ and is driven negative by assuming $\zeta_1 < 0$.

To discuss this latter possibility, consider the terms with coefficients ζ_i in Eq. (1). The VEV v_s gives the axion decay constant $f_a = v_s/6$ ($N_{\text{DW}} = 6$ in this model) and hence $2 \times 10^9 \text{ GeV} \lesssim v_s \lesssim 10^{12} \text{ GeV}$. The singlet couplings $\zeta_{1,2,3}$ must therefore be very small: $\zeta_{1,2,3} \leq O(m_h^2/v_s^2)$, where m_h is the observed Higgs mass. For CW breaking to proceed, it is necessary for ζ_4 to be even smaller: $\zeta_4 \leq O(\zeta_{1,2,3}^2)$. As mentioned above, this region of parameter space is natural since the couplings ζ_i are forbidden in the shift symmetry limit, $S \rightarrow S + \delta$ [1], and thus are multiplicatively renormalised. The stronger constraint on ζ_4 is consistent with radiative corrections,

as can be seen by noting the couplings are also forbidden by a partial scale symmetry $S \rightarrow \lambda S$, where $\zeta_{1,2,3}$ scale as λ while ζ_4 scales as λ^2 . If the symmetry is broken (perhaps by gravity) by a term scaling as λ , the relative ordering of ζ_4 results.

Even though the S couplings are all extremely small, CW breaking in the S sector is still possible. It is convenient to consider the phenomenologically relevant limit in which the term proportional to ζ_2 provides the dominant CW term. It is in this limit that the additional Higgs states coming from the second Higgs doublet are heavy enough to have escaped detection to date [12].⁴ In this limit the potential, including the dominant one-loop correction, can be written as:

$$V(\phi_1, \phi_s) \approx \frac{\lambda_1}{8} (\phi_1^2 + \alpha \phi_s^2)^2 + \frac{1}{64\pi^2} (\zeta_2 \phi_s^2)^2 \left[\ln \left(\frac{\zeta_2 \phi_s^2}{\bar{M}^2} \right) - \frac{3}{2} \right], \tag{3}$$

where $\alpha = \zeta_1/\lambda_1$, $\bar{M}^2 = 2M^2 e^{-16\pi^2(\zeta_4 - \lambda_1 \alpha^2/2)/\zeta_2^2}$, and M is the scale at which the couplings are defined. This has a minimum at (minimisation of a similar single Higgs potential is discussed in [1]):

$$v_s^2 = \frac{e\bar{M}^2}{\zeta_2}, \quad v_1^2 = -\alpha v_s^2. \tag{4}$$

Finally, v_2 is driven by the term proportional to ζ_3 in Eq. (1), giving:

$$v_2 \approx -\frac{\zeta_3}{\zeta_2} v_1. \tag{5}$$

In the region of parameter space considered here, mixing between states is small and the observed Higgs h is approximately ϕ_1 . Similarly, the other neutral Higgs H and the pseudo-dilaton are approximately ϕ_2 and ϕ_s , respectively. Then a straightforward calculation gives:

$$\begin{aligned} m_h^2 &\approx -\zeta_1 v_s^2 \approx \lambda_1 v_1^2, \\ m_H^2 &\approx \zeta_2 v_s^2/2, \\ m_s^2 &\approx \zeta_2^2 v_s^2/8\pi^2. \end{aligned} \tag{6}$$

Determining the charged Higgs masses is more subtle as they only acquire mass via the term proportional to ζ_3 in Eq. (1). This happens because all the other terms are functions of $|H_1|^2$ and $|H_2|^2$, so:

$$\partial^2 V / \partial \phi_{1,2}^+ \partial \phi_{1,2}^- \propto \partial V / \partial \phi_{1,2} = 0. \tag{7}$$

As a result, we find the “uneaten” charged Higgs state has a mass:

$$m_{H^\pm}^2 \approx -(v_1/v_2)(\zeta_3/2)v_s^2 = \zeta_2 v_s^2/2. \tag{8}$$

Finally we turn to the phases of the fields. One combination,

$$\theta_Z \propto \theta_1 v_1 + \theta_2 v_2, \tag{9}$$

provides the longitudinal component of the Z boson. An orthogonal combination given by:

$$\theta_A = (-\theta_1/v_1 + \theta_2/v_2 + 2\theta_s/v_s)/N, \tag{10}$$

⁴ We treat the effect of the ζ_3 term perturbatively as it drives the H_2 VEV, which we have assumed to be in the perturbative regime.

where:

$$N^2 = 1/v_1^2 + 1/v_2^2 + 4/v_s^2, \tag{11}$$

gets mass from the ζ_3 term. Its mass is given by:

$$m_A^2 = -(\zeta_3/2)v_s^2 v_1 v_2 N^2 \approx -(\zeta_3/2)v_s^2 v_1/v_2 = \zeta_2 v_s^2/2. \tag{12}$$

The orthogonal state to θ_Z and θ_A is the axion. The axion only gets its mass from QCD effects, as usual.

To summarise, we have ($\zeta_1 < 0$):

$$\begin{aligned} m_H^2 = m_{H^\pm}^2 = m_A^2 &= -\frac{\zeta_2}{2\zeta_1} m_h^2, \\ m_s^2 &= \frac{\zeta_2}{4\pi^2} m_H^2 = -\frac{\zeta_2^2}{8\pi^2 \zeta_1} m_h^2. \end{aligned} \tag{13}$$

2.2. The KSVZ model

In the KSVZ model, the SM states are PQ singlets. However, the SM singlet field S interacts with some new heavy quark $X_{L,R}$, which is vector-like with respect to the SM gauge group but carry PQ charge, via the Yukawa interaction

$$L_{KSVZ} = -f \bar{X}_L S X_R - f^* \bar{X}_R S^\dagger X_L. \tag{14}$$

Imposing classical scale invariance, the scalar potential has the relatively simple form

$$V(H, S) = \frac{\lambda}{2} |H|^4 + \eta_1 |S|^2 |H|^2 + \eta_2 |S|^4, \tag{15}$$

where H is the SM Higgs doublet.

Following from the non-observation of additional coloured states up to the TeV range and the need to keep the Higgs light, one sees from Eqs. (14) and (15) that the largest coupling to the S field is f and the associated dominant loop correction to the S potential involves the new heavy quark. As a result, the loop correction contributes to the potential with a relative minus sign compared to that of Eq. (3) in the DFSZ case. This does *not* give rise to one-loop EW breaking because, if it triggers EW breaking, it drives the Higgs VEV to an unacceptably large scale. Avoiding this problem requires an additional CW radiative correction with the opposite sign to dominate. Such a term could arise if there are additional SM singlet fields. It could also possibly be engineered at the two-loop level by fermion loops, similar to a model discussed in [8]. We do not explore these possibilities further here.

3. Phenomenological implications

The DFSZ model requires the extension of the SM spectrum to include a second doublet of Higgs fields and a complex singlet S which contains the axion a and the pseudo-dilaton ϕ_s . The ultra-weak couplings ζ_i ensure that for collider experiments the phenomenology of the model is just that of the Type II two Higgs doublet model (2HDM) with the common mass scale of the additional Higgs states $\{H, A, H^\pm\}$ determined by the ratio $R \equiv m_H/m_h \simeq \sqrt{\zeta_2/|2\zeta_1|}$. In the

2HDM, additional Higgs states with masses of roughly 350 GeV or above, which corresponds to $R \gtrsim 3$, are allowed in significant regions of parameter space [12].⁵

At the same time, an approximate upper bound $R \lesssim 5$ comes from the requirement that one does not re-introduce the little hierarchy problem due to the coupling between the light and heavy Higgs sectors. This requires the masses of the heavy Higgs states to be smaller than $O(600 \text{ GeV})$.

In the usual implementation of the DFSZ model, the pseudo-dilaton ϕ_s is very heavy with a mass of $\mathcal{O}(v_s)$. The novel feature of the model discussed here is that ϕ_s is very light. From Eqs. (4) and (6), we have:

$$\zeta_1 = -\frac{m_h^2}{v_s^2} \approx -1.6 \times 10^{-20} \left(\frac{10^{12} \text{ GeV}}{v_s} \right)^2, \quad (16)$$

and hence from Eqs. (4) and (13):

$$\begin{aligned} m_s^2 &= -\frac{\zeta_1}{2\pi^2} \left(\frac{m_H}{m_h} \right)^4 m_h^2 \\ &\simeq 32 \left(\frac{10^{12} \text{ GeV}}{v_s} \right) \left(\frac{R}{3} \right)^2 \text{ eV}. \end{aligned} \quad (17)$$

Since the pseudo-dilaton is light and couples to quarks through its mixing with the SM Higgs, one way to detect it is through fifth force experiments. However, using the estimate for the coupling strength of the pseudo-dilaton to protons:

$$\alpha_5 \sim \frac{1}{2\pi} \left[\left(\frac{2m_u + m_d}{v_s} \right) \right]^2 \quad (18)$$

and $\sim 1/m_s$ for the effective range of the dilaton exchange force, it can be seen that the pseudo-dilaton lies outside the region excluded by Casimir-force and neutron scattering experiments [13].

The axion couples to electromagnetic fields through the axial anomaly in the usual way, $\sim c(a/v_s)(\alpha/4\pi)F_{\mu\nu}\tilde{F}^{\mu\nu}$. Likewise, the dilaton couples as $\sim c'(\phi_s/v_s)(\alpha/4\pi)F_{\mu\nu}F^{\mu\nu}$, with $c, c' \sim \mathcal{O}(1)$. A detailed analysis of the detectability and limits from the electromagnetic coupling for the dilaton goes beyond the scope of this paper. It is possible that future terrestrial “5th-force”, nuclear and RF-cavity experiments can be devised to look for the pseudo-dilaton directly, but this remains unexplored. It is possible that future terrestrial experiments can be devised to look for the pseudo-dilaton directly, but this remains unexplored. At present, the only way to constrain the pseudo-dilaton is through its cosmological influences, which we turn to a discussion of now.

4. Cosmology of the pseudo-dilaton

If the S field acquires its VEV before inflation, the energy density of the pseudo-dilaton will be diluted away. This is the case if the dilaton mass is larger than the Hubble parameter during inflation, which requires a low scale of inflation:

⁵ Note that the convention in studying the Type II 2HDM is to have H_2 couple to the up-type quarks rather than H_1 . Therefore when applying 2HDM limits to this model, one should use the definition $\tan \beta \equiv v_1/v_2$ rather than the usual $\tan \beta \equiv v_2/v_1$.

$$V_{\text{inf}}^{1/4} \lesssim 10^5 \left(\frac{10^{12} \text{ GeV}}{v_s} \right)^{1/2} R \text{ GeV}, \tag{19}$$

and if the reheat temperature is sufficiently low such that the PQ symmetry is not restored after inflation. On the other hand, if the PQ symmetry breaking occurs after inflation, there will be energy stored in the dilaton potential that will be released after inflation (the Polonyi problem [14]) in the form of dilaton oscillations. We shall consider both cases in turn, starting with the latter case.

4.1. High scale inflation

The energy stored in the dilaton potential depends on the initial value (VEV) of the dilaton. For the case that the Hubble parameter during inflation is much larger than the dilaton mass, the dilaton will perform a random walk of step length $H_{\text{inf}}/2\pi$ in each Hubble time. The maximum dilaton energy corresponds to the largest initial value of $\langle \phi_s \rangle$, which in turn corresponds to the case of the maximum Hubble parameter during inflation, $H_{\text{inf}} \sim 10^{14}$ GeV, consistent with the BICEP2 result [15]. To be conservative, let us consider this extreme case since all others will have a smaller amount of energy stored in the dilaton and will be more weakly constrained. For 70 e-folds of inflation, one may expect the initial value of the dilaton field to be given by $\langle \phi_s \rangle_i \sim 10^{14}$ GeV.

After inflation and reheat, $\langle \phi_s \rangle$ begins to oscillate when its effective mass becomes larger than the Hubble parameter. In the presence of a thermal bath, ϕ_s obtains a large thermal mass [16]

$$m_{s,\text{th}}^2 \simeq \frac{\xi_2}{6} T^2, \tag{20}$$

where we have neglected all but the largest coupling of ϕ_s to thermalised particles. For a sufficiently high reheat temperature, the thermal mass (20) dominates the dilaton potential and the dilaton oscillates about a zero VEV when it begins to roll. The roll begins when $m_{s,\text{th}} \sim H$, corresponding to the temperature

$$T_{\text{roll}} \approx 5 \times 10^7 R \left(\frac{10^{12} \text{ GeV}}{v_s} \right) \text{ GeV}. \tag{21}$$

As the universe expands, the energy density in the dilaton at the beginning of the roll,

$$\rho_{s,\text{roll}} \simeq \frac{\xi_2 T_{\text{roll}}^2}{12} \langle \phi_s \rangle_i^2, \tag{22}$$

redshifts as radiation, *i.e.* $\propto T^4$ [16]. This is faster than the matter redshift that one might expect because the temperature-dependent thermal mass also redshifts. As the temperature drops down to $T \sim 20R$ GeV, the thermal mass of the dilaton becomes comparable in size to the 1-loop term in the potential (3) and the minimum at $\langle \phi_s \rangle = v_s$ appears. However, the tunnelling rate to the true vacuum is low and the dilaton continues to oscillate about a zero VEV.⁶

As the temperature of the universe drops below $T \sim 10R$ GeV the energy density of the universe is dominated by the potential energy in the Higgs and dilaton fields, giving rise to a period of thermal inflation which continues until the EW symmetry is ultimately broken. This

⁶ The amplitude of the oscillations scales $\propto T$ from its initial value $\langle \phi_s \rangle_i$ at T_{roll} . At $T \sim 20R$ GeV, the amplitude of the oscillations is also too small to reach the minimum at v_s .

occurs at the temperature when QCD becomes non-perturbative and drives the quark condensate, which in turn gives masses of $\mathcal{O}(100 \text{ MeV})$ to the W and Z bosons as well as the Higgs. Once the temperature drops below the masses of these bosons, the stabilising thermal mass term for the dilaton rapidly vanishes due to the Boltzmann suppression [16] and the dilaton rolls to its true zero-temperature minimum at v_s , giving the EW and Higgs bosons, as well as the rest of the SM particles, their correct zero-temperature masses. Thermal inflation thus stops after approximately $\ln(10R \text{ GeV}/200 \text{ MeV}) \sim 5$ e-folds of inflation.⁷ This short period of thermal inflation does not affect the density perturbations coming from the initial stage of slow-roll inflation and still allows for successful baryogenesis.

Once EW symmetry is broken, the potential energy in the Higgs field reheats the thermal plasma. Since the Higgs' couplings to the plasma are $\mathcal{O}(1)$, the reheating is efficient and gives a reheat temperature of $T_{\text{reh}} \sim 10R \text{ GeV}$. Meanwhile, the potential energy in the dilaton,

$$\rho_s \simeq \frac{\zeta_2^2 v_s^4}{256\pi^2} \simeq \frac{R^4 m_h^4}{64\pi^2}, \quad (23)$$

is released as a coherent oscillation of the field that redshifts as matter, *i.e.* $\propto T^3$.

It is worth emphasising that the relatively low scale of electroweak breaking $T \sim \Lambda_{\text{QCD}}$ is a consequence of the fact that thermal effects keep the initial vevs of the Higgs and dilaton vevs displaced from their zero-temperature values. However, once they roll to the zero temperature minima their vevs remain large during reheating to temperatures of $\mathcal{O}(M_W)$ due to the large potential barrier that develops. Thus neither the Higgs reheating process nor the coherent oscillation of the dilaton restore the thermally-induced minimum at $\langle \phi_s \rangle = 0$.

The energy density (23) is large enough that it will quickly dominate the energy density of the universe, thereby ruining late-time cosmology, unless it is somehow dissipated. This indeed happens because of a resonant enhancement of the scattering rate of the coherent state of zero momentum oscillating dilatons on the thermal background.

To illustrate this consider the process $s + c \rightarrow c \rightarrow \text{SM states}$ involving the scattering of the dilaton off the distribution of charm quarks. Since the dilaton mass is so small, the intermediate c is nearly on-shell and its propagator is dominated by its thermal width $\Gamma_c \simeq G_F^2 m_c^5 / (192\pi^3)$.⁸ Since this width is small, there is an enhancement of the scattering rate that leads to a thermal dissipation rate of the dilaton given by [17,18]

$$\Gamma_s \simeq \frac{\sqrt{2} m_c^4}{\pi^{3/2} v_s^2 \Gamma_c} \left(\frac{T}{m_c} \right)^{1/2} e^{-m_c/T}. \quad (24)$$

This rate exceeds the Hubble expansion rate $H \sim T^2/M_{\text{Pl}}$ over some range of temperatures $T_* < T \lesssim m_c$ for $v_s \lesssim 5 \times 10^{14} \text{ GeV}$. Thus the dilaton oscillations are dissipated for all v_s of interest.

Note that the inverse dilaton production processes, such as $g + q \rightarrow q \rightarrow q + s$ where g is a gluon and q is a thermalised quark, do not have a resonant enhancement because none of the reactants are zero momentum coherent states. Due to the low reheat temperature $T_{\text{reh}} \sim 10R \text{ GeV}$, the number density of the top quark is exponentially suppressed and it is the bottom

⁷ Note that the fluctuations of the dilaton during thermal inflation, $\delta\phi_s \sim H_{\text{inf}}/2\pi \sim 10^{-16} R^2 \text{ GeV}$, are too small to displace the dilaton away from zero.

⁸ Here we neglect the finite temperature corrections and use the zero temperature width. This is valid for temperatures $T \lesssim m_c$, at which the dissipation rate is sufficiently large anyway.

quark scattering that gives the largest rate of dilaton production.⁹ An estimate of this rate for $T \gtrsim m_b$ is:

$$\Gamma_s^{\text{prod}} \simeq \frac{9\zeta(3)}{\pi^2} \left(\frac{m_b}{v_s}\right)^2 \alpha_s T, \tag{25}$$

which produces a dilaton population:

$$\frac{n_s}{n_s^{\text{eq}}} \sim \frac{\Gamma_s^{\text{prod}}}{H} \Big|_{T=m_b} \sim 0.4 \left(\frac{2 \times 10^9 \text{ GeV}}{v_s}\right)^2. \tag{26}$$

If the dilaton is sufficiently long lived, it is non-relativistic today with an abundance:

$$\Omega_s \sim 0.3 \left(\frac{R}{3}\right)^2 \left(\frac{7 \times 10^9 \text{ GeV}}{v_s}\right)^3. \tag{27}$$

To constitute dark matter, however, the dilaton must be stable on cosmological timescales. The dominant direct decay mode of the dilaton is to two axions with the decay rate:

$$\Gamma_{s \rightarrow aa} = \frac{1}{32\pi} \frac{m_s^3}{v_s^2} \simeq \frac{R^6 m_h^6}{64\sqrt{2}\pi^4 v_s^5}, \tag{28}$$

giving a lifetime:

$$\tau_s \simeq 3.4 \times 10^{18} \left(\frac{3}{R}\right)^6 \left(\frac{v_s}{7 \times 10^{10} \text{ GeV}}\right)^5 \text{ s}. \tag{29}$$

Constraints on decaying dark matter require the lifetime to be on the order of 100 Gyr (3×10^{18} s) or longer [19]. Thus for $v_s \sim 7 \times 10^9$ GeV for which a significant dilaton population is produced, the dilaton is unstable on cosmological time scales and cannot be dark matter. Conversely, for $v_s \gtrsim 7 \times 10^{10}$ GeV for which the dilaton is sufficiently stable, dilaton production is negligible.

The axion, however, provides a very plausible cold dark matter candidate. The energy density in the coherent oscillations (zero mode) of the axion through vacuum realignment is [20]:

$$\Omega_a h^2 \simeq 0.236 \theta_i^2 f(\theta_i) \left(\frac{v_s/N_{\text{DW}}}{10^{12} \text{ GeV}}\right)^{7/6}, \tag{30}$$

where $N_{\text{DW}} = 6$ for this model, θ_i is the initial misalignment angle, and the function $f(\theta_i) = [\ln(e/(1 - \theta_i^2/\pi^2))]^{7/6}$ encodes the anharmonic effect. Meanwhile, the higher momentum axion modes and the axions produced in the decay of strings and domain walls contribute a comparable amount to the energy density as vacuum realignment [21]. For $v_s \sim 10^{12}$ GeV, the axion can therefore provide all of the dark matter.

There remains the question of how an unacceptable energy density from the domain walls produced after the PQ breaking transition can be avoided. Since $N_{\text{DW}} = 6$, the energy density in stable domain walls is many orders greater than the critical energy density for closing the universe and completely unacceptable. However, small PQ breaking can cause the walls to decay and hence avoid the problem while preserving the axion solution to the strong CP problem.

⁹ Resonantly enhanced scattering processes such as $s + c \rightarrow c \rightarrow c + s$ from the zero momentum state itself must also be considered as sources of dilaton production [18]. However, the rate of such processes are suppressed relative to (24) by a factor of $(m_c/v_s)^2$ and hence they are negligibly small.

To see how this can happen, we note that the most general potential $V(H_1, H_2, S)$ includes the terms $\lambda_5(H_1^\dagger H_2)^2 + \zeta_5 S^2 H_2^\dagger H_1 + \zeta_6 S^4 + h.c.$ that break the PQ symmetry and splits the degeneracy of the $Z(N_{\text{DW}})$ discrete symmetry that leads to the domain wall problem. Note that these couplings multiplicatively renormalise and so, following the discussion above, we conclude they can naturally be arbitrarily small. Indeed, with PQ breaking of a similar magnitude to scale breaking ($\lambda_5 \sim \zeta_{1,2,3}$) these terms are in the range needed to solve the domain wall problem without disturbing the axion solution of the strong CP problem [22].

In summary, the large thermal mass of the dilaton produces a period of thermal inflation with approximately 5 e-folds after the usual slow roll inflation. For all v_s of interest, the interactions of the light dilaton with the thermal bath dissipate the energy in its coherent oscillations. A significant relativistic population of dilatons is produced in the region of parameter space with $v_s \lesssim 7 \times 10^9$ GeV, but the dilaton is too short-lived in this region to be dark matter; the dilaton can therefore only have a negligible contribution to dark matter. The axion, however, can comprise all of the dark matter for $v_s \sim 10^{12}$ GeV.

4.2. Low scale inflation

In the case that Eq. (19) is satisfied and the reheat temperature is sufficiently low ($T_{\text{reh}} \lesssim 100$ GeV) that the dilaton does not obtain a thermal mass that forces it to roll to $v_s = 0$, the PQ symmetry remains broken during and after slow roll inflation. As a result, the energy density in the dilaton oscillations are driven exponentially small and the axion field gets homogenised by the expansion of the universe during the inflationary phase, thereby preventing the formation of domain walls [21]. The usual result (30) for the axion contribution to the energy density from vacuum realignment still holds¹⁰ and the axion can provide all of the dark matter for $v_s \sim 10^{12}$ GeV and $\theta_i \sim 1$.¹¹ The dilaton, however, never plays a significant role in cosmology.

5. An ultra-weak DFSZ SUSY model

Classical scale invariance of the low-energy theory does not apply if there are heavy states coupled to the Higgs, such as Grand Unified states, with mass below the scale at which the SM couplings are defined. These introduce radiative contributions to the Higgs mass that are proportional to the mass of the heavy GUT states. Unlike the radiative corrections to mass simply proportional to the cut-off scale, these GUT corrections involve a logarithmic dependence on the scale at which they are measured and thus are physical. To avoid the hierarchy problem, the model discussed above must therefore not have a stage of Grand Unification.

It is possible to include a stage of Grand Unification in a scale invariant theory by super-symmetrising the model so that the contribution to the Higgs mass coming from interactions with the heavy GUT states, although present, are acceptably small. As we sketch below, CW breaking in the ultra-weak sector associated with the axion can readily be extended to a supersymmetric theory.

The states of the DFSZ model neatly correspond to the non-SUSY states of the ($N = 1$) NMSSM, so a supersymmetric version of the model can be constructed easily. After imposing

¹⁰ The contribution from the quantum fluctuations of the axion field during inflation, which are included by making the replacement $\theta_i^2 \rightarrow \theta_i^2 + \sigma_\theta^2$ in Eq. (30) where $\sigma_\theta^2 \simeq (H_I/2\pi f_a)^2$ [20], are negligible for H_I satisfying Eq. (19).

¹¹ As discussed in [20], with moderate fine-tuning to give $\theta_i \simeq \pi$, the axion can provide all of the dark matter for smaller values v_s due to the anharmonic effect.

the PQ symmetry, the allowed couplings are more restricted than those of the NMSSM and correspond to those recently discussed in [23]. The only term in the superpotential W involving the S field is:

$$W = \zeta_1 \hat{S} \hat{H}_1 \hat{H}_2, \quad (31)$$

where the scalar components of the super fields \hat{S} and $\hat{H}_{1,2}$ are the S field and the Higgs doublets.

Due to the constraints of supersymmetry, the model is classically scale invariant in the absence of SUSY breaking. We are interested in the case that ζ_1 is ultra-weak, which is natural due to the underlying shift symmetry when ζ_1 is zero. Allowing for SUSY breaking, the only other terms involving just these fields are the soft terms¹²:

$$V(H_{1,2}, S) = m_s^2 |S|^2 + m_1^2 |H_1^2| + m_2^2 |H_2^2| + T_{cl} S H_1 H_2. \quad (32)$$

The quartic scalar terms coming from Eq. (31) are positive semi definite, so the only possibility for dynamical SUSY breaking is through the soft terms. Including radiative corrections, m_s^2 can be driven negative by radiative corrections proportional to $\zeta_1^2 m_{1,2}^2$. This triggers v_s at a scale close to the point at which the mass is zero, which can be very large, as required. However, this requires that the starting value of m_s^2 should be ultra-small relative to $m_{1,2}^2$. An ultra-small mass is natural if there is an underlying shift symmetry, which can readily happen if, for example, SUSY is broken in a hidden sector and SUSY breaking is communicated to the S field by gravitational effects while the SM states receive their SUSY breaking masses via gauge mediation. In this case, the soft S mass and the graviton will be much lighter than the SUSY breaking masses $m_{1,2}^2$ in the visible sector. The dimensional transmutation mechanism in the UW sector provides an economical and elegant origin for the axion decay constant that does not require the inclusion of an O’Raifeartaigh term involving an explicit mass scale [23].

The SUSY phenomenology of the model is essentially that of the minimal supersymmetric SM, the MSSM (with gauge mediation), because the additional couplings of the Higgs to the singlet sector are ultra-weak and hence insignificant, apart from providing the origin of the μ term of the MSSM, $\mu = \zeta_1 v_s$, cf. Eq. (31). In this case, EW breaking proceeds in the usual way through radiative corrections that, due to the top Yukawa coupling, drive the soft Higgs mass squared negative [24–26].

The LSP is the axino, the fermion component of S , with a mass:

$$m_{\hat{S}} = \frac{\mu v_1 v_2}{v_s^2} \sim 10^{-9} \left(\frac{10^{12} \text{ GeV}}{v_s} \right)^2 \text{ eV} \quad (33)$$

generated by the see-saw mechanism through its coupling to the Higgsinos. The decay of the lightest MSSM SUSY state to the gravitino or axino is so slow that it does not occur within the detector and does not change the MSSM phenomenology. The dark matter component that ends up in the axino depends on the MSSM parameter choice and has been discussed extensively elsewhere.

Due to the quartic couplings associated with the superpotential term in Eq. (31), the Higgs obtain S dependent masses as in the non-supersymmetric DFSZ model. As a result, Higgs oscillations are driven by the dilaton oscillations in the manner discussed above. The energy in the dilaton fields is converted to energy in the SM sector at a time before nucleosynthesis and does not significantly change the usual MSSM cosmology.

¹² Soft terms can be generated in a classically scale invariant theory through spontaneous breaking, for example via gaugino condensation.

6. Summary and conclusions

The discovery of a Higgs scalar with properties very close to that predicted by the SM, together with the absence of any indication for physics beyond the SM, has led to a re-evaluation of the need for such physics to solve the hierarchy problem. Formally, as a pure field theory, the SM has no hierarchy problem because the radiative corrections to the Higgs mass squared that are quadratically dependent on the cut-off are not physical; only the renormalised mass is measurable, so any value of m is possible and only the empirical choice $m = 0$, which corresponds to classical scale invariance of the theory, is special.

With this motivation, we discussed how the SM could result from a classically scale invariant theory that also addresses the major questions left unanswered by the SM. While there has been extensive discussion of the possible origin of baryogenesis, dark matter and inflation, very little attention has been paid to the strong CP problem in such theories. In this paper, we showed how a scale invariant version of the DFSZ model can spontaneously generate the large PQ scale through an ultra-weakly coupled sector involving a complex SM singlet scalar field S . As discussed in [1], such an ultra-weak sector involving gauge single fields is technically natural due to an underlying approximate shift or scale symmetry.

Due to the ultra-weak couplings, the DFSZ extension of the SM contains an anomalously light pseudo-dilaton as well as the usual axion, which come from the complex scalar S . Despite the ultra-weak couplings, there is no Polonyi problem associated with the S field due to a resonant enhancement of the scattering of the coherent S state off the thermal background after the PQ and EW phase transition are triggered. Unusually, the PQ phase transition occurs at the EW scale in this model. Meanwhile, the dilaton production cross section does not have the resonant enhancement and its abundance is typically negligible. Dark matter can be in the form of axions produced via a combination of vacuum alignment and decay of axion domain walls. Such decay is possible due to additional PQ breaking terms, which can be consistent with the axion solution to the strong CP problem as long as they are also ultra weak and have a strength comparable to the scale breaking terms.

Due again to the ultra-weak couplings of the singlet fields, the phenomenology of the model is that of the usual two-Higgs doublet extension of the SM. The most significant constraint on the additional heavy Higgs states comes from the requirement that the little hierarchy problem is not re-introduced. It may be possible to search for the ultra-light dilaton along the lines suggested in [27], but this remains to be studied.

Finally, we outlined the construction of a scale-invariant SUSY version of the model that can accommodate a stage of Grand Unification without re-introducing the hierarchy problem. It provides a simple origin for the μ term and the LSP is the axino, the fermion component of the super field that contains the DFSZ complex scalar field S . However, since the decay of the lightest MSSM state to the LSP is extremely slow, the collider phenomenology of the model is just that of the MSSM with gauge mediated SUSY breaking.

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